Seventy-Seven Theses: Next Steps and the Way Forward in the Modified Cosmological Model

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Abstract

The purpose is to lay out a plan for future inquiry pertaining to the modified cosmological model (MCM) and its overarching research program. The material is modularized as a catalog of open questions that seem likely to support productive research work. The main focus is quantum theory but the material spans a breadth of physics and mathematics. Cosmology is heavily weighted and some Millennium Prize problems are included. A comprehensive introduction contains a survey of falsifiable predictions and associated experimental results. Listed problems include original ideas deserving further study as well as investigations of others’ work when it may be germane. A new elliptic curve application is presented which has not appeared in previous work. With a few exceptions, the presentation is high-level and qualitative. Formal analyses are mostly relegated to the future work which is the topic of this paper. Sufficient technical context is given that third parties might independently undertake the suggested work units.
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Introduction

This paper is a long list of thesis problems in physics and mathematics. In the way that a previous review [1] was written to broaden the horizons of the modified cosmological model (MCM), the present purpose is to pinpoint within those horizons ideas that should be brought forward to completion.

Review and Main Results

For much of the previous development of the MCM, this writer had already exited the academic environment which is most conducive to initial surveys of topics concluding in original contributions at the level of a PhD thesis. The fixation of the research program on the absolute bare bones of the rudiments of the fundamentals has come at the expense of such “PhD level” work. This provides fodder for detractors. While the fractional distance program in real analysis [2] must exceed the requirements for a PhD in math, this writer has rarely taken research in physics to a conclusive calculation, and never one at the level of a PhD thesis. In the way that mathematicians are sometimes said to be concerned with the existence of solutions more so than with finding them, it follows that this writer’s thesis equivalent [2] is in real mathematical analysis. The presumed existence of solutions has sufficed throughout the MCM’s development, contrary to what is most common in physics. First and foremost, however, this writer is a physicist. Physics ultimately requires real solutions for experimental applications. While it was hoped for many years that others would jump at the chance to write the papers in which such solutions are written, history has taken a different tack. The present paper describes many of the open and untreated questions that have arisen in the development of the MCM. While this material may not be low-hanging fruit at the level of Newton’s apple falling on his head, the material described here is low-hanging enough that it was noticed along the way. A fair portion of what is described in this paper should lead to experimentally falsifiable predictions even while any eventual treatment by this writer may not take these problems all the way to that conclusion.

The first numerical prediction of the MCM was a characteristic length scale for new physics in Reference [3]. As it is the aim to tie up all the loose ends in physics with a new model of cosmology (and ontology), we applied the structure of the MCM to the
foremost unsolved problem in classical mechanics: the precession of spinning discs. If any theory will be a theory of everything, certainly it will lay to rest the open questions in classical mechanics. The calculation published in Reference [3] gives a length scale for new physics on the order of $10^{-4}$m. While the calculation yielding $10^{-4}$m was very simple, precession is not a manifestly complicated problem. The result was remarkable in that $10^{-4}$m is neither the nano-scale of quantum mechanics nor the macro-scale of classical mechanics. Instead, an intermediate meso-scale value was obtained in the regime where the catch-all losses due to friction are usually called on to scoop up everything not classically deterministic or quantum mechanical. Furthermore, Arkani-Hamed and others have already written extensively about the open question of new physics at the sub-millimeter scale [4, 5]. The entire mechanism surrounding the characteristic scale calculation was well-defined but possibly not as well motivated as would usually be expected in professional publications. One reason for this is that this writer is not a professional. As an amateur contributor, he is not constrained by the professional community standards which sometimes make it difficult to put highly speculative ideas to paper. Still, the $10^{-4}$m result was remarkable. So, while the MCM is sometimes criticized as lacking falsifiable predictions, the $10^{-4}$m characteristic scale for new effects is exactly in the last small holdout of length scale where new effects are not yet categorically ruled out. If $10^{-30}$, $10^{-10}$, $10^{0}$, or $10^{10}$m was determined as the scale for the proposed mechanism [3], then we could know without any further thinking that the mechanism is unphysical. To the contrary, the computation shows that experiment allows the idea, in part, at least. If the work of physics is to rule out theories, which are only ideas or formalized ideas, the calculation in Reference [3] shows that the MCM passes at least one hurdle of not being ruled out. The hurdle was not high but first hurdles rarely are.

The best physical prediction to come of the MCM is that there should not exist any spin-0 fundamental particles such as the Higgs boson. This prediction is directly falsifiable in a way that exceeds the possibility for new effects on a certain scale. The prediction is perfectly well motivated. It is as clean and concise as any prediction in the history of physics. Following a brief review of Kaluza–Klein (KK) theory, the MCM unit cell was constructed in Reference [6]. The purpose of the construction was to build on previous work in the MCM so as to address some of the failures of KK theory detailed by Overduin and Wesson [7]. (See Section 18.) Namely, the so-called cylinder condition requires that 4D physics in spacetime must not depend on the fifth coordinate. This condition is generated naturally when the realm of physics is taken as a 4D Poincaré section (slice) of a 5D space for some constant value of the fifth coordinate.
coordinate [6]. Another problem is that KK theory only allows solutions in which the electromagnetic (EM) strength tensor $F_{\mu\nu}$ vanishes. While one 5D KK metric tensor contains an EM potential 4-vector and a dual EM potential 4-vector, the MCM uses two such metrics containing twice as many EM degrees of freedom. This doubling of the degrees of freedom should be sufficient for $F_{\mu\nu} \neq 0$ solutions. While much work remains to formalize the MCM at the level of Kaluza’s and Klein’s original works, the MCM unit cell was assembled [6] to address in principle the main problems with KK theory.

Soon after the initial construction of the MCM unit cell [6], it was demonstrated that the unit cell is a good candidate to answer the fundamental question of quantum field theory (QFT) [8]. That question asks why we have the particles we have and not some other particles. The standard model of particle physics is pretty good for determining what our particles do but it does nothing to address to the fundamental question about why we have the standard model particles to begin with. The spectrum of lattice vibrations in the unit cell of the MCM lattice is identical to the known spectrum of elementary particles. Even such nuance as the eight varieties of gluons arises in the MCM lattice from simple classical mechanics. Each particle is given as a different kind of spring in a 5D lattice of masses connected by springs. The ultimate goal of QFT is to generate the true spectrum of fundamental particles from the theory itself without having to force agreement with experiment by the imposition of an empirical model: the standard model. The MCM unit cell very nearly replicates all of the particles in the standard model. The only disagreement with the standard model is in the scheme of the fundamental bosons which are like the masses when matter particles are like springs, or vice versa. The standard model supposes that there exists a spin-0 fundamental particle: the famous scalar boson following from the work of famous people such as Higgs, Englert, Brout, Guralnik, Hagen, and Kibble [9–11]. The MCM scheme does not, in its current incarnation, permit the existence of any such spin-0 particles. So, the MCM answer to the fundamental question of QFT is plainly falsifiable. Posed in early 2013, this prediction followed closely on the heels of CERN’s discovery of a new particle (the Higgslike particle) in the summer of 2012 [12, 13]. If that particle is found have spin-0, then the MCM is wrong and needs to be revised or scrapped. If that particle is the Higgs boson, or if it is any possible variety of Higgs boson, it will have spin-0. The objective existence of a spin-0 fundamental particle would send an important MCM result back to the drawing board. More than simply causing a rescission of a prediction, the entire structure of the model would be cast into doubt. As it stands, the MCM is supposed
to generate the fundamental particles as lattice vibrations in an almost (but not quite) trivial model of lattice cosmology.

Though many detractors of the MCM cite an alleged mountainous body evidence proving that the Higgslike particle does not, and can not, have spin-1, Ralston has shown that spin-1 was not ruled out [14] by the initial observations at CERN. Famous particle theorist Arkani-Hamed has also stated plainly in a talk [15] that spin-1 is not ruled out for the Higgslike particle. Ralston, in his analysis of the decay channels reported from CERN, cites “model-independent Lorentz invariance” as allowing spin-1. However, in the ten years since the particle was discovered, this writer has not seen any treatment of the model independent amplitudes cited by Ralston. Instead, the ATLAS collaboration at CERN rules out “some specific models” of spin-1 [16], “several alternative spin scenarios” [17], and “alternative hypotheses for spin” [18]. CMS reports that, “all tested spin-one boson hypotheses are excluded,” [19] and, “any mixed-parity spin-one state is excluded” [20]. In further contradiction to the claims of certain detractors of the MCM, Particle Data Group (PDG)—the de facto bottom-line authority on the state of the art in particle physics—reports that the spin of the Higgslike particle was not yet determined as of 2020 [21]. PDG writes the following.

“Whereas the observed signal is labeled as a spin-0 particle and is called a Higgs Boson, the detailed properties of $H^0$ and its role in the context of electroweak symmetry breaking need to be further clarified.”

“The observation of the signal in the $\gamma\gamma$ final state rules out the possibility that the discovered particle has spin 1, as a consequence of the Landau–Yang theorem. This argument relies on the assumptions that the decaying particle is an on-shell resonance and that the decay products are indeed two photons rather than two pairs of boosted photons, which each could in principle be misidentified as a single photon.”

Regarding the latter excerpt, experiment trumps theory. Experiments are carried out mainly with the intention to falsify theories. Landau–Yang would go out the window if an experimental result was found to disagree with it. While Landau–Yang is well trusted, theory can never rule out reality. PDG’s published reasoning is specious. Ralston writes the following regarding the supreme dominion of experiment over theory [14].

“The Landau–Yang theorems are inadequate to eliminate spin-1. Theoretical prejudice to close the gaps is unreliable, and a fair consideration based
on experiment is needed. A spin-1 field can produce the resonance structure observed in invariant mass distributions, and also produce the same angular distribution of photons and \( ZZ \) decays as spin-0. However spin-0 cannot produce the variety of distributions made by spin-1. The Higgs-like pattern of decay also cannot rule out spin-1 without more analysis. Upcoming data will add information, which should be analyzed giving spin-1 full and unbiased consideration that has not appeared before.”

It is unusual that almost ten years have gone by since the particle was discovered and the “unbiased consideration” has not yet appeared in the literature, to the knowledge of this writer. As in References [16–20], the considerations published to date are biased under the suppositions of one model or another. While it seems impossible, the literature appears to suggest that the model-independent case has not been considered. What does seem possible is that the model-independent case has been considered and the result has been withheld due to politics. Indeed, we suggest that the particle is labeled as a spin-0 particle and called a Higgs boson mainly to further a false impression that the MCM prediction for spin-1 has been ruled out. Usually physicists are zealously and notoriously reluctant to jump to conclusions, but not in this case. In that vein, just months after the MCM prediction for spin-1, Ellis and You wrote the following [22].

“...There are many indirect and direct experimental indications that the new particle \( H \) discovered by the ATLAS and CMS Collaborations has spin zero and (mostly) positive parity, and that its couplings to other particles are correlated with their masses. Beyond any reasonable doubt, it is a Higgs boson[.]”

This excerpt may contain the only reference in the entirety of the physics literature to the formal standard of proof in USA jurisprudence: reasonable doubt. A more common standard in physics is given by the motto of the Royal Society: *Nullius in verba*. It means “take nobody’s word for it.” While Ellis and You make this bold and patently unscientific claim in the abstract of their paper, they immediately back off from the outrageous overstatement in the paper’s first sentence.

“...It has now been established with a high degree of confidence that the new particle \( H \) with mass \( \sim 126 \) GeV discovered by the ATLAS and CMS has spin zero.”

This paper of Ellis and You is remarkable not only for its reference to some ill-defined and unquantifiable standard of reasonable doubt in place of physics’ usual
5σ criterion, but also because it was the first citation of the Royal Swedish Academy of Sciences in their technical write-up regarding the 2013 Nobel Prize in physics [23]. The prominent citation by the Royal Swedish Academy of Sciences can be construed as an endorsement of the false supposition that the Higgslike particle is the Higgs boson beyond a reasonable doubt. Aside from the reasonable doubt cast by the MCM prediction [8], Ralston has reported that an entirely indeterminate amount of doubt remains. PDG cites an uncertain number of photons and a questionable assumption about the on-shell condition as reasonable sources of doubt. Almost two years after Ellis and You published, CMS reported with atypical bluntness that it was still important to study the spin-1 case experimentally because the observed state may be that one [20].

“Despite the fact that the experimental observation of the $H \rightarrow \gamma\gamma$ decay channel prevents the observed boson from being a spin-one particle, it is still important to experimentally study the spin-one models in the decay to massive vector bosons in case that the observed state is a different one.”

It is not clear whether CMS suggests (i) the existence of a second, different particle at $\sim$125GeV, (ii) that the observed one is different than the one ruled out by the Landau–Yang theorem, or (iii) that the final state is different than $\gamma\gamma$. CMS’ obtuse language about “a different one” is consistent with a theme of sidestepping the spin-1 issue in the literature. Even while CMS emphasizes the importance of experimental study, they still call the $H \rightarrow \gamma\gamma$ decay a fact while PDG reports that this channel is not yet established as a fact [21]. Assuming it is a fact, as it may be, CMS does not state their reliance on the assumed perfection of the Landau–Yang theorem to find that such a decay prevents spin-1.

If reasonable doubt were to have some meaning in physics, then it could only be the usual standard of 5σ. However, there does not exist any literature claiming to have ruled out spin-1 at that level. Certain models of spin-1 have been ruled out to certain levels but the model-independent, objective property of spin-1 has never been ruled out for the Higgslike particle at any high significance, and never at 5σ. Most likely, the reference to the reasonable doubt standard of USA jurisprudence was used to establish in a court of USA law, for some (nefarious) reason, that this writer’s prediction is wrong. In fact, spin-1 has not been ruled out. Any publication claiming that spin-1 has been ruled out will be found to have ruled out only certain models of spin-1 divorced from model-independent Lorentz invariance [14]. Ten years later, one would think the particle’s discoverers would have determined its spin. To this writer’s knowledge, no other particle’s spin was so elusive that it could not be
determined even ten years after the initial discovery. In the opinion of this writer, and at the expense of the intended factual nature of this paper, the Higgslike particle has been determined to have spin-1, and CERN withholds the result because it supports the MCM over work which is better loved in the academic mainstream.

In Reference [24], another falsifiable MCM prediction was posed. It was suggested that one might observe variations in the value of the fine structure constant $\alpha$ correlated with the delay between an event and its detection in some apparatus. The unstated but implicit reasoning was that the state space of things which existed in the past is not the same as the state space of things which exist in the present. Therefore, observables might depend on how far in the past an event occurred prior to its detection. This was already the case for an earlier MCM result regarding dark energy [25]. Though the MCM unit cell was not constructed until about a year after the delay prediction appeared in Reference [24], the unit cell elucidates the motivation and complements it with further motivation. Usually, signals from events in the past are thought to propagate into detectors along a path in topological Minkowski space. In the MCM, in addition to the different state space in the past supposed in Reference [24], the past is not totally Minkowski in the unit cell. Due to the fifth dimension, one may speak of earlier chronological times which are totally Minkowski, as well as earlier chirological times in which the past is anti-de Sitter. Propagation through some non-Minkowski geometry would cause deviations from the predictions for pure Minkowski propagation, and these deviations would be correlated with the amount of time spent in the non-Minkowski geometry. This prediction is not so precise as the prediction that the Higgslike particle should have spin-1. Still, it is a strong prediction. If such delay correlations are not be observed, then the fundamental ideation behind the prediction would be falsified. The predicted correlations were observed, however!

