

Velocity triangles

- Velocity triangles are used to illustrate the flow in the turbine blading
- Changes in the flow direction and velocity are easy to understand with the help of the velocity triangles
- Note that the velocity triangles are drawn for the rotor inlet and outlet at a certain radius (usually mean diameter)

The Euler turbomachinery equation

Tangential force acting on the rotor:

 $F_u = m(c_{1u} - c_{2u})$

Torgue (change in the moment of momentum):

$$
M = F_u \cdot r = m(r_1 \cdot c_{1u} - r_2 \cdot c_{2u})
$$

Power on the shaft:

$$
P = M \cdot \omega = m(\omega \cdot r_1 \cdot c_{1u} - \omega \cdot r_2 \cdot c_{2u})
$$

$$
P = m(u_1 \cdot c_{1u} - u_2 \cdot c_{2u})
$$

The Euler turbomachinery equation

Power output per unit mass:

$$
a_u = \frac{P}{m} = u_1 \cdot c_{1u} - u_2 \cdot c_{2u}
$$

which is called the Euler turbomachinery equation (Leonard Euler, 1707-1783)

In an axial flow turbine $u_1 = u_2$ and equation can be written:

$$
a_u = u \cdot \Delta c_u
$$

The Euler turbomachinery equation is applicable also to blowers, pumps and compressors.

Turbine stage in h,s diagram

Stator: $\Delta h' = h_0 - h_1 = \frac{1}{2} \cdot (c_1^2 - c_0^2)$

Rotor:

fixed coordinates: $\Delta h'' = h_1 - h_2 = a_u - \frac{1}{2}(c_1^2 - c_2^2)$

rotating coordinates: $\Delta h'' = h_1 - h_2 = \frac{1}{2} \cdot [(w_2^2 - w_1^2) - (u_2^2 - u_1^2)]$

 \Rightarrow a_u = ½ [(c₁² - c₂²) + (w₂² - w₁²) - (u₂² - u₁²)]

 a_u is the available specific work

Degree of reaction

The degree of reaction for a turbine stage is defined as:

h h $\Rightarrow R = \frac{\Delta h^{\prime\prime}}{\Delta h^{\prime} + \Delta h^{\prime\prime}} = \frac{\Delta h^{\prime\prime}}{\Delta h}$ $R = \frac{\text{static enthalpy change in the rotor}}{R}$ '' static enthalpy change in the stage

By substituting enthalpies we get:

$$
R = \frac{\left(w_2^2 - w_1^2\right) - \left(u_2^2 - u_1^2\right)}{\left(c_1^2 - c_0^2\right) + \left(w_2^2 - w_1^2\right) - \left(u_2^2 - u_1^2\right)}
$$

Degree of reaction

Assumptions:

 $c_0 = c_2$, absolute velocity is the same before and after the stage $u_1 = u_2$, axial flow turbine - constant peripheral velocity $c_{1a} = c_{2a}$, axial velocity component is constant

Degree of reaction becomes:

$$
R = \frac{\left(w_2^2 - w_1^2\right)}{\left(c_1^2 - c_2^2\right) + \left(w_2^2 - w_1^2\right)}
$$

Degree of reaction

Impulse turbine

If $w_1 = w_2$ or $(w_2^2 - w_1^2) = (u_2^2 - u_1^2)$, R becomes zero. This special case is called an impulse stage.

Reaction turbine

Any case when $R\neq 0$, is a reaction stage. A special case of a reaction turbine is R=0.5 ,which leads to symmetrical velocity triangles and is a very common design.

Impulse turbine

- Best efficiency when $u/c_1 = \cos \alpha_1/2 \approx 1/2$
- Optimum value of α_1 about 20°
- High peripheral velocity (rotation speed) needed when c_1 is high.
- Rotation speed can be lowered by dividing the velocity drop to two or more similar impulse stages (Curtis-turbine)

Reaction turbine

- Best efficiency for R=0.5 reaction stage is achieved when $u/c_1 = \cos \alpha_1 \approx 1$
- Optimum value of α_1 about 20°

Impulse - Reaction

- Δc_u is higher in impulse turbines, which means that an impulse stage does more work than a reaction stage with the same peripheral velocity
- Smaller number of impulse stages than reaction stages needed for a certain enthalpy drop

- More friction losses in an impulse turbine due to larger change in flow direction (angle between w_1 and w_2)
- Pressure difference over the rotor in a reaction turbine
	- an axial force to the direction of the flow
	- more leakage special balancing valve is needed

Impulse - Reaction

- Manufacturers nowadays try to combine the best features of both turbine types
- Usually the first stage is impulse-type (control stage) and the rest are reaction stages with R=0.5

Twisted blades

- Energy transfer (u Δc _u in Euler´s equation) to the rotor should be uniformly distributed along the blade lenght
- Due to centrifugal force the static pressure of the working fluid increases with increasing radius
- The pressure difference over the stator is therefore highest at the blade root and lowest at the blade tip

Twisted blades

- Fluid with density *ρ* and peripheral velocity component $c_{\rm u}$ passes through an circular area of $A=2\pi r dr$.
- The mass per unit widht: $dm = \rho \cdot 2 \cdot \pi \cdot r \cdot dr$
- Centripetal force acting on the mass:

$$
F_c = a \cdot dm = \rho \cdot 2\pi \cdot r \cdot dm \cdot \frac{c_u^2}{2}
$$

Twisted blades • Between radii r and r+dr, the centripetal force makes a pressure difference: *r c dr dp r c* $r \cdot dp = \rho \cdot 2\pi \cdot r \cdot dr$ *u u* 2 2 1 $2\pi \cdot r \cdot dp = \rho \cdot 2$ $\Rightarrow -\cdot \frac{dp}{l} =$ $\cdot r \cdot dp = \rho \cdot 2\pi \cdot r \cdot dr \cdot$ ρ $\pi \cdot r \cdot dp = \rho \cdot 2\pi$

• This is the static pressure - centripetal force balance requirement

Turbine´s throughput

Usually $p_1 >> p_2$ and we can write the turbine equation simply:

$$
m = K \cdot \sqrt{\frac{p_1}{v_1}}
$$

The "turbine constant" K in the equations represents the turbine's throughput. With the help of the K-value calculated at design point, it is possible to get information about the turbine´s operation at partial load.

Steam turbine power control

• Throttle control

- steam flow is choked before the turbine
- pressure drops, but enthalpy stays constant
- pressure difference over the turbine decreases, which leads to smaller mass flow
- "shorter" expansion
- low efficiency at partial load

Steam turbine power control

- Throttling by a control stage
	- the control stage (of impulse type) is divided into segments, which all have their own choke valves
	- all valves are open at full power and they close one at a time when the load decreases
	- better efficiency at partial load compared to throttle control

Steam turbine power control

- Throttling by a control stage
	- Not suitable for small volume flow rates, because the first stage blades would be too short (low efficiency)
	- The control stage must have low degree of reaction (near zero), otherwise the pressure difference over the rotor would cause uncontrolled flow and more losses

Characteristic for gas turbines

- The pressure ratio in compressor stages much lower than in turbine stages (more stages in the compressor than in the turbine)
- No bleeds in the turbine

Gas turbine power control

- the blading of a gas turbine operates all the time with full mass flow (täyssyöstö)
- power output is controlled by adjusting the fuel mass flow into the combustion chamber
- flue gas temperature (enthalpy) changes
- adjustable guide vanes in the first compressor stages in order to enhance efficiency at part load