Arithmetic Progressions and Prime Numbers

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Let's consider the simple arithmetic progression (n + 1), where n takes non-negative integer values (n = 0, 1, 2, ...). This progression generates all natural numbers. If we define c = n + 1, then $c^2 = (n + 1)^2$ is a second-degree polynomial in n.

To analyze the distribution of prime numbers, we define three disjoint arithmetic progressions:

- a(n) = 3n + 1
- b(n) = 3n + 2
- c(n) = 3n + 3

More generally, we can use independent variables:

- f(x) = 3x + 1
- g(y) = 3y + 2
- h(z) = 3z + 3

Consider the product of two terms from these progressions, for example, K = f(x)g(y). This product generates a quadratic curve. Specifically, if we choose terms from two different progressions (e.g., f(x) and g(y)), K represents a hyperbola. If we choose two terms from the same progression, we obtain a parabola.

Example: K = (3x + 1)(3y + 2). We choose this progression because the only prime number in (3n + 3) occurs when n = 0.

Conditions for Square Numbers

For $K = c^2$, where c is a natural number, the following conditions must be met:

- K must be a perfect square ($K = p^2$).
- K must be a perfect square ($K = q^2$).
- If K = pq, where p and q are natural numbers, then the prime factors of p and q must have even exponents in their prime decomposition. That is: p = 2^(n1) * 3^(n2) * 5^(n3) * ... q = 2^(m1) * 3^(m2) * 5^(m3) * ... Where $n_i + m_i$ is an even number for all i.

If these conditions are met, then $(3x + 1)(3y + 2) = c^2$. More generally, the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = c^2$ has positive integer natural solutions. In the quadratic case, all conics are classified under projective transformations.

Generalization to Higher Exponents

To obtain natural numbers of the form $x^n + y^n = c^n$, we use the trivial arithmetic progression (n + 1). Then, $c^n = (n + 1)^n$, which is a polynomial of degree n:

f(x)=anxn+an−1xn−1+… +a2x2+a1x+a0

For degrees greater than 2, the intersection curves do not belong to a single family like conics. They can have different genera, singularities, and irreducible components. Therefore, there is no general way to reduce f(x) to $x^n + y^n = c^n$, which suggests that there are no positive integer solutions for n > 2 since f(x) has positive integer solutions and from f(x) I can not reduce to $x^n + y^n = c^n$,.