The main gist communicated here to the reader is that all of the verifiable ideation in the MCM has survived: the specific things and the less specific things. More than 99% of new theories can be rejected immediately due to some obvious physical problem so it is a great accomplishment of the MCM not to be one of those theories. Often times, laypersons hear that new theories are a dime a dozen, which is true, but this glosses over a further notion which is more relevant in the present case. A new theory which can survive even a cursory check is a diamond in the rough. Almost none of them make it past a single hurdle. Ones that do are often absurdly convoluted. Quintessence and the chameleon field are examples of ridiculous convoluted theories not so ridiculous that they are immediately discarded. Even the popular theory of
early cosmological inflation, which is not easy to rule out, is rather convoluted. To
the contrary, the MCM is elegant, intuitive, and beautifully simple, though not yet
mathematically formalized with new equations of motion. There is no trivial way
to immediately rule out the MCM as is the case for almost all new theories. This
testifies to the very high quality of the work. Beyond the lack of an easy rejection,
the MCM’s predictions have multiple experimental confirmations.

The Babar experiment concluded in 2008. Although the primary analysis of the
data generated by the experiment had also concluded by the time of the MCM delay
prediction [24], the search for the such correlations in the Babar data was undertaken
immediately following the MCM publication. Not astonishingly, the MCM prediction
was immediately borne out when the Babar collaboration published their observation
of time reversal symmetry violation in the \( B^0 \) meson system [26]. While the Babar
analysis did not exactly search for the delay correlations in the value of \( \alpha \) which
had been suggested, the result follows. Since physics is Hamiltonian, meaning that
everything is determined once any two things are determined, the value of \( \alpha \) which
can be extracted from the delay correlations published in Reference [26] will depend
on the delay. The observation of time reversal symmetry violation is easily the 21st
century’s second biggest discovery in particle physics after the Higgslike particle. This
discovery is a direct experimental verification of the structure of the MCM. During
the primary data analysis stage following Babar’s data collection stage, no one had
the idea to check for correlations with delay. After it was suggested that the MCM
would be such that delay correlations should exist, someone at Babar checked and
found a signal they had missed. Before the MCM, no one had any reason to expect
such correlations. After the MCM prediction, time reversal symmetry violation was
discovered and the history of physics was changed forever. An amazing result popped
out of a multi-million dollar experiment which almost missed it. If the Higgslike
particle is eventually reported to have spin-1, then the biggest and second biggest
discoveries in particle physics in the 21st century will be among the MCM’s small
handful of falsifiable predictions. Not only that, the MCM also predicts (among even
more things) the dark energy effect [25] whose discoverers were awarded the 2011
Nobel Prize in physics: Perlmutter, Schmidt, and Riess. So, there is a decent volume
of ordinary physics output recorded in the publications constituting the MCM. The
lack of an easy falsification among these predictions makes the MCM better than 99%
of similar attempts to bushwhack a new path. The confirmation by Babar makes the
MCM the best new theory on the market today, bar none. However, Babar does not
credit the ideation for delay correlations to Reference [24] and the ordinary scientific
proceedings are retarded.

The predictions above, and others mentioned below, are intermingled with other content in the MCM publications. Some of that content is highly non-standard. Why the weird tone? (See Appendix D.) After this writer became convinced that his work was wrongfully blacklisted against appearing even on the unreviewed arXiv, a tone was adopted which could never be published, even if the absence of blacklisting. Even in the presence of outstanding original work, the tone in many papers is such that it could never appear in the usual venue for the dissemination of scientific information in physics. Although the MCM’s many grand successes form an independent middle finger, the non-standard content and tone was added as a second middle finger so that this writer could be seen giving two middle fingers to the establishment which prefers the political mechanisms of the USA to the actual practice of science. In part, the present paper lays out a series of problems whose write-ups should be sufficiently technical that the tone of the papers cannot be confused or conflated with the results. As mentioned above, the technical treatment of the problems should rise in many cases to the level of a PhD thesis. In the absence of such clearly demonstrated technical mastery or a complementary voluminous set of calculations, it has been easy for detractors to conflate the author’s prose with the objective results.

This writer has not been able to publish even on arXiv: the unreviewed—yet censored—preprint repository in which absolute trash is published every day (along with many fair or outstanding research papers in physics and mathematics.) Before the non-standard tone was adopted, Reference [25] was submitted to arXiv in September 2009. The typesetting and graphics were substandard, the tone was ordinary, and the content was top-tier. For some reason most likely related to a payment routed through Cyprus to Paul Manafort in October of 2009, the paper was rejected for publication on arXiv. (For placement of Reference [25] on the spectrum of what is acceptable in the physics preprint literature, compare to Reference [27].) Some detail relating to the publication status of Reference [25] is found in Reference [28]. The overall lack of peer-review for the MCM, which is a superset of the problem with arXiv, provides more fodder for detractors. Even the most outlandish and easily disproven models of alternative physics have extensive online documentations including Wikipedia articles and various forum discussions, e.g.: Timecube. The relative invisibility of the MCM suggests that the publication blacklist exceeds blacklisting in the traditional publication venues goes so far as the total prohibition of this writer’s intention to communicate results. As a scientist, a physicist’s trade is to ply the scientific method whose final step is communicate results. The fake internet bubble
in which this writer appears to communicate results (while ultimately failing to do so, for the most part) has had a stronger negative impact on this writer’s career than any number of figurative middle fingers ever could. Still, this writer’s research does get communicated, somehow. Reference [25] is now called SCP-001 in certain corners of the internet where the MCM is known to exist.

The supposition, or allegation, that the MCM has not passed peer-review is false. Before moving on to a review of the MCM unit cell and its labeling, followed by a review of the MCM scheme of fundamental particle we will summarize the extensive peer-review of the MCM and its glowing yet uncredited receptions. Originally, the MCM was a work in phenomenology. Given certain results, a model was constructed to accommodate them in Reference [25]. The optical effect described as dark energy is explained therein without an anomalous (and borderline unphysical) acceleration of the expansion of the 3D universe. Instead, accelerating expansion in the time part of 4D spacetime is identified as the cause of the observed optical effects. This was the kernel of the idea that things in the past should not be exactly as they are in the present. In Reference [29], inquiry into the structure of the past was taken all the way back to the cosmological beginning. Since a famous theorem of Arnowitt, Deser, and Misner [30,31] proves that the 0-component of the universe’s 4-momentum must be non-zero, the usual model of big bang cosmology cannot conserve 4-momentum. However, physics usually requires that momentum is conserved. In the way that Pauli was able to deduce the existence of the neutrino from a quantity of missing momentum, it was deduced that a big bang would have had to spawn two universe moving oppositely through time if it was a momentum conserving process. If the energy of one universe is positive-definite, then the other universe (whose time has a minus sign on it) would be negative-definite. This is required to conserve 4-momentum, as is usual in physics. Shortly after the proposition for negative time published in November 2011 [29] (which was a restatement of the same idea published in 2009 [25]), Rubino and McLenaghan et al. conducted an experiment regarding negative frequency in quantum optics [32]. Since frequency is merely inverse time, and since the experiment was reported only months after negative time was found to resolve the momentum problem in big bang cosmology, we suggest that the ideation for the experiment of Rubino and McLenaghan et al. followed after a review of the early work in the MCM [25, 29]. Short of experimental verification, it is the highest and most valid form of peer-review that one man’s research should influence another man’s research direction. Many papers passing ordinary, administrative peer-review go on to accumulate zero citations. Papers well received by the community of experts
in that area go on to acquire citations. If not for the apparent USA-sponsored blacklist of this writer, it is suggested that Rubino and McLenaghan et al. might have cited Reference [29] as the direct motivator of their search for physical negative frequency modes. What peer-review can be higher than to have one’s work received and built upon? The answer cannot be a layer of dust atop an unknown but peer-reviewed CV item.

Spawning new scientific inquiry among one’s peer community is nearly the highest form of peer-review. It far surpasses the administrative peer-review which is widely hated by academics [33] and yet revered as holy by those who are only indirectly aware of the mechanism. Surpassing even positive reception in one’s community, the highest mark in peer-review is experimental confirmation. Rubino and McLenaghan et al. write the following about their discovery of negative frequency resonant radiation [32]. Negative frequency resonant radiation (NRR) is a direct confirmation of the theory of negative time at the heart of the MCM.

“[F]requency conversion processes may be understood in terms of energy transfer between specific modes [sic]. However, to date only the positive frequency branch of the dispersion has been considered when this actually also has a branch at negative frequencies. This branch is usually neglected or even considered meaningless when, in reality, it may host mode conversion to a new frequency. The fact that a mode on the negative branch of the dispersion relation may be excited has a number of important implications, beyond the simple curiosity of the effect in itself. Indeed, light always oscillates with both positive and negative frequencies, but the negative-frequency part is directly related to its positive counterpart and seems redundant. On the other hand, light particles, photons, have positive energies and are associated with positive frequencies only. A process such as that highlighted here, that mixes positive and negative frequencies will therefore change the number of photons, leading to amplification or even particle creation from the quantum vacuum. In this work we show how alongside the usual resonant radiation spectral peak observed in many experiments, a second, further blue-shifted peak is also predicted. This new peak may be explained as the result of the excitation of radiation that lies on the negative frequency branch of the dispersion relation. We first explain why this radiation should be observed and then provide experimental evidence of what we call ‘negative frequency resonant radiation’ in both bulk media and photonic crystal fibres.”
So, although the existence of these modes had been known for a long time, no one had ever thought to look for them until the theory of negative time was published in References [25, 29]. Perhaps history will show that this was only a coincidence. In any case, we suggest that the negative frequency experiment was motivated by a review of the MCM, and that the experiment confirmed the negative time hypothesis through the observation of negative frequency optical modes. To the extent that Reference [32] cites the possibility for “amplification,” consider the following from a follow-on publication of Rubino et al. in late 2012.

“\[W\]e may derive a photon number balance equation by generalizing [sic] to the case of a moving scatterer. We find that:

\[|RR|^2 - |NRR|^2 = 1,\]

where \(|RR|^2\) and \(|NRR|^2\) are the photon numbers of the [resonant radiation] and [negative resonant radiation] modes normalized to the input photon number, \(|IN|^2\). The negative sign in front of the \(|NRR|^2\) photon number is a direct consequence of the fact that the NRR-mode has negative frequency in the comoving reference frame [sic]. So the difference between the normalized number of photons has to be equal to the photon number in the input mode. As a consequence, the total output photon number, \(|RR|^2 + |NRR|^2 > 1\), i.e. we have amplification [emphasis added]. The scattering process mediated by the traveling [relativistic inhomogeneity] will amplify photons as a result of the coupling between the positive and negative frequency modes.”

As we have previously commented on the eccentric citation of Ellis and You to the legal standard of doubt in USA jurisprudence, the note at the top of Reference [34] is also eccentric. It is the only instance of such a note that this writer (who does not usually browse the experimental quantum optics literature) has come across. The note directs that correspondence and requests for materials should be addressed to co-author Faccio. The eccentric note in anticipation of correspondence is given because Reference [34] reports that the authors discovered free energy. The negative frequency optical mode which follows directly from the negative time mode in the MCM—following logically and chronologically—revealed the holy grail of physics: a feasible mechanism for the construction of devices whose coefficients of efficiency exceed unity. While the MCM did not predict the application in quantum optics directly, it follows because negative frequency is inverse negative time. It is suggested that this writer’s peers saw that it follows, did the experiment, and confirmed the
physics. So, the MCM has yet again passed the true bar of review by peers but not yet the false bar of administrative peer-review by the docents of a politicized bureaucracy. The MCM has been experimentally confirmed at least twice. If the Higgslike particle has spin-1, it will be at least three times. Next to experimental confirmation, administrative peer-review is meaningless. If it was suggested that objects on Earth tend to fall in the downward direction, no one would ask if the claim has passed peer-review. For the MCM, however, the fact that it has not passed peer-review in the most artificial and useless sense is cited as problematic. Following the work on NRR [32, 34], Lockheed abruptly announced their near-term plans for truck-sized nuclear fusion reactors. Fundamentally, Lockheed was front-running their expectation for the mass production of NRR power generators which would be truck-sized because they are only optical tables in a box. After Lockheed’s initial press releases, the West Texas oil contract inexplicably cratered in 2014 and it has not recovered as of 2021.\footnote{During the preparation of this paper, the WTI oil contract reached highs not seen since 2014.} The blacklist on the MCM was extended by the powers that be to cover the only hope by which humanity might ever escape its shackles of toil. The results regarding free energy are now known in certain corners of the internet as “golf rumors.” The quoted name follows from men at their country clubs talking about the NRR result before the full violence of USA political machine squashed such talk.

The discovery of negative frequency resonant radiation by Rubino and McLennaghan \textit{et al.} [32, 34] suggests that the MCM has passed peer-review with flying colors. So does the result about time reversal symmetry violation published by Babar [26]. Both of these results obviously connect to the MCM’s requirement for negative time, through negative frequency and time reversal respectively. Both results are experimental confirmation of the MCM in excess of affirmative peer-review by positive reception leading to follow-on work. Additionally, there are no known results which conflict with the MCM predictions for new effects at $10^{-3}$m, that the Higgslike particle should have spin-1, or any other features of the MCM. The many mechanisms described in Reference [25] are each individually likely to be parlayed into several further direct experimental confirmations of the MCM. Additionally, there are many mathematical confirmations. For example, the MCM search for quantum gravity shows that Einstein’s equation for general relativity may be derived from the physics of a quantum particle in a box. A number alike to the fine structure constant to within 0.4\% is characteristic of this box. Other examples of affirmative review by peers include Ashtekar’s response paper [28, 35]. Wilczek’s 2012 quantum time crystals [36,37]
follow from the 2011 $\hat{M}^{3}$ operator developed in Reference [24].\footnote{This writer became aware of viXra in the summer of 2012 and the viXra submission dates of References [24,25,29] do not reflect the initial publication dates.} The MCM unit cell is the unit cell of a time crystal in the most intuitive way. Almost all of Finkelstein’s arXiv publications are MCM response papers (see Section 33.) Mochizuki’s Hodge theater is just the MCM dressed in a thick coat of jargon. Hairer’s $3M$ Breakthrough Prize-winning regularity structure [38] is just the MCM unit cell dressed in another coat of jargon. When Hairer’s colleague reported that Hairer’s work must have been done by aliens [39], it was a jibe regarding how obviously Hairer had used the MCM and its $\hat{M}^{3}$ operator without citation. Apparently, those on the far side of the blacklist see something akin to aliens between them and this writer. The RBM model in the autodidactic universe of Alexander et al. [40] is plainly the process given by $\hat{M}^{3}$. The list of such glowing yet uncited peer-reviews goes on and on. Many such topics are posed as open problems or avenues for future inquiry among the theses listed in this paper.

**The MCM Unit Cell**

This section contains a glossary of symbols pertaining to the MCM unit cell (Figure 1), and remarks on its most prominent features. Greek tensor indices run from 0 to 3. Latin indices run from 0 to 4.

- $A^{\mu}$ is the electromagnetic potential 4-vector. This object has its usual meaning and it is almost always assumed to be zero. This facilitates consideration of the simplest cases which can be extended to $A^{\mu} \neq 0$ later.

- $A^{\mu}_{\pm}$ is an electromagnetic potential 4-vector in $\Sigma^{\pm}$. Usually, descriptions of the MCM assume an $A^{\mu}_{\pm}=0$ ground state.

- $\Sigma^{\pm}$ are semi-infinite 5-spaces consisting of all points at which the fifth coordinate takes some positive or negative value respectively.

- $\chi^{A}_{\pm}$ are the 5D coordinates in $\Sigma^{\pm}$. Coordinates written with $\chi$ are called *abstract coordinates* to distinguish them from *physical coordinates* written with $x$. Different coordinate charts’ distances are measured with different metrics.

- $\chi^{\mu}_{\pm}$ are the abstract coordinates of $\Sigma^{\pm}$ at some constant value of $\chi^{4}_{\pm}$.

- $\chi^{A}_{\varnothing}$ or $\chi^{\mu}_{\varnothing}$ are the hypothetical coordinates to the right of $\Omega$ and to the left of $\mathcal{A}$, as in the lower representation of Figure 1. In previous usage, $\chi^{4}_{\varnothing}$ has referred to a single point added to splice $\Omega_{1}$ with $\mathcal{A}_{2}$, as $\mathcal{H}$ splices $\Sigma^{\pm}$. In that case, the
Figure 1: The MCM unit cell is the fundamental element of a cosmological lattice. $\mathcal{H}$ is a Minkowski space representing the observable universe. $\Sigma^\pm$ do not include their shared boundary at $\mathcal{H}$. It is expected that the $\chi^-$ coordinates are left handed if the $\chi^+$ coordinates are right handed. The second figure with $\Sigma^\pm$ joined on $\mathcal{H}$ is most properly the unit cell in the sense of crystallography but often unit cell refers to the representation centered on $\emptyset$. 
χ_∅ coordinates are 4D and written χ_μ. However, the mechanism for connecting Ω_1 to Ω_2 is one of the major outstanding problems in the MCM (see Section 1 and Section 7.) Since the level of aleph (Section 90) changes at ∅, meaning that ∅ marks the progression from one neighborhood of fractional distance to the next [2], the pointlike property of χ_4 is on one level of aleph may be resolved in greater detail on another level of aleph. For this reason, it is supposed that χ_4 might span a 5-space.

- x^μ are the physical, relativistic coordinates of the geometric manifold H, a Minkowski space. Distance between the points specified with x^μ is given by the metric g_μν.

- x^μ± are the physical coordinates of gravitational manifolds located in Σ± at constant values of χ_4±. These manifolds are also charted in the abstract χ^μ± coordinates so it is required to distinguish between the physical coordinates x^μ± and the abstract coordinates χ^μ±.

- g_μν is the metric of Minkowski space. Sometimes we might write g_μν = η_μν + h_μν with η_μν the flat Lorentzian metric and h_μν some small perturbation. Almost always, we assume h_μν = 0.

- g_4±_AB is the 5D metric in Σ±. Tilded metric always describe distance in the abstract χ coordinates. Ignoring some ± scripting g_4±_AB is the Kaluza–Klein metric

\[
\tilde{g}_4^{AB} = \left( \begin{array}{cc}
\tilde{g}_{\alpha\beta} + \kappa^2 \phi^2 A_\alpha A_\beta & \kappa^2 \phi^2 A_\alpha \\
\kappa^2 \phi^2 A_\beta & \phi^2
\end{array} \right).
\]

The MCM use case for the KK metric identifies the square of the scalar field ϕ with a function of the piecewise disconnected fifth dimension χ_4. Taking the simplest case of A_μ± = 0 and using the identity function f(χ_4±) = χ_4±, we have

\[
\tilde{g}_4^{AB} = \left( \begin{array}{cc}
\tilde{g}_{\alpha\beta} & 0 \\
0 & \chi_4^±
\end{array} \right).
\]

Since χ_4± is positive or negative in Σ± respectively, g_4±_AB has Lorentzian signature {± ± ± ± ±} in Σ+ and pseudo-Lorentzian signature {± ± ± ± ±} in Σ−. In full generality (but setting κ = 1), we have

\[
\tilde{g}_4^{AB} = \left( \begin{array}{cc}
\tilde{g}_{\alpha\beta} + \chi_4^± A_\alpha^± A_\beta^± & \phi^2 A_\alpha^± \\
\chi_4^± A_\beta^± & \chi_4^±
\end{array} \right).
\]
\( g_{\mu\nu}(\chi^4_{\pm}) \) is the Lorentzian metric of a submanifold of \( \Sigma^\pm \) defined by some constant value of \( \chi^4_{\pm} \). This metric describes distances in the physical \( x^\mu_{\pm} \) coordinate charts. In \( \Sigma^+ \), \( g_{\mu\nu}(\chi^4_{\pm}) \) is the dS\(_4\) metric and in \( \Sigma^- \) it is the AdS\(_4\) metric. \( g_{\mu\nu}(\chi^4_{\pm}) \) is the metric induced from \( \tilde{g}_{AB} \) on the hyperboloid

\[
-(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = f(\chi^4_{\pm})
\]

embedded in 5D Minkowski space. The means by which \( A^\mu_{\pm} \neq 0 \) might disturb the uniform curvature dS\(_4\) or AdS\(_4\) remains to be clarified. However, usually we assume \( A^\mu_{\pm} = 0 \). Then, each slice of constant \( \chi^4_{\pm} \) in the abstract coordinates has a corresponding maximally symmetric dS\(_4\) or AdS\(_4\) metric with de Sitter parameter \( f(\chi^4_{\pm}) \) which is inversely proportional to the Ricci scalar on that slice. \( g_{\mu\nu}(\chi^4_{\pm}) \) describes distances measured in the physical \( x^\mu_{\pm} \) chart on each slice of constant \( \chi^4_{\pm} \).

- \( \mathcal{H} \) is Minkowski charted in \( x^\mu \). Up to a topological issue of global closure or openness, Minkowski space is the low curvature limit of de Sitter space and/or anti-de Sitter space. The \( \mathcal{H} \)-brane stitches together \( \Sigma^\pm \) at \( \lim \chi^4_{\pm} \to 0^\pm \) in a smooth continuum of increasing curvature.

- \( \Omega \) is a specific worldsheet in \( \Sigma^+ \) located at \( \chi^4_{\pm} = \Phi \) with \( \Phi \) being the golden ratio. In the physical coordinates (with \( A^\mu_{\pm} = 0 \)), \( \Omega \) is a dS\(_4\) space with open topology and positive uniform curvature. In Figure 1, \( \Omega \) spans some width of the horizontal coordinate but this in only a representation showing to show the spherical geometry. Formally, \( \Omega \) is a single sheet at one value of \( \chi^4_{\pm} \), as is \( \mathcal{H} \). The difference is that the physical metric on \( \mathcal{H} \) is the flat Minkowski metric while that on \( \Omega \) is the dS\(_4\) metric \( g_{\mu\nu}(\Phi) \). This brane is located at a finite distance from \( \mathcal{H} \) in the abstract coordinates but it is understood \( \Omega \) is also associated with the physical \( t_{\text{max}} \): the latest possible time.

- \( \mathcal{A} \) is a specific worldsheet in \( \Sigma^- \) located at \( \chi^4_{\pm} = -1 \). In the physical \( x^\mu_{\pm} \) coordinates (with \( A^\mu_{\pm} = 0 \)), \( \mathcal{A} \) is an AdS\(_4\) space of closed topology and negative uniform curvature. In Figure 1, \( \mathcal{A} \) spans some width of the horizontal coordinate but this is only a representation emphasizing the the hyperbolic geometry of the induced metric. Whereas \( -1 = -\Phi^0 \), it may prove more useful to place \( \mathcal{A} \) at \( \chi^4_{\pm} = -\Phi^{-1} \). Previous work in the MCM has been such that the distance from \( \mathcal{H} \) to \( \Omega \) should be either \( \Phi \) or \( \Phi^2 \) times the distance between \( \mathcal{A} \) and \( \mathcal{H} \). These conventions place \( \mathcal{A} \) at \( \chi^4_{\pm} = -1 \) and \( \chi^4_{\pm} = -\Phi^{-1} \) respectively. This brane is associated with physical \( t_{\text{min}} \): the earliest time.
• $\emptyset$ is the worldsheet of a black hole, also called a black brane. The topological singularity manifests in the physical coordinates so we have not introduced an $x_\emptyset^\mu$ chart. However, $\emptyset$ is a 4D surface or of a 5D bulk between $\Omega_1$ and $A_2$. $\emptyset$ may be an object between $\Omega$ and $A$ or it may be their union. $\emptyset$ is supposed to facilitate the forward connection of $\Sigma^+$ into the following instance of $\Sigma^-$ by joining the respective open and closed topologies on a topological singularity. Usually, there would be no smooth connection from the Lorentzian $\{-+++-\}$ metric in $\Sigma^+$ to the pseudo-Lorentzian $\{-++++\}$ metric in $\Sigma^-$. The topological singularity at $\emptyset$ is intended to stitch these disparate topologies together, and the resolution of the abstract coordinates inside the singularity is given to facilitate some smooth affine continuation. Often, we speak of the de Sitter parameter which determines a hyperboloid as governing the curvature on the slices of $\Sigma^\pm$. However, the Ricci scalar is a better characterization of curvature in maximally symmetric spacetimes such as Minkowski space, $dS_4$, and $AdS_4$. As an example of how one might impose a physical topological singularity without destroying the abstract coordinates, consider

$$R_A = \tan\left(\frac{\pi \chi^4}{2}\right), \quad \text{and} \quad R_\Omega = \tan\left(\frac{\pi \chi^4_+}{2\Phi}\right).$$

Here, the Ricci scalar $R$ goes to $\pm \infty$ at $\chi^4 = -1$ and $\chi^4_+ = \Phi$. In geometrizing the KK metric with $\phi^2 = f(\chi^4_{\pm})$, this convention reflects the choice of some function besides the identity function. The de Sitter parameter in maximally symmetric spacetime is a simple linear function of the Ricci scalar.

• $\{\mathcal{H}', \mathcal{A}', \Omega'\}$ is a rigged Hilbert space, also called a Gelfand triple. In previous work, we have used the convention that $\{\mathcal{A}', \mathcal{H}', \Omega'\}$ is a rigged Hilbert space. Reasons for the change of notation are given in Section 1 and Section 10.

• $\mathcal{A}'$ is a Hilbert space. In the MCM, the relevant Hilbert space is almost always, if not always, taken as the infinite dimensional Hilbert of position space wavefunctions. In that case, $\mathcal{A}'$ is, or is essentially, the $L^2(\mathbb{R}^3)$ space of square integrable functions.

• $\mathcal{H}'$ is a subdomain of Hilbert space $\mathcal{H}' \subset \mathcal{A}'$. Under certain conditions related to unbounded observable operators with continuous spectra, such as the position operator $\hat{x}$, there exist states in $\mathcal{A}' = L^2$ for which certain ordinary quantum mechanical identities fail [41,42] (see Section 10.) $\mathcal{H}'$ is the subdomain of $\mathcal{A}'$ in which things like the expectation value and certainty formulae are guaranteed.
to be well behaved.

- \( \Omega' \) is a dual (antidual) space of \( \mathcal{H}' \) such that \( \{ \mathcal{H}', \mathcal{A}', \Omega' \} \) is a rigged Hilbert space. Eigenstates of operators with continuous spectra are usually non-normalizable Dirac \( \delta \) functions which do not exist in \( \mathcal{A}' \) or \( \mathcal{H}' \). Such eigenstates, usually position eigenstates, belong to the state space \( \Omega' \) satisfying \( \mathcal{H}' \subset \mathcal{A'} \subset \Omega' \).

- \( \varnothing' \) is a hypothetical state space for states in the \( \varnothing \) black brane. Since \( \varnothing \) is a singularity, the space of states in it might be taken as the empty space. However, it may be required to resolve \( \varnothing \) to a higher degree for the propagation of information from unit cell to the next.

The standard cosmological model (SCM) describes a 4D spacetime: the universe. The SCM is cited as some generalized picture of the Friedmann–Lemaître–Robertson–Walker cosmology or the more modern ΛCDM model. Both are more specific than what is required to describe the MCM as an extension of an informally labeled SCM. Indeed, the MCM is more quantum mechanical in nature now than cosmological, and the exact details of an underlying standard cosmology are out of scope. The SCM universe is the \( \mathcal{H} \)-brane: a worldsheet in the center of the unit cell (Figure 1.) \( \mathcal{H} \) is Minkowski space spanned by \( x^0 \) and \( x^i \) in the usual way. It is a flat 4-space in the Lorentzian topology. Its \( x^i \) 3-spaces are Euclidean. \( \Sigma^{\pm} \) only contain positive and negative values of \( \chi^4_{\pm} \) respectively and \( \mathcal{H} \) is a 4D surface joining two semi-infinite 5-spaces \( \Sigma^{\pm} \) at the place where \( \chi^4_{\pm} \) would be equal to zero.

The main jumping off point for separating the MCM from the SCM was the implementation of a cyclic cosmology. Cyclic cosmology is a variant of big bang cosmology in which one assumes a big crunch at the end of things, and that the crunch serves as a big bang for a new epoch (cycle) of cosmology. Sometimes it is said that cyclic cosmology is unphysical due to the observed thermodynamic state of the universe but all such issues can be sidestepped in a number of ways. For instance, an epoch of cosmological deflation dual to the epoch of inflation would suffice. Furthermore, there is little reason to think that cosmology is so well understood that theoretical arguments about the universe’s equation of state can categorically rule out exotic behaviors on cosmological time scales. Beyond that, the present incarnation of the MCM is not necessarily a model of big bang cosmology in any guise at all because the periodicity first assigned to the vertical \( x^0 \) direction has been implemented along the horizontal \( \chi^4 \) direction. This writer considers it an open question whether or not the MCM in its current incarnation is a model of big bang cosmology in any form (see Section 8.) In other words, it is not yet fully determined if the periodicity in
\( \chi^4 \) has replaced the supposed \( x^0 \) periodicity or if it has complemented it. In the original MCM jargon [25, 29], however, big bangs and big crunches are replaced with *big bounces*. The bouncing is a periodicity in the \( x^0 \) direction: vertical on the page of Figure 1. This writer was introduced to cyclic cosmology under the guise of loop quantum cosmology (LQC) [43] but the MCM contains nothing specific to LQC which is not found in all other models of cyclic cosmology.

For the present version of the MCM unit cell, the main modification to the SCM is the fifth embedding dimension \( \chi^4 \) [6]. It was added a few years after the 2009 publication of the paper which gives the MCM its name: “Modified Spacetime Geometry Addresses Dark Energy, Penrose’s Entropy Dilemma, Baryon Asymmetry, Inflation and Matter Anisotropy” [25]. The fifth dimension was implemented following a review of Kaluza–Klein theory (see Section 18) in which Overduin and Wesson write the following [7].

“Kaluza’s achievement was to show that five-dimensional general relativity contains both Einstein’s four-dimensional theory of gravity and Maxwell’s theory of electromagnetism. He however imposed a somewhat artificial restriction (the cylinder condition) on the coordinates, essentially barring the fifth one a priori from making a direct appearance in the laws of physics. Klein’s contribution was to make this restriction less artificial by suggesting a plausible physical basis for it in compactification of the fifth dimension. This idea was enthusiastically received by unified-field theorists, and when the time came to include the strong and weak forces by extending Kaluza’s mechanism to higher dimensions, it was assumed that these too would be compact. This line of thinking has led through eleven-dimensional supergravity theories in the 1980s to the current favorite contenders for a possible ‘theory of everything,’ ten-dimensional superstrings.”

Klein supposed that the fifth dimension might not contribute because it is a compactified at an unobservably small scale. The MCM unit cell is purposed to motivate the so-called cylinder condition by requiring that physics takes place only on surfaces of constant \( \chi^4 \). Namely, \( \mathcal{H} \) is taken as the small \(|\chi^4_\pm|\) limit of \( \Sigma^{\pm} \). Derivatives with respect to the fifth dimension can’t contribute in \( \mathcal{H} \) due to an effective condition \( \chi^4_\pm = 0 \). All derivatives with respect to a constant vanish.

A shortcoming of KK theory highlighted by Overduin and Wesson [7]—the main one which prevented the success of KK theory in its effort to unify gravitation with classical electromagnetism—is that the only allowable solutions require a vanishing electromagnetic strength tensor \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). It is hoped that by doubling the
number of electromagnetic degrees of freedom from four, as in Equation (19.1), to eight, as in Equation (19.2), will provide a workaround by which $F_{\mu\nu} \neq 0$ solutions can be extracted in $\mathcal{H}$ as the limit of two disconnected KK theories in $\Sigma^\pm$ (see Section 18.) The MCM unit cell reflects the ground state condition in which $A_{\mu}^\pm = 0$ but is expected that the non-zero solutions can be implemented trivially. The result will be that the $\mathcal{H}$-, $\mathcal{A}$-, and $\Omega$-branes will lose their shared character of maximal symmetry as $A_{\mu}^\pm$ vary away from zero. In other words, non-vanishing electromagnetic fields will perturb the maximally symmetric character of the respective AdS$_4$, M$_4$, and dS$_4$ spacetimes. In the $A_{\mu}^\pm = 0$ ground state, the fifth dimension $\chi^4$ charts a continuum of increasingly curved, maximally symmetric spacetimes between $\mathcal{A}$ and $\Omega$. This serves as a toy model upon which one would build more realistic applications. To make use of the expanded degrees of EM freedom in $\Sigma^\pm$, one must use $A_{\mu}^\pm$ to define $A_{\mu}$ in $\mathcal{H}$. Here, we associate the EM potential 4-vectors in $\Sigma^\pm$ with the advanced and retarded potentials of classical electromagnetism (see Section 19.)

The idea to have the physics of the observable universe $\mathcal{H}$ defined by two 5D theories [6] reflects a principle called holographic duality. This idea was made famous by Maldacena’s demonstration of a “correspondence” between a 4D conformal field theory and a 5D AdS space [44]. The MCM flavor of “holographic duality” between the physics of a 4D surface and the surrounding 5D bulk is much simpler than Maldacena’s famous AdS/CFT duality but the duality is is holographic nonetheless. The mechanism reflects exciting new thinking. It is well-defined according to the most scrupulous standards. Usually, holographic duality between a surface and a bulk is considered to be such that the surface is a 3D boundary on the outside of a 4D universe. Sometimes one considers the 2D surface on the outside of a 3D black hole. The fresh new idea for holographic duality in the MCM unit cell is to sandwich a surface between two bulks instead of on the outside of just one bulk. This idea alone greatly separates the MCM from competing theories. It cannot be overstated that the MCM has accomplished what other theories have not accomplished due in large part to this original thinking in the red-hot area of bulk-boundary physics. Although the writer was not acquainted with Randall–Sundrum (RS) models (see Section 21) when constructing the unit cell [6], it is obvious that it is a third class of RS model not considered by RS. The two famous RS1 and RS2 models put branes at one side of a bulk or another—at infinity or at zero in their given coordinates—but they do not consider the case of a brane set between two bulks. In the MCM, the $\mathcal{H}$-brane is at the origin, and the $\mathcal{A}$- and $\Omega$-branes are located at plus and minus infinity in an appropriate parameterization.
Before continuing on to the MCM particle scheme, the reader’s attention is called to the reality that certain labeling conventions in the unit cell are chosen intuitively from a few possible permutations. The purpose in this program is to facilitate easy discussion that would be clouded by repeated clarifications for caveats about all possible permutations. Usually, the number of permutations is very low if it is not the stated one, the alternative would reflect nothing more than a sign change. For instance, it is assumed that the trip from $H_1$ to $H_2$ goes through $\Omega$ and then $A$ but this is subject to a change of sign convention if needed. The fifth dimension is only timelike in $\Sigma^-$ and this a hard constraint on the total functioning of the model. If $\chi_4^4$ was needed to be timelike instead, one would add a minus sign. In the end, we will require that binding energy is negative in $H$ and that its entropy tends to increase. The assignment of $dS$ and AdS slices to $\Sigma^\pm$ is only a sign convention.

The association of the state spaces $A'$, $H'$, and $\Omega'$ to a rigged Hilbert space \{A', H', \Omega\} is slightly more nuanced than metric signature convention for $\Sigma^\pm$, but only slightly. In this paper, we will not continue in the previous convention that $A' \subset H' \subset \Omega$ for an RHS. We will use \{H', A', \Omega\} instead. However, it might be better to speak about a generalized rigged Hilbert space \{S_1, S_2, S_3\} so as not to assume which state space is attached to which brane. In generalized $S_1$ picture, $S_2$ is Hilbert space, $S_1$ is a subdomain of Hilbert space, and $S_3$ is $S_1$’s dual (or antidual) space. $S_3$ contains $S_2$ as a subspace. In the previous convention \{A', H', \Omega\}, $S_2 = H'$ is attached to the $H$ brane but it is reasonable to think that we might attach the $S_1$ subdomain to the geometric space $H$ where measurements are made. In the structure of rigged Hilbert space, $S_1$ is the subdomain of $S_2$ on which all uncertainty formulas and expectation values for observables are well-defined [41, 45]. Therefore, the lowest nested part of RHS is the most physical space and should be associated with $H$ rather than $A$. However, even the hypothetical assignment of $S_3$ to $H$ gives a reasonable convention in which particles live at exact positions in $H$ while uncertainty is offloaded into the knowledge of the observer. These issues are mostly reducible to the representation of the $\hat{M}^3$ operator which cycles states through rigged Hilbert space. The open issues regarding that representation are summarized in Section 1. Construction of a rigged Hilbert space of MCM states is the problem given in Section 10.

The MCM Particle Scheme

Reference [25] poses a solution to the mystery of the matter asymmetry [46]. That mystery asks why the universe is made of mostly matter instead of mostly anti-
matter. The question is very similar to the question about the non-conservation of 4-momentum at the big bang. If nature is thought to conserve baryon number and 4-momentum, then why should the big bang not conserve both? It was suggested in Reference [25] that two universes leaving a big bang, or a big bounce, should be understood as an ordinary particle pair in the sense of pair creation by vacuum fluctuations. It is not known why any particular fluctuation occurs but the particle production process is better understood than some alleged cosmological big bang process for a single universe with an anomalous baryon number and increment of momentum. In the pair picture, the forward and reverse time universes are a particle and an anti-particle. One has a positive baryon number and positive $p^0$. The other has a negative baryon number and negative $p^0$. The MCM model of particles [8] follows from this notion that a universe, or one quantum of spacetime, is like a fundamental particle. In the unit cell, the universe given positive baryon number $B$ is the $H$-brane [6,8,25]. It is spanned by $x^0$ and $x^i$. The present particle scheme supposes that all fundamental particles are quanta of spacetime spanned by a spatial unit vector $\hat{x}^0$ and a temporal one: $\hat{x}^0$ or $\chi^4$. Given the two types of time in the MCM, we immediately arrive at the 12 well known members of the three generations of matter particles, as in Figure 2.

Looking at the MCM unit cell (Figure 1), space $x^i$ points into the page. Chronos $x^0$ points up and chiros $\chi^4$ points horizontally. The spanning bases for planar spacetimes are $x^0 x^i$ and $\chi^4 x^i$. $\chi^0$ and $x^0$ point in the same direction and don’t span a geometric plane. The basis vectors can form left- or right-handed coordinate systems with the third member of \{ $x^0$, $x^i$, $\chi^4$ \} so there exist four distinct varieties of spacetime quanta in the MCM: space crossed with either of chronos or chiros, each in left- and right-handed varieties. The planes of space crossed with the very well studied $x^0$ flavor of time are taken as the relatively well-behaved leptons. Space crossed with the exotic new chirological time $\chi^4$ is taken as a quark. We suggest that quantum electrodynamics (QED) is simple relative to quantum chromodynamics (QCD) because $x^0$ is simple relative to $\chi^4 \cong \{ \chi^4_+ , \chi^4_0 , \chi^4_- \}$. The three color flavors of each quark are distinguished by the three varieties of $\chi^4$.

Having established two leptons and two quarks, the three generations of each are associated with the $H'$, $A'$, and $\Omega'$ state spaces. In the end, the primary distinction among the three generations may be attributed most directly to the three different lattice positions \{ $A$, $H$, $\Omega$ \} or to the three different state spaces \{ $H'$, $A'$, $\Omega'$ \}. The three generations of leptons are increasingly massive and we would like to associate this property with the $S_1 \subset S_2 \subset S_3$ structure of RHS. Since electrons are stable
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Figure 2: The MCM particle model (right) compared to the standard model of particle physics (left.) Each instance of $x^0$ or $\chi^4$ refers to a spacetime spanned by $x^i$ and either $x^0$ or $\chi^4$. The scalar Higgs boson is an outlier in the standard model. There are are no such outliers in the modified model.

in $\mathcal{H}$ while muons and taus are not, this suggests the present RHS convention for \{$\mathcal{H}'$, $\mathcal{A}'$, $\Omega$\}. Since we have associated each generation of matter particle with a state space in RHS, the increasing masses and of the generations should be associated with the increasing $\mathcal{H}' \subset A' \subset \Omega'$ structure. It is the first generation particles which are stable in $\mathcal{H}$ and this gives us good reason to assign the smallest state space to the $\mathcal{H}$-brane, and to the lightest generation of matter particles. Another consideration for the MCM states space structure regards lepton universality. The standard model predicts that each lepton flavor should be identical to the others up to its mass. However, modern experiments suggest this is not the case. The proton radius puzzle observed in the muonic hydrogen system [47] is an example of experimentally determined non-universality among lepton flavors. By putting each of the MCM matter particle generations (flavors) in a different state space, we motivate lepton non-universality in principle, as required for agreement with experiment.

For the construction of the MCM model of particles, we have relied to some degree on phenomenological considerations but it suffices for the claim of a first principles \textit{(ab initio)} derivation of the particle spectrum that the unit cell has permutations of its objects generating two pairs of particles in three varieties, that one of those pairs of particles may be distinguished by three further varieties of QCD color charge, and that the fundamental bosons are well accommodated too. It is known that the 12 fundamental matter particles are spin-$\frac{1}{2}$ fermions so we assign that property to each MCM quantum of spacetime by supposition. The force carrying particles of the
standard model are known to have spin-1 so the MCM force carriers are assembled from pairs of matter particles. This is done in part because $\frac{1}{2} + \frac{1}{2} = 1$, and in part because forces are usually transmitted between fermionic matter particles, certain weak and strong interactions being exceptions.

Being the most ordinary and well understood force carrying particle, the photon $\gamma$ is the $x^0x^0$ particle at the top of Figure 1’s stack of elementary bosons. The most complicated and least understood elementary boson is the gluon $g$ associated to the $\chi^4\chi^4$ connection. Here, we find more support for the MCM particle scheme. It is known from experiment that there exist eight varieties of gluon and it is a triumph of the MCM model that we obtain eight such varieties. Quark flavor is associated with the three varieties of $\chi^4$; gluons are associated with connections between quarks. The nine connection permutations are $++, +\emptyset, +-, \emptyset+, \emptyset\emptyset, --, --+, --\emptyset$, and $-\emptyset$. Removing $\emptyset\emptyset$ on some qualitative grounds, we are left with eight varieties of gluon.

Why should $\emptyset\emptyset$ not be associated with a gluon? There are many possible reasons but it is hoped that the reason will fall out from future inquiry. Since $\chi^4$ has no length, the $\emptyset\emptyset$ gluon has no moment, in some sense, while the other eight connections have non-vanishing moments, in that sense. Another reason might be that the other eight gluons have some sort of connection to $\mathcal{H}$ through $\Sigma^\pm$ while $\emptyset\emptyset$ does not, and is therefore not observable or not directly observable. Another possibility is that there are indeed nine gluons and a nine gluon model would improve on the current state of QCD. In particular, one might take the $\emptyset\emptyset$ connection as a sterile gluon in the manner that sterile neutrinos are sometimes thought to exist. In general, the total picture of strong force QCD physics is complicated, not very well understood, and has a lot of room for improvement.

Ignoring a hypothetical Higgs boson, the only remaining standard model particles requiring placement in the modified model are the $W^\pm$ and $Z^0$ bosons. These are accommodated by either of the two remaining connections: $x^0\chi^4$ or $\chi^4x^0$. Randomly choosing the former permutation, we originally assigned $W^\pm$ as $x^0\chi^4_\pm$ and $Z^0$ as $x^0\chi^4\emptyset$ [8]. Here again, there exists further ideological support. The weak force governs interactions between leptons and particles made of quarks so the admixture of the $x^0$ and $\chi^4$ elementary fermions in the $x^0\chi^4$ weak boson connection is philosophically robust. Obviously, we have only randomly chosen the $x^0\chi^4$ connection for $W$ and $Z$, and we might have chosen $\chi^4$. In either case, the MCM predicts at least one more spin-1 elementary particle, possibly three, in the other of $x^0\chi^4$ and $\chi^4x^0$. However, there exists another possibility not mentioned in the first iteration of the MCM particle scheme [8]. One issue in the proposed model is that we have associated the $W^\pm$
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particle-anti-particle pair with $\chi^4_\pm$ while we have not placed anti-gluons in the $\chi^4\chi^4$ connection. If the $\pm$ scripting does not specify the anti-particle for gluon, then neither should it for $W$. Therefore, we might (should) associate the $Z$ and $W$ particles with only two of $x^0\chi^4_+, x^0\chi^4_-$, and $x^0\chi^4_{\pm}$. In that case, we would suppose that the Higgslike particle is the third member of the $x^0\chi^4$ connection, that $x^0\chi^4$ and $\chi^4 x^0$ are indistinguishable, and that the Higgslike spin-1 particle completes the smorgasbord.

Whatever the exact details are, the modified model predicts that there should be no spin-0 fundamental particles. Therefore, the Higgslike particle must have spin-1. If the Higgslike particle is eventually determined to have with spin-1, that will be strong evidence that time and effort should be spent on wrapping up MCM’s loose ends.

1 The $\hat{M}^3$ Operator and its Equation

1.1 Overview

The fundamental equation of classical mechanics $\vec{F} = \partial_t \vec{p} = m\partial^2_t \vec{x}$ is postulated in Newton’s laws. The fundamental equation of quantum mechanics (QM), $i\hbar \partial_t \psi = \hat{H} \psi$, is usually implemented as a postulate. In both cases, the differential operators $\partial_t$ and $\partial^2_t$ (or the $\partial^2_\vec{x}$ in $\hat{H}$) are used in postulated equations. In the MCM, we would like to obtain a new equation for $\hat{M}^3 := \partial^3_t$ such that the discrepancies between classical reality and quantum theory are lessened or remedied. Various postulates or hypotheses for the functioning of $\hat{M}^3$ have appeared in the body of MCM publications. The number of variations on these guesses approaches the number of papers written about them. In the end, the postulate should be the only expression consistent with the requirements (up to the form of the representation.) At that time, putting the correct equation to paper should be effortless. For this reason, previous work in the MCM has more closely attended that which $\hat{M}^3$ needs to do than the formal statement and study of a postulate. So, we have identified $\hat{M}^3$ as an appropriate operator for what should be some new equation of motion for a theory of everything. What should that equation be?

To begin with, $\hat{M}^3$ describes the actions of a physicist. Although the extant quantum theory requires a physicist’s actions to implement wavefunction collapse upon measurement, the usual approach to QM ignores the rest of what the physicist does. In efforts to better understand quantum theory, philosophical considerations sometimes fixate upon an artificial distinction between a quantum state and an ideal measuring apparatus. It is asked how an ideal measurement can be made when de-
tectors are necessarily quantum mechanical themselves. The new idea in the MCM which generates a requirement for $\hat{M}^3$ seeks to separate the physicist from his experiment rather than to separate a hypothetical ideal detector from its quantum subject matter. In the MCM, measurement is made ideal as a psychological process divorced from anything quantum mechanical. Observation is psychological and many experiments—the double-slit and delayed-choice quantum eraser experiments, for example [48]—show that observation is supremely weird within the existing framework. Regarding such epistemological issues that don’t impede one’s ability to compare experiments to predictions, physics may be differentiated between work in the esoteric fundamentals and work in the more glamorous applications [49]. Is it a step too far to suppose that there exists a better framework? Perhaps there is one to which the current theory is only an approximation? Is it wrong not to shut up and calculate? To these ends, we have identified $\hat{M}^3$ as a good operator for what should be a true revolution in the arena of the fundamentals.

The psychological process for $\hat{M}^3$ was defined as follows in Reference [3].

“To test any theory[,] two measurements must be made. Call these measurements $A$ and $B$ corresponding to events $a$ and $b$. The boundary condition set by $A$ will be used to predict the state at $b$. To make this prediction[,] the observer applies physical theory to trace a trajectory from $A$ to the future event $b$. Before the observer can verify the theory, sufficient time must pass that the future event occurs. Once this happens[,] a retarded signal from $b$ reaches the observer in the present and a second measurement $B$ becomes possible. [F]rom the present[,] the observer traces a path into the future. Once that future becomes part of the observer’s past, a signal reaches the observer in the present and the theory can be tested. A three-fold process.

Present $\rightarrow$ Future $\rightarrow$ Past $\rightarrow$ Present$^1$ .”

The process of $\hat{M}^3$ starts at $A$. Some event $a$ has already occurred, the signal from $a$ has reached the observer, and the observer has represented the condition of $a$ as some abstract expression. For instance, a detector has registered a particle at some point in space, or more formally in some region of spacetime,$^2$ and then the detector told the observer what it saw. The observer says, “Given my observation $A$,

\footnote{The $\rightarrow$ symbol was chosen only so as to use a different symbol for this word-level idea than the $\rightarrow$ symbol appearing in Equations (1.1) and (1.2.)}

\footnote{Whether an apparatus detects the particle at a point or merely within some region is an interesting and open question. In the end, all that is known is that the observer cannot glean more information from the apparatus than the region of spacetime in which the particle is detected.}
I predict by theoretical construction that a subsequent event $b$ will occur, which I will observe at $B$. This prediction is the first step of $\hat{M}^3$. It is an abstract prediction Present$\leftrightarrow$Future. The next step requires a time translation of the observer to some time later than the time associated with $b$. Since we expect $\hat{M}^3$ to operate on states rather than the observer, the observer’s time translation might be implemented as a translation of $a$ and/or $b$ to an earlier time. This is the second step Future$\rightarrow$Past. The third step is a reconnection to the psychological level when the signal from $b$ comes to the observer’s attention at $B$: Past$\rightarrow$Present. It is hoped that a new equation which reflects this process will improve quantum theory. Feynman states the general idea [50].

"[T]here is always hope that [a] new point of view will inspire an idea for the modification of present theories, a modification necessary to encompass present experiments."

The usual formulation of QM provides no dynamical mechanism for wavefunction collapse, also called state reduction or projection. The MCM adds some extra steps which might accommodate such a mechanism. Presently, collapse is inserted into QM to force agreement with experiment as needed. If dynamical collapse is achieved, quantum theory will be greatly improved. Isham writes the following regarding this most glaring gap begging for improvement [51].

"[T]he idea of a reduction of the state vector is often invoked in more realist approaches in which the state vector is deemed to refer to a single system. The reduction is then assumed to occur after a single (ideal) measurement, and has nothing to do with system selection in a series of repeated measurements. From this perspective, the overall time development of a state of a single system consists of sharp jumps produced by the act of measurement, separated by periods of deterministic evolution governed by the Schrödinger equation[.]

"The major problem is to understand the origin of these sudden changes in the state. In particular, can they be obtained from the existing quantum formalism, or does the reduction of the state vector have to be added to the general rules of quantum theory as a fundamental postulate? This problem is particularly acute in any approach to quantum theory that aspires to demote ‘measurement’ from playing a fundamental part in the formulation of the theory. In this case, there is a strong motivation to try to derive the state reduction vector from the existing formalism; albeit, perhaps, only as..."
an empirically useful approximation to the actual development of the state in time.

"The nature of the problem depends in part on the perceived referent of the state. If the state is held to quantify our knowledge of the system, then the reduction process is arguably analogous to the conditioning procedure in classical probability in which the addition of extra information about what is actually the case changes our state of knowledge. On the other hand, if the state vector is held to refer to the system itself, then the idea of reduction is frequently tied to the 'uncontrollable disturbance' thesis. This raises the obvious question of the possibility of understanding the nature of this effect in direct physical terms. In particular, what type of interaction serves as an 'ideal measurement'?

"One approach to this problem is to ask again about the significance of the fact that actual measuring devices are made of quantum atoms. Is it possible to understand a state reduction as the outcome of some dynamical evolution in which object and apparatus are both regarded as quantum-mechanical systems? Indeed, even within the minimal, pragmatic approach to quantum theory there is good reason for asking what type of interaction between two systems is to be regarded as a bona fide measurement of one by the other. The concept of measurement plays a fundamental role in the formulation of quantum theory, and therefore deserves to be understood further."

After introducing notation associating quantum states with the elements of the unit cell (Subsection 1.2), we will present cases that $\hat{M}^3$ should be useful for implementing dynamical state reduction (Subsection 1.8), explaining the origin of the fine structure constant (Subsection 1.7), and promoting the metric from a disconnected background object to dynamical one via a new theory of quantum gravity (Subsection 1.10). Use cases for certain new mathematical tools related to fractional distance analysis [2] are developed (Subsection 1.5), as are a few other open issues for $\hat{M}^3$. Since it remains hard to motivate the MCM value for the fine structure constant $\alpha_{\text{MCM}}$, we will not phrase the present problem of $\hat{M}^3$ in terms of the original motivation [24]. Instead, we will lay out the current best understanding of $\hat{M}^3$ and some problems which are found to deserve further development. Regarding our intention to supplement the existing framework of quantum theory with $\hat{M}^3$, Finkelstein writes the following [52].

"Quantum theory began with ad hoc regularization prescriptions of Planck

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1 A review of Reference [24], and the motivations therein, is found in Appendix A.
and Bohr to fit the weird behavior of the electromagnetic field and the nuclear atom[,] and to handle infinities that blocked earlier theories. In 1924[,] Heisenberg discovered that one small change in algebra did both naturally.”¹

Heisenberg stated the following about his own program in his 1933 Nobel address.

“Quantum mechanics [sic] arose, in its formal content, from the endeavor to expand Bohr’s principle of correspondence to a complete mathematical scheme by refining his assertions.”

Similarly, it remains to expand the MCM principles to a complete mathematical scheme by refining the assertions. To wit, we have found a value $\alpha_{\text{MCM}}$ that falls out of some mostly standard quantum mechanical language but we have neither connected that language to the full quantum theory nor explained the 0.4% discrepancy with $\alpha_{\text{QED}}$. Also, there exists an idea for how state reduction might be implemented more naturally in the MCM than it is in QM [53] but we have not written down any Eureka-level solutions.

On the problem of quantum gravity, we say it is a hard problem because there does not exist a robust mathematical language in which the objects of one theory can be put into an equation with the objects of the other.² General relativity (GR) is a theory of points in spacetime but the state of being located at a point cannot be measured and does not exist in Hilbert space. Quantum states are fuzzy but GR does not admit fuzziness. Far removed from a theory of gravitons or questions about the curvature of spacetime as a disconnected background to quantum theory, the general problem of quantum gravity is that there does not exist a good framework in which it is possible to put the equivalence relation $=\text{between two separate statements of gravitational and quantum dynamics.}$ For instance, the equivalence of the inertial mass in classical mechanics and electrodynamics allows us to combine the Lorentz

---

¹Heisenberg’s famous $\hat{p}\hat{q}-\hat{q}\hat{p}\neq 0$ quantum algebra was a small change in notation but it reflects a giant leap in the ability of humans to understand the natural world. After all, the idea that $3\times 2$ under certain circumstances might not equal $2\times 3$ was a radical departure from thousands of years of previous mathematical thinking. Heisenberg’s change of algebraic structure is the origin of the phrase “a quantum leap” meaning “a huge or sudden increase or advance of something.”

²There is some machinery in QFT by which a certain tensor field $\varphi_{\mu\nu}$ (called a graviton field) can couple in its two indices to a stress-energy tensor $T^{\mu\nu}$. The QFT graviton can be used to reproduce a few experimental results but those come only under a host of simplifications, hand-waving, and cumbersome constraints. The QFT graviton is ugly, not beautiful, and it is only useful for small perturbations on Minkowski space. In the opinion of this writer, furthermore, there is little reason to think that the hypothetical quantum force carrier of the gravitational force is real because there is no gravitational force. Gravitation is geometry in curved spacetime. It is a fact that a rank-2 tensor field can couple to $T^{\mu\nu}$ in QFT but it is not well established that this confluence of tensor indices is well suited to the general problem of quantum gravity. After all, this coupling has been known for decades but there is no consensus on what a working theory of quantum gravity might look like or how one might demonstrate gravitons’ existence through observation. Indeed, there is no consensus on the existence of gravitons due in some part to the weakness of the theoretical framework of $\varphi_{\mu\nu}$ for applications in gravitation.
force law with arbitrary mechanical forces. On the other hand, there is no Schrödinger equation for the metric and there is no way to put probability amplitude into a stress-energy tensor which is mutually dynamical with Schrödinger evolution. The MCM mechanism for quantum gravity (Subsection 1.10) offers an original and exciting mathematical language in which quantum objects might interact with gravitational objects. However, it very much remains to establish this new language as a complete mathematical framework.

1.2 The Ontological Basis

To describe what $\hat{M}^3$ does, we must first define its constituency of related objects. The guidelines for $\hat{M}^3$ are

$$\hat{M}^3 : H'_1 \to \Omega'_1 \to A'_2 \to H'_2,$$

and

$$\hat{M}^3 : H_1 \to \Omega_1 \to A_2 \to H_2.$$

Equation (1.1) describes abstract algebraic translation across an RHS. Equation (1.2) describes geometric translation through coordinate space. For $\hat{M}^3$ to do both, we must reconcile the properties $H'_1 \subset A' \subset \Omega'$ and $H \not\subset A \subset \Omega$. The former reflects the nested structure of rigged Hilbert space. The latter reflects the geometric arrangement of the unit cell. Reconciliation requires a somewhat subtle treatment of the elements of a state space.

Let $\psi_k : D \to \mathbb{C}$ be a function and let $\times$ be an inner product. Then

$$\mathcal{H}' = \{\psi_1, \psi_2; \times\}$$

$$\mathcal{A}' = \{\psi_1, \psi_2, \psi_3; \times\}$$

$$\Omega' = \{\psi_1, \psi_2, \psi_3, \psi_4; \times\}$$

at least approximates an RHS if it does not satisfy the definition directly. One simple way to break the nested subset structure is to append labels as

$$\mathcal{H}'_H = \{\psi_1, \psi_2; \times, H\}$$

$$\mathcal{A}'_A = \{\psi_1, \psi_2, \psi_3; \times, A\}$$

$$\Omega'_0 = \{\psi_1, \psi_2, \psi_3, \psi_4; \times, Omega\}$$

$$\implies \mathcal{H}'_H \not\subset \mathcal{A}'_A \not\subset \Omega'_0.$$

(1.4)
Suppose $\mathcal{D}_H$, $\mathcal{D}_A$, and $\mathcal{D}_\Omega$ are three non-intersecting subsets of $\mathbb{R}$ such that

\[
\begin{align*}
\psi_k \in \mathcal{H}' &\implies \psi_k : \mathcal{D}_H \to \mathbb{C} \\
\psi_k \in \mathcal{A}' &\implies \psi_k : \mathcal{D}_A \to \mathbb{C} \\
\psi_k \in \Omega' &\implies \psi_k : \mathcal{D}_\Omega \to \mathbb{C}.
\end{align*}
\]  

(1.5)

The usual definition of a function is as a binary relation between two sets so it follows, for instance, that $\psi(x) = \sin(x)$ is the same function regardless of which $\mathcal{D}$ is its domain. However, if

\[
\begin{align*}
\mathcal{H}' &\ni \psi : [0, 2\pi] \to [-1, 1] \\
\mathcal{A}' &\ni \psi : [4\pi, 6\pi] \to [-1, 1] \\
\Omega' &\ni \psi : [8\pi, 10\pi] \to [-1, 1],
\end{align*}
\]  

(1.6)

then the different functions are not exactly the same. This invokes a nuanced technical issue which we will revisit broadly in Section 27 pertaining to a criticism of Scholze and Styx against Mochizuki’s inter-universal Teichmüller theory. Regarding the matter of $\hat{M}^3$, it is not relevant whether the identity of a function formally depends on the identity of its domain. So, while the nested RHS structure of $\{\mathcal{H}', \mathcal{A}', \Omega'\}$ is such that $\psi \in \mathcal{H}'$ implies $\psi \in \mathcal{A}'$ and $\psi \in \Omega'$, we will do physics in the way that ignores unnecessary mathematical nuance. We will drop the subscripts and call $\{\mathcal{H}', \mathcal{A}', \Omega'\}$ an RHS even though we have added an implicit labeling scheme such that, for instance, $\psi \in \mathcal{A}'$ implies $\psi = \psi(x_A)$ where $x_A \in \mathcal{D}_A$. It suffices to say that MCM wavefunctions must have their associated manifold specified so we can know which coordinates chart the wavefunctions’ domains. For this purpose, we have introduced what is called the ontological basis $\{\hat{e}_H, \hat{e}_A, \hat{e}_\Omega\}$ with notation such that

\[
\begin{align*}
\psi \in \mathcal{A}' &\iff |\psi\rangle = |\psi\rangle \hat{e}_A = |\psi; \hat{e}_A\rangle = \psi(x^-_A) \\
\psi \in \mathcal{H}' &\iff |\psi\rangle = |\psi\rangle \hat{e}_H = |\psi; \hat{e}_H\rangle = \psi(x^+_i) \\
\psi \in \Omega' &\iff |\psi\rangle = |\psi\rangle \hat{e}_\Omega = |\psi; \hat{e}_\Omega\rangle = \psi(x^+_i).
\end{align*}
\]  

(1.7)

Although it exceeds the present discussion, we suppose the existence of a fourth basis element $\hat{e}_\varnothing$ such that $\psi \hat{e}_\varnothing = \psi(x^\mu_\varnothing)$. Now that we have developed the requisite objects, we may supplement the abstract notation of Equations (1.1) and (1.2) with
an ordinary operator algebra. Letting \( \hat{M}^3 \equiv \hat{M}_3 \hat{M}_2 \hat{M}_1 \), we have

\[
\begin{align*}
\hat{M}_1 |\psi; \hat{e}_{\Omega_1}\rangle &= c_1 |\psi; \hat{e}_{\Omega_1}\rangle \\
\hat{M}_2 |\psi; \hat{e}_{\Omega_1}\rangle &= c_2 |\psi; \hat{e}_{A_2}\rangle \\
\hat{M}_3 |\psi; \hat{e}_{A_2}\rangle &= c_3 |\psi; \hat{e}_{\Omega_2}\rangle \\
\end{align*}
\]

\[\implies \hat{M}^3 |\psi; \hat{e}_{H_1}\rangle = c_3 c_2 c_1 |\psi; \hat{e}_{H_2}\rangle \tag{1.8}\]

\( \hat{M}_1 \) executes \( H_1 \rightarrow \Omega_1 \), \( \hat{M}_2 \) executes \( \Omega_1 \rightarrow A_2 \), and \( \hat{M}_3 \) executes \( A_2 \rightarrow H_2 \) so that \( \hat{M}^3 \) executes \( H_1 \rightarrow H_2 \) via the given intermediate steps.

In practice, the MCM cosmological lattice is infinite in extent. Each unit cell resides on a higher level of aleph, and at a later chronological time than all leftward unit cells. However, by an expected normalization convention such that the present level of aleph is always the 1st (0th) level marked by \( \hat{\Phi}^1 (\hat{\Phi}^0) \), we might drop the subscripts to treat the problem as cyclic algebraic group. In other words, it may be useful to consider \( \hat{M}^3 : \mathcal{H} \rightarrow \mathcal{H} \) in place of the non-cyclic \( \hat{M}^3 : \mathcal{H}_1 \rightarrow \mathcal{H}_2 \). In practice, any given \( \mathcal{H}_k \) will have infinite number of lower and higher \( \mathcal{H}_j \)'s and it is expected that the normalization of the wavefunction to satisfy a probability interpretation should be attended with a normalization of the level of aleph such that the present observation defines the \( \mathcal{H}_0 \)-brane specified with \( \hat{\Phi}^0 \). Such a normalization procedure requires that we first complete the work needed to formally define \( \hat{M}^3 \).

The defining property of a set of basis vectors is usually the linear independence of the basis’ elements. The particular basis \( \{ \hat{e}_{H}, \hat{e}_A, \hat{e}_{\Omega}, \hat{e}_{\varnothing} \} \) is called ontological due to the specification of certain non-unit magnitudes for its elements. Usually the name “basis vector” is taken synonymously “unit vector” but not so for the ontological basis vectors. Choosing the number theoretically significant magnitudes \{2, \pi, i, \Phi\} in some order for \{\|\hat{e}_H\|, \|\hat{e}_{\Omega}\|, \|\hat{e}_A\|, \|\hat{e}_{\varnothing}\|\} is supposed to generate certain properties of the natural world by association with the structure of the unit cell. The present convention is

\[
\begin{align*}
\hat{e}_{H} &= \hat{\Phi} \\
\hat{e}_A &= \hat{i} \\
\hat{e}_{\Omega} &= \hat{\pi} \\
\hat{e}_{\varnothing} &= \hat{2} \\
\end{align*}
\]

\[\|\hat{\Phi}\| = \Phi \quad \|\hat{i}\| = 1 \quad \|\hat{\pi}\| = \pi \quad \|\hat{2}\| = 2 \tag{1.9}\]

In the original scheme for the ontological basis, the \( \mathcal{H} \)-brane was labeled with \( \hat{\pi} \). This was chosen intuitively and adding the hat to \( \pi \) was also proffered as solution to the Ehrenfest paradox [54]. The problem of whether or not such a solution exists, and
whether or not such a hypothetical solution can survive in the present convention is treated in Section 32. The present convention is such that $\hat{\Phi}$ marks the $H$-brane. In this way, it directly marks the $n$th level of aleph for a state as $|\psi; \hat{\Phi}^n\rangle$ and the hat on $\Phi$ avoids a requirement for modifying known wavefunctions to be proportional to $\Phi$. When $\Phi$ is pulled out of $\psi$ with the hat, $\psi$ is only proportional to 2, $\pi$, and $i$: three numbers which do not appear out of place in the analytical representations of quantum states.

The ontological basis was used to define the non-unitary property of $\hat{M}^3$ as

$$
\begin{align*}
\hat{M}_1 |\psi; \hat{\Phi}^1\rangle &= \Phi |\psi; \pi\rangle \\
\hat{M}_2 |\psi; \pi\rangle &= \pi |\psi; i\rangle \\
\hat{M}_3 |\psi; i\rangle &= i |\psi; \Phi^2\rangle
\end{align*}
\right\} \implies \hat{M}^3 |\psi; \hat{\Phi}^1\rangle = i\pi \Phi |\psi; \hat{\Phi}^2\rangle .
$$

(1.10)

If $\langle \psi; \hat{\Phi}^1 | \psi; \hat{\Phi}^1 \rangle = 1$, then $\langle \psi; \hat{\Phi}^2 | \hat{M}_1 \hat{M}_1 | \psi; \hat{\Phi}^1 \rangle = \Phi^2$. This is what is meant when it is emphasized that $\hat{M}^3$ is not a unitary operator: the $c_k$ appearing in Equation (1.8) are not all such that $|c_k| = 1$. It is hoped that such non-unitary properties will have applications to the hierarchy problem which asks about the origin of very large and very small numbers in physics.

---

NEED TO CLARIFY THIS AFTER REVIEWING TENSOR

Furthermore, coordinate transformations of the type $x^\mu \hat{e}_1 \rightarrow x^\mu \hat{e}_2$ may have novel consequences when $|\hat{e}_1| \neq |\hat{e}_2|$.  

---

Equation (1.10) places the same $\psi$ in $H_2$ as was found in $H_1$. This makes an appeal to a time independent Schrödinger equation for some stationary state $\psi$. A more general evolutionary process would be take $|\psi; t_1; \hat{\Phi}^1\rangle$ and return $|\psi; t_2; \hat{\Phi}^2\rangle$. For this, we expect $\hat{M}^3$ to be complemented by the time evolution operator $\hat{U}$. Such issues are outlined in Subsection 1.6. To the extent that $\hat{U}$ is an exponential operator, future work may call for the exponentiation of $\hat{M}^3$ yielding

$$
e^{\hat{M}^3 |\psi; \hat{\Phi}^1\rangle = \Phi |\psi; \hat{\Phi}^2\rangle .
$$

(1.11)

Such an equation might be interpreted as a 180° phase shift coupled to an increase in the level of aleph specified by $i\pi$ and $\Phi$ respectively. It has been supposed that transmission through the singularity at $\emptyset$ might be modeled as specular reflection in which the 180° phase shift is ordinary, and the increased level of aleph tells us that we have not truly reflected, but have instead been transmitted to the forward unit.
Responding to the accurate observation that the ontological basis is chosen as a wild guess, it is pointed out that no less than three physically important dimensionless numbers fall out of the chosen basis without much complexity added in the path of computation.

- The fine structure constant \( \alpha_{\text{MCM}} \) can be generated with these numbers. It differs from the accepted experimental value \( \alpha_{\text{QED}} \). \( \alpha_{\text{MCM}} \) is treated briefly in Subsection 1.7 and less briefly in Section 3 where the 0.4% discrepancy is discussed.

- The dimensionless constant \( 8\pi \) from Einstein’s equation appears in a natural way as well (see Subsection 1.10.)

- The classical EM coupling constant \( (4\pi)^{-1} \) appears in what is called the on-logical resolution of the identity:

\[
1 \equiv \hat{1} = \frac{1}{4\pi} \hat{\pi} + \frac{\varphi}{4} \hat{\Phi} + \frac{1}{8} \hat{2} - \frac{i}{4} \hat{i} .
\]

EXPLAIN MOVING HATS

In the following subsection, we will show that MCM states transform as tensors. Informally, if one want to transform the coordinates the \( \hat{e}_\mu \)-site to those of the \( \hat{e}_\nu \)-site, one simply multiplies by the appropriate identity and moves the hat by choice. For instance

\[
x \hat{\Phi} = x \hat{\Phi} \hat{1} = x \hat{\Phi} \frac{\pi}{\pi} = \frac{\Phi}{\pi} x \hat{\pi} .
\]
1.3 Tensor States

It was stated in Reference [3] that MCM states specified with the ontological basis are tensor states. Proof that such states satisfy the tensor transformation law has not appeared previously so it will be given here. This section deviates from the theme of open problems to present a complete result. Wavefunctions satisfy the axioms of a vector space as follows.

- The sum (superposition) of two state vectors is another state vector.
- The (inner) product of two states is a non-state scalar.
- For a scalar $c$ and a state $|\psi\rangle$, the product $c|\psi\rangle$ is another state vector. For physics, $c_1|\psi\rangle$ and $c_2|\psi\rangle$ are interpreted as the same state due to their linear dependence. In practice, normalization for the probability interpretation washes out any information encoded on a or b.
- The vacuum state $|0\rangle$ is the zero vector $\vec{0}$.

If there exist axioms of a tensor space, they are no so well known as the axioms of a vector space. To show that something is a tensor, one demonstrates the tensor transformation law of which the vector transformation law is the simplest case. However, it is not immediately intuitive that quantum states satisfy the vector transformation law in the usual sense of coordinate transformations. The coordinates of state space play little to no role in the ordinary practice of QM but the structural framework for such a demonstration can be useful in enhancing ones understanding of the theory. We will illuminate a little remarked upon feature of state spaces: they are coordinate spaces exactly like $\mathbb{R}^n$.

To the extent that $\mathbb{R}^3$ is spanned by $\{\hat{x}, \hat{y}, \hat{z}\}$, an $N$-dimensional quantum state space is $\mathbb{R}^N$ spanned by $\{\hat{e}_1, \hat{e}_2, ..., \hat{e}_N\}$. The $\mathbb{R}^N$ structure of state space requires us to treat the spanning basis vectors as static objects though they are the main objects of interest in QM. The $\mathbb{R}^N$ picture of a static basis is useful for envisioning the time evolution of quantum states. Given $\hat{A}|a_k\rangle = a_k|a_k\rangle$ and

$$|\psi, t\rangle = c_k e^{-iE_k t/\hbar}|a_k\rangle,$$  \hspace{1cm} (1.13)

one understands that $|\psi, t\rangle$ is a vector sweeping through the $\mathbb{R}^N$ given by $k \in \{1, 2, ..., N\}$. The $|a_k\rangle$ “eigenbasis” is exactly the geometric spanning basis of the space of states written in that basis. Time evolution is further simplified by the unitarity condition:
the tip of the $|\psi, t\rangle$ vector always lies on the unit sphere such that

$$\langle \psi, t|\psi, t\rangle = 1 = \sum_k c_k^* c_k .$$

(1.14)

At first glance, we can tell that ordinary states and MCM states are vectors and tensors respectively from

$$|\psi\rangle = \sum_k a_k|\psi_k\rangle \implies \psi = a_k\psi_k ,$$

(1.15)

and its generalization as

$$|\psi\rangle \hat{e}_\mu = \sum_k a_k|\psi_k\rangle \hat{e}_\mu = a_{ik}\psi_k\hat{e}_\mu \equiv \psi_{i\mu} .$$

(1.16)

A one index tensor $\psi_i$ is a vector and a vector with an extra index $\psi_{i\mu}$ is a tensor. However, this is not a formal demonstration of the transformation law.

An informal statement of the vector transformation law would be, "Let $\hat{R}(\hat{n}, \phi)$ be a rotation operator and let $|\psi\rangle = |a\rangle + |b\rangle$ so that $\hat{R}|\psi\rangle = \hat{R}|a\rangle + \hat{R}|b\rangle$ obviously preserves the ‘angle’ between $a$ and $b$, meaning that $\hat{R}|a\rangle$ and $\hat{R}|b\rangle$ are still orthogonal." If two orthogonal objects belong to a vector space, then they will remain orthogonal under coordinate transformations. If $\psi$ is not written in the position space representation, however, then the detail are modestly more complicated. The rotation operator, which is only one example of a coordinate transformation, must pertain to the coordinates of state space. As it is usually understood, $\hat{n}$ is some rotation axis in $\mathbb{R}^3$ but it does not makes sense to rotate a state around such an axis when the state is not written in the position basis. Instead, we must generalize to the case where $\hat{n}$ points in a direction defined by the spanning eigenvectors of the space instead of the $\{\hat{x}, \hat{y}, \hat{z}\}$ basis of $\mathbb{R}^3$. Indeed, we must generalize to case of arbitrary coordinate changes in state space and not only rotations. This line of reasoning is not used much in QM but it can be useful in enhancing ones understanding of the structure of the theory. It may be unfamiliar to some to think of a function like $\psi(x)$ as $\hat{e}_n$ but, in fact, this is exactly the structure of state space.$^1$ As a thinking device, one might consider the 2D space of electron spin states

$$\hat{x} \equiv \hat{e}_1 = |\uparrow\rangle , \quad \text{and} \quad \hat{y} \equiv \hat{e}_2 = |\downarrow\rangle .$$

(1.17)

These states transform as spinors under rotations of the lab frame (physical space)

\footnote{We omit some nuance differentiating the Hilbert space of abstract states from a formal function space.}
but they transform as vectors under rotations of state space. The state space spanned by these eigenstates is just $\mathbb{R}^2$. The time evolution of states in this space is visualized as the tip of vector moving on the unit circle.

First we will show that vectors in $\mathbb{R}^N$ satisfy the vector transformation law

$$v'^\mu = v^\nu \frac{\partial x'^\mu}{\partial x^\nu} .$$

For some vector $v^\mu$ in $\mathbb{R}^N$, we have

$$\vec{x} = \sum_k x^k \hat{e}_k \implies x = x^\mu \hat{e}_\mu , \quad \text{and} \quad x^\mu = x^\mu .$$

On the left is the usual vector notation: the vector is the sum of its components times the respective unit vectors. On the right is the same vector written in the tensor index notation.

Let there be a coordinate transformation

$$x'^\mu = f(x^\mu) = T^\mu_\nu x^\nu .$$

such that we want to find the components of $\vec{x}$ in the primed coordinates: $x'^\mu$. The prime on $x$ denotes that we are giving $\vec{x}$ in terms of the transformed coordinates. Taking the derivative of Equation (1.20) with respect to $x^\nu$ gives

$$\frac{\partial x'^\mu}{\partial x^\nu} = T^\mu_\nu .$$

Therefore, Equation (1.20) satisfies the vector transformation law: Equation (1.18).

To demonstrate the same transformation with states in Hilbert space instead of $\mathbb{R}^N$ vectors, we write the vector transformation law as

$$\psi'^\mu = \psi^\nu \frac{\partial x'^\mu}{\partial x^\nu} .$$

In this case, it may not be immediately obvious what are the $x_j$, and $x_k$, or what is meant by $\psi$ and $\psi'$. Noting that a general state vector is written as

$$|\psi\rangle = \sum_k c_k |a_k\rangle \quad \iff \quad \psi^\mu = |a_\mu\rangle .$$

we see that $x_k \rightarrow c_k$ and $\hat{e}_k \rightarrow |a_k\rangle$. To understand what $\psi'$ is, first one is reminded that the same state can be written in the eigenbasis of any operator. If $|\psi\rangle$ is the same
state expanded in the eigenbases of either $\hat{A}$ or $\hat{B}$, then it is very easy to understand that $|\psi\rangle$ is the same state when we transform the eigenbasis of $\hat{A}$ or $\hat{B}$. This has no physical significance, however, and is the reason why the vector transformation law for quantum states is not usually demonstrated. Introducing the transformation

$$|a'_k\rangle = \hat{T}|a_k\rangle ,$$

and we will choose to apply the rotation $\hat{R}(\hat{e}_n, \phi)$ for some constant $n$. When the vector transformation law is satisfied, we will conclude that it holds for arbitrary transformations. The coordinate transformation is

$$x_j = \hat{R}x_k , \text{ which implies } \frac{\partial x_j}{\partial x_k} = \hat{R} \ .$$

Note that $\hat{R}$ is a matrix and the indices balance out with $\hat{R} = R_{jk}$. Applying the rotation to the arbitrary state vector $|\psi\rangle$ we obtain $|\psi'\rangle = \hat{R}|\psi\rangle$ which satisfies the vector transformation law: Equation (1.22).

For a two-index tensor, the tensor transformation law is

$$\psi_{\mu\nu} = \psi_{\kappa\lambda} \frac{\partial x^\kappa}{\partial x'^\mu} \frac{\partial x^\lambda}{\partial x'^\nu} .$$

Using the definition $\psi_{\mu\nu} = \psi_{\mu} \hat{e}_\nu$, we have already shown that one of the indices transforms correctly. The coordinate transformations relevant to the other index are those specified by the hat swapping operations. We will use the transformation from the $\hat{\Phi}$ coordinates to the $\hat{\pi}$ coordinates:

$$x'^\mu \hat{\pi} = T_{\mu\nu}x^\nu \hat{\Phi} = \varphi \pi x^\nu \hat{\Phi} .$$

It follows that

$$\frac{\partial x^\lambda}{\partial x'^\nu} = \frac{\Phi}{\pi} .$$

This is exactly what we get with the hat swapping procedure

$$|\psi; \hat{\Phi}\rangle = \frac{\pi}{\pi} |\psi; \hat{\Phi}\rangle = \frac{\Phi}{\pi} |\psi; \hat{\pi}\rangle$$

Thus is demonstrated that MCM states satisfy the tensor transformation law.

HOW DOES SIN(X) TRANSFORM?
1.4 Maximum Action

Quantum and classical probabilities differ in that the intermediate steps of quantum motion between two measurements cannot be inferred.\(^1\) If a classical ball rolls down a ramp, its wavefunction is collapsed the whole time. It has a definite position at each instant during the motion, even if one looks away while the ball is rolling. For a quantum particle moving on some analogous potential energy landscape, the position of the particle is not knowable while one is looking away. This is the heatlike diffusion of the wavefunction (probability amplitude) given by the Schrödinger equation. When one looks, the wavefunction collapses but contrary to the classical case, the wavefunction diffuses while one looks away. The longer one looks away from a quantum particle, the more likely it is to be found away from the path of classical motion. The main beauty of Feynman’s formulation of non-relativistic quantum mechanics [50] is to show that the probability amplitude for the particle having followed one path or another is a fuzzy distribution peaked around the classical trajectory. If the quantum ball is observed at a higher potential and then at a lower one without the benefit of any intermediate measurements, it may not have followed the path which minimizes the action. Indeed, the most common interpretation of QM is that a quantum particle does not follow any path between measurements. Between measurements, the measured position eigenstate is said to undergo decoherence [56] such the eigenstate evolves to a superposition of position states centered on the classical trajectory. Decoherence is the quantum analogue of thermodynamic information loss into a heat bath, and this correspondence is well reflected in the Schrödinger equation being on the form of the classical heat equation.

The purpose in writing $\hat{M}^3$ as three separate operations is to hard-code into the motion stops on $\Omega$ and $\mathcal{A}$ between successive $\mathcal{H}_k$. Though we only make measurements at $\mathcal{H}_k$, the MCM postulates that there exists definite knowledge that the state must have been located on $\Omega$ and $\mathcal{A}$ between $t_0$ and $t_1$. Using intuitive notation such that $t_0 < t_a < t_b < t_0$, we know that MCM states “collapse” to $|\psi, t_a; \hat{\pi}\rangle$ and $|\psi, t_b; \hat{i}\rangle$ between measurements $A$ and $B$ corresponding to states $|\psi, t_0; \hat{\Phi}^0\rangle$ and $|\psi, t_1; \hat{\Phi}^1\rangle$. The intermediate states’ positions being given principally in the abstract $\chi^4$ coordinate avoids any discrepancy with the empirically verifiable probabilities. Effectively, $\hat{M}^3$ enforces a pseudo-classical probability on the motion but only in the abstract

\(^1\)See the beautifully written Sections 2-4 of Feynman’s famous “Space-Time Approach to Non-Relativistic Quantum Mechanics” [50] for a summary of non-classical probability. Section I.2 in Zee [55] also gives an excellent statement of the issues.
coordinates of the unit cell, not in the physical coordinates of $\mathcal{H}$. The motion is not determined in the bulk of $\Sigma^\pm$ but it is determined at the labeled branes.

It is a conjecture of the MCM that quantum and classical motions differ in that they satisfy the action principle through the maxima and minima of the action respectively (see Section 9.) It is taken for granted that motion along any path totally within $\mathcal{H}$ must be associated with some finite action. Therefore, the path across the unit cell is associated with infinite action. For the purposes of physics, what is usually called finite action may be defined as action less than some natural number of finite action increments. In the language of fractional distance [2], a natural number quantity action is called an action in the neighborhood of the origin. Infinite action is automatically left for motion across the unit cell and the circumstance of fractional distance is such that any action in the neighborhood of infinity will be one which takes the state out of $\mathcal{H}$. Action in the neighborhood of infinity makes an immediate appeal to the correspondence principle: when the action is large compared to $\hbar$, the motion should correspond to the classical motion. In other words, large action impedes the diffusion of the wavefunction. Thus, the correspondence principle imposes knowledge that the particle must have landed on $\Omega$ and $\mathcal{A}$ during a transit of the unit cell between two measurements.

In the example of the ball on a ramp, the action of the ball’s motion is always large relative to $\hbar$ due to the classical ball’s macro-scale mass. One often considers large as the limit in which $\hbar \to 0$ but here we consider $S \to \infty$. In the $\hbar \to 0$ limit, one obtains the classical motion identically but the restrictions of KK theory are understood to require that the Ricci tensor must vanish in the bulk of $\Sigma^\pm$, and that a quantum of matter-energy should not be found with a definite position inside the bulk. Therefore, one might make an appeal to finite action in the neighborhood of infinity to avoid violations of the KK framework. The arithmetic of numbers in the neighborhood of infinity [2] is well suited to studying variations in the form $S = \infty \pm \delta S$ but $S \to \infty$ is a prime example of the “infinities that blocked earlier theories.” $S = \infty$ is a mathematical non-starter for the calculus of variations. The study of maximum action has been historically impossible for this reason. Action in the form $\infty \pm \delta S$ does not require total wavefunction collapse within the bulk of $\Sigma^\pm$ when $\delta S \neq 0$ so the utility of such an action must be examined. Furthermore, some framework must be developed such that the end state of large action motion across the unit cell agrees with the small action end state of $\mathcal{H}$-only Schrödinger evolution. Namely, classical evolution across the unit cell would intersect the forward $\mathcal{H}$-brane at

\footnote{The full restrictions of KK theory require in depth analysis, as in Section 18. It is the preliminary understanding that position eigenstates for massive particles in the bulk of $\Sigma^\pm$ are not allowed.}
the classical prediction and this must be modified such that the intersection deviates from the classical result according to the probability distribution predicted by the Schrödinger equation. This result is an expected utility of the non-unitary property of $M^3$. The rescaling of coordinates as, for instance, $x \Phi x$ might introduce the phase factors required to skew the trajectory away from the classical probability. Thoughts on this mechanism are developed somewhat further in Subsection 1.8.

### 1.5 Fractional Distance

The labeled branes of the MCM unit cell are separated by finite distance in the abstract coordinates. To avoid mutual interactions, and specifically to avoid gravitation, we would like to place the labeled branes at infinite physical distance with respect to one another. One exciting utility of numbers in the neighborhood of infinity [2] is that the gravitational interaction goes to zero across any finite distance in the neighborhood of infinity. Indeed, the MCM requirement for branes separated by infinite, yet analytically tractable distances was the progenitor of the work which led to the discovery of fractional distance and an exciting corollary regarding the Riemann hypothesis [2]. The main output of the inquiry into fractional distance was a new algebraic object $\mathcal{\infty}$, called algebraic infinity, developed to support certain mechanisms proposed for the MCM. Informally, this object is already in wide use in physics. For example, in QFT one often writes the integral over all of spacetime as

$$
\int d^4x = \int d^3x \int dx^0 = VT ,
$$

such that $V$ is the volume of all of space and $T$ is an infinite amount of time which later cancels with another $T$ somewhere else. This common method in physics is exactly replicated with $T = \mathcal{\infty}$ and the attendant arithmetic axioms given in Reference [2]. Indeed, $\mathcal{\infty}$ functions exactly like the infinity physicists use regularly without the permission of mathematicians. Prior to the introduction of $\mathcal{\infty}$, levels of aleph were introduced in Reference [53]. This concept is the area of the MCM in which the most technical progress has been made. Levels of aleph are now identified as successive neighborhoods of fractional distance labeled $\mathbb{R}_X$ in Reference [2]. In the physical picture of the MCM, these neighborhoods are a sequence of concatenated unit cells.

The potential utility for levels of aleph was described at first as giving a new structure to the exponential function. The original infinite series expansion [53]

$$
e^{ikx} = \sum_{n=0}^{\infty} \frac{(ikx)^n}{n!} = \sum_{n=0}^{\aleph_0} \frac{(ik_0x)^n}{n!} + \sum_{n=\aleph_0}^{\aleph_1} \frac{(ik_1x)^n}{n!} + \sum_{n=\aleph_1}^{\aleph_\infty} \frac{(ik_2x)^n}{n!} + ... ,
$$

1
has been formalized as the big exponential function [2]

\[ E^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} , \quad \text{with} \quad n \in \mathbb{N}_\infty \cup \{0\} \]

\[ \implies E^{ikx} = \sum_{n \in \mathbb{N}_0} \frac{(ik\chi)^n}{n!} + \sum_{n \in \mathbb{N}_\chi} \frac{(ik\chi)^n}{n!} + \sum_{n \in \mathbb{N}_{\chi'}} \frac{(ik\chi)^n}{n!} + \ldots \quad (1.32) \]

Contrary to the preliminary statement given by Equation (1.31), \( e^x \) is the \( n \in \mathbb{N}_0 \) term of \( E^x \). The place for such an extended series in physics remains to be exactly nailed down but the series itself has been made completely formal. The general idea would be such that expanded series in quantum theory, such as certain well known powers series in \( \alpha \), would be better interpreted as contributions from different levels of aleph. In some intuitive way, one would associate the enumerated loop corrections of QFT with levels of aleph, and that would lead to an enhanced understanding of theory. Furthermore, levels of aleph was integral in solving the Riemann hypothesis. The architecture [57] of the later direct contradictions [2, 58–60] was totally reliant on odd and even levels of aleph conceptualized as the intermediate and maximal neighborhoods of infinity. Though levels of aleph were not cited in the computing the characteristic length scale \( 10^{-4} \text{m} \) [3] (see Section 52), the general idea was that contributions from other levels of aleph tunnel into the normalized \( \hat{\Phi}^0 \) level to alter the expected \( F_{\text{net}} \hat{\xi} = \vec{0} \) Newtonian force diagram.

Another useful result coming from the analysis of fractional distance was the discovery of the set \( \mathbb{F} \) containing all non-arithmatic or immeasurable real numbers. If the various piecewise \( \chi^4_+, \chi^4_-, \chi^4_\emptyset \) are concatenated to make a smooth curve from \( \mathcal{H}_k \) to \( \mathcal{H}_{k+1} \) in one affine parameter, call it “\( \chi^4 \),” then the location of \( \emptyset \) along that curve is given by some \( \chi^4 \in \mathbb{F} \). In other words, if the neighborhood of \( \chi^4 \) around \( \mathcal{H}_k \) is parameterized as \( \mathbb{R}_{\chi^4} \), the higher level of aleph on the far side of \( \emptyset \) is a neighborhood of greater fractional distance \( \mathbb{R}_{\chi'} \) such that \( \chi' > \chi^4 \). We have demonstrated a highly nuanced framework of analysis surrounding the requirement that \( \mathbb{R}_{\chi^4} \) and \( \mathbb{R}_{\chi'} \) be enumerable with successive integers as are, for instance, \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) [2]. In this parameterization of the path between two \( \mathcal{H} \)-branes, \( \emptyset \) becomes a sort of topological obstruction because \( \chi^4 \in \mathbb{F} \) is a non-arithmatic number. In this way, \( \emptyset \) is similar to the topological obstruction at \( \mathcal{H} \) where \( \chi^4_+ \) are not defined. The obstruction at \( \mathcal{H} \) is important for separating the KK theories in \( \Sigma^\pm \) and this other obstruction serves the same purpose. This likeness of the \( \mathcal{H} \)- and \( \emptyset \)-branes is reflected in the representa-

\[ \text{The subscript on } k \text{ identifies the wavenumber as a tuple with some value on each level of aleph.} \]
tions of the unit cell dislocated at one or the other, as in Figure 1. One of the main
difference is that a matching condition on $\Sigma^\pm$ at $H$ ensures continuity but there is
an expected discontinuity at $\emptyset$ in the form of a phase shift, a time reversal, or some
such thing which remains to be more precisely determined.

The likeness between $H$ and $\emptyset$ in the unit cell reflects the likeness between $\hat{0}$ and
$\infty$ in fractional distance analysis. In Reference [58], the existence of the neighborhood
of infinity was invoked through a non-contradiction operator $\hat{N}_{\text{CP}}$. The argument in
that line is that if there exists a Euclidean line segment $AB$ with a chart $x \in \mathbb{R}^+$, then
for every $x=b$ near $A$ (which is implicitly $x=\hat{0}+b$) there must exist an $x=\infty-b$ near
$B$. This follows from the invariance of $AB$ under the permutations of its endpoints.
In the fuller analysis of Reference [2], the neighborhood of infinity was shown to
exist as a consequence of Hilbert’s discarded axiom. In that analysis, intermediate
neighborhoods of infinity were demonstrated: for every $x=b<n \in \mathbb{N}$, there exists an
uncountable infinity of other $x=\mathbb{R}_X \pm b$ such that $X \in (0,1)$. Each $X$ specifies some
neighborhood of fractional distance: all numbers of the form $x=\mathbb{R}_X \pm b$ are such that
$x/c_\infty = X$. Two successive neighborhoods of fractional distance\(^1\) are separated by a
non-arithmatic number $x \in \mathbb{F}$.

One idea requiring further development is that we should replace the expected time
evolved trajectory across an infinite number of unit cells parameterized by $\chi^4 \in \mathbb{R}$
(the total cosmology of the cosmological lattice) with an infinite number of them
parameterized by $\chi^4 \in \mathbb{T}$ where $\mathbb{T}$ is the transfinite continuation of $\mathbb{R}$. In the way that
adding two points at $\pm\infty$ closes $\mathbb{R}$ as $\mathbb{R}$, $\mathbb{T}$ adds those points and then concatenates
infinite copies of $\mathbb{R}$ joined on the infinities which are easily labeled as $\infty = \aleph_1$, $2\infty = \aleph_2$,
$3\infty = \aleph_3$, etc. This scheme has several potential benefits. Rather than placing the
$\emptyset$-branes at $\mathcal{F}(n) \in \mathbb{F}$, we put them at $\aleph_n \in \mathbb{T}$. This sidesteps the paradox related
to the integer enumeration of the elements of $\mathbb{F}$. Every non-arithmatic number along
the smooth parameter $\chi^4$ is relegated to the bulk of $\Sigma^\pm$ where it is less likely to
directly interfere in the functioning of $\hat{M}^3$ to execute $H \rightarrow \Omega \rightarrow A \rightarrow H$. Contrary
to the lack of arithmetic defined for $x \in \mathbb{F}$, we have already defined the complete
system of arithmetic for $x = n\infty$ when $n \in \mathbb{N}$ [2]. The scheme by which one would
execute the parameterization $\chi^4 \in \mathbb{T}$ is outlined in Figures 3-6. Going beyond infinity
is not allowed in real mathematical analysis but neither is going onto the complex
plane as is already standard in physics. Going beyond infinity is only the longitudinal
continuation of $\mathbb{R}$ in the way that going onto $\mathbb{C}$ is the transverse continuation.

\(^1\)The issue of sequential neighborhoods of fractional distance is non-trivial because it implies the countability of
an uncountable set. As a corollary, the existence of a least positive real number is implied. The paradox is avoided,
however, by a certain scheme related to levels of aleph and the embedding $\mathbb{R} \subset \mathbb{T}$. See Reference [2] for fuller details.
Figure 3: The structure of $\mathbb{R}$ as developed in Reference [2]. The number $\aleph_{X}$ is the midpoint of the interval $\mathbb{R}_{X}$ consisting of all numbers having fractional distance $X$ with respect to infinity. If $x \in \mathbb{R}_{X}$, then $x/\infty = X$, as in Reference [2]. $F_{X}$ is an immeasurable (non-arithmatic) number separating the neighborhood of fractional distance $X$ from the next greater neighborhood of fractional distance labeled $\mathbb{R}_{Y}$. The main property of non-arithmatic numbers is that arithmetic is not defined for them in the usual way. A secondary property is that the non-arithmetics function as the naturals on a higher level of aleph. There, the naturals’ arithmetic is the basis from which all other arithmetic operations are extended, as per usual. The existence of the naturals is assumed before defining $\mathbb{R}$, and they are output again as the non-arithmatics used to define arithmetic for transfinite number systems in turn.

Figure 4: This figure shows the real number line separated only between the neighborhood of the origin $\mathbb{R}_{0}$ and the maximal neighborhood of infinity $\mathbb{R}_{1}$. In the objects of Figure 3, the neighborhood of the origin $\mathbb{R}_{0}$ terminates at $F_{0}$. Since it is not possible to do arithmetic with non-arithmatic numbers [2], we should introduce some coordinate chart that ignores the intermediate neighborhoods of fractional distance. We propose to introduce a coordinate transformation such that, for instance, every $x = b \in \mathbb{R}_{0}$ is associated with some $x' = \infty - b \in \mathbb{R}$. In the scheme where $\Omega$ and $\mathcal{A}$ are collocated with $\varnothing$, an intractable $x \in F_{0}$ at the $\mathcal{H} \rightarrow \Omega$ step of $M^{3}$ is made tractable by a coordinate transformation to $x' = \infty$. Arithmetic, and by extension calculus, is well defined for $\infty$. This change coordinates must be the one associated with the the $\hat{e}_{\mu}$ ontological basis, as in Subsection 1.3.
Figure 5: Even levels of aleph are sewn together with odd levels, and vice versa. To avoid the identification of one particular unit cell as being on the level of aleph containing the absolute origin, which is consistent with the relativistic principle of no absolute rest frame, one would associate the $\hat{\Phi}^0$ level of aleph with some intermediate $F_n$. While the fractional distance analysis in Reference [2] used the Euclidean metric on $\mathbb{R}$ such that $\text{len} \mathbb{R}_X = \text{len} \mathbb{R}_Y$ for any $\mathcal{X}, \mathcal{Y} > 0$, the MCM idea for levels of aleph is such that $\mathcal{X} < \mathcal{Y}$ implies $\text{len} \mathbb{R}_X < \text{len} \mathbb{R}_Y$. In fact, the ratio of the scale of successive unit cells is expected to be at least infinite. This mechanism and its details require further clarifications.

Figure 6: This figure shows the parameterization scheme rewritten in terms of MCM objects. With the understanding that $\Omega$ and $\mathcal{A}$ are collocated at $\emptyset$, or with the understanding that they must be measurably displaced from it, smooth affine parameterization will fail when $\emptyset$ is located at $F_0$ because the expression $F_k \pm \Delta \chi^4$ is undefined. The coordinate stitching of Figure 5 places $\emptyset$ at $\infty$ such that we obtain defined expressions in the form $k \infty \pm \Delta \chi^4$. The span of each $\Sigma^+$ or $\Sigma^-$ is doubly charted relative to an origin of coordinates in the labeled branes to its left and right. It remains to develop the language by which $\hat{\Phi}^k$ will reduce the coordinates of the $\mathcal{O}^{(k)}$ neighborhood of infinity down to the usual lab coordinates in the neighborhood of the origin.
In the pure mathematical analysis of fractional distance appearing in Reference [2], the metric along $\mathbb{R}$ was taken as the Euclidean metric. However, the application in the MCM for successive levels of aleph to exist on different scales requires that the metric be such that $\text{len} (\mathbb{R}_x) \neq \text{len} (\mathbb{R}_y)$. If the forward scale is greater, the diffusion of probability amplitude into the future rather than the past is like thermodynamic expansion into vacuum. If the forward scale is lesser, it is like gravitational attraction toward a singularity in the future. One mechanism or the other should be used to generate a physical basis for the observed arrow of time and the time asymmetry of the Schrödinger equation. The non-unitary character of $\hat{M}^3$

$$\hat{M}^3|\psi, \Phi\rangle = i\pi \Phi |\psi, \Phi^1\rangle \quad \Rightarrow \quad (\hat{M}^3)^\dagger \neq 1 , \quad (1.33)$$

is given to implement this change of scale. In addition to scale changes according to integer factors of $\infty$, the ratio of the scale of successive levels of aleph is irrational because the constant $i\pi \Phi$ is related to the change of scale. In part, this is supposed in part to address the Ford paradox (Section 75), and also in part for applications toward wave-particle duality (Subsection 1.8.)

1.6 Translation, Evolution, and the Schrödinger Equation

1.6.1 $\hat{M}^3$ as a Translation Operator

The usual quantum theory implements time evolution between measurements as diffusion (or oscillation) followed by collapse. The MCM supplements the usual theory with intermediate steps of translation to $\Omega$ and $A$ between measurements in $\mathcal{H}_k$ and $\mathcal{H}_{k+1}$. Therefore, given

$$\hat{M}^3|\psi, \Phi^k\rangle = \hat{M}_3 \hat{M}_2 \hat{M}_1 |\psi, \Phi^k\rangle = i\pi \Phi |\psi, \Phi^{k+1}\rangle , \quad (1.34)$$

one might take $\hat{M}_k$ as an ordinary translation operator $\hat{J}_k$ such that for $k \in \{+, -, \emptyset\}$ we would have

$$\hat{M}_k \equiv \hat{J}_k (\Delta \chi_k^4) = c_k \exp \left\{ \frac{-i\hat{p}_k \Delta \chi_k^4}{\hbar} \right\} , \quad \text{with} \quad \hat{p}_k = -i\hbar \partial_k . \quad (1.35)$$

In this way, the operator $\hat{M}^3 \equiv \hat{M}_- \hat{M}_\emptyset \hat{M}_+$ will send states across the unit cell as $\mathcal{H} \rightarrow \Omega \rightarrow A \rightarrow \mathcal{H}$ but there are a number of problems with $\hat{M}^3$ so defined. These deficiencies provide guidance toward a better analytical representation.

- The unit cell is such that for $\mathcal{H}$ located at $\lim \chi^4_{\pm} \rightarrow 0$, we have (presumably) $A$ at
\( \chi^4 = -1 \) and \( \Omega \) at \( \chi^4 = \Phi \). This allows us to define appropriate \( \hat{J} \) with \( \Delta \chi^4 = \Phi \) and \( \Delta \chi^4 = 1 \). (The latter is supplemented by an understanding that \( \Delta \chi^4 \) is defined according to the scale of the forward level of aleph.) However, the step \( \Omega \rightarrow A \) may be more like a time reversal or reflection operation than a translation operation. If \( \Omega \) is a black hole and \( A \) is a white hole connected by a zero distance wormhole (the case in which \( \Omega \) and \( A \) are collocated at \( \emptyset \) rather than bounding a region containing it), a reversal of the time arrow is probably all that is needed to execute \( \Omega \rightarrow A \). However, it is not yet determined whether \( A \) and \( \Omega \) bound the region containing \( \emptyset \) or if they are merely collocated there. (These cases are better described in Section 7.) Therefore, it is not clear that the \( \Omega \rightarrow A \) step involves any translation at all. If it does, simple translation cannot tell the whole story because the metric signature changes between \( \Sigma^\pm \). Waves (or heatlike solutions) cannot be simply transmitted through such an obstruction in the topology.

- With subscripts running over \( \{+, -, \emptyset\} \), one would assume \( [\hat{p}_j, \hat{p}_k] = 0 \), and that \( [\hat{M}_j, \hat{M}_k] = 0 \) as a consequence. If these operators commute, then we should be able to reorder them but that is not consistent with the overall idea. For instance, the \( \hat{M}_2 \) operator executing \( \Omega \rightarrow A \) should only act on states in \( \Omega \). It would not make sense for it to act on other states.

- \( \hat{J} \) executes equal-time parallel transport. (See Appendix B for a review of \( \hat{J} \).) Since the observation in \( \mathcal{H}_{k+1} \) necessarily takes place at some time \( t_{k+1} \) later than the \( t_k \) associated with the measurement in \( \mathcal{H}_k \), the translation operator alone is not sufficient to accomplish the task. The state \( \hat{M}^3|\psi; \hat{\Phi}^k\rangle = |\psi; \hat{\Phi}^{k+1}\rangle \) must show up in \( \mathcal{H}_{k+1} \) with a time that agrees with \( \hat{U}(t_{k+1}, t_k)|\psi, t_k\rangle = |\psi, t_{k+1}\rangle \). In other words, MCM time evolution must incorporate Schrödinger evolution as a simultaneous process during transit of the unit cell. Static transport by \( \hat{M} \propto \hat{J} \) cannot agree with experimental results. States in \( \mathcal{H}_{k+1} \) must be time evolved in \( x^0 \) with respect to states in \( \mathcal{H}_k \).

1.6.2 The Schrödinger Equation

The translation operation is a mathematical operation. State translation is nothing but a change of coordinates. The energy landscape has no role to play whereas everything that is properly physical must depend on an energy function. At minimum, a good equation for \( \hat{M}^3 \) must contain both time and space derivatives. The Schrödinger
equation

\[ \frac{i\hbar}{\partial t} |\psi, t\rangle = \hat{H} |\psi, t\rangle = \left[-\frac{\hbar^2}{2m} \nabla^2 + \hat{V}\right] |\psi, t\rangle, \quad (1.36) \]

provides an excellent template for what a physical equation looks like. The presence of time and space derivatives appear remedies the problem of equal-time parallel transport cited above. Time evolution is implemented as spatial motion much different than instantaneous mathematical translation. While the Schrödinger equation incorporates the requisite elements of physics lacking in Equation (1.35), it may or may not be sufficient for MCM evolution on its own. If it is, \( \hat{M}^3 \) will show up as a new energy term in \( \hat{V} \). If it isn’t, \( \hat{M}^3 \) will show up in the time derivative part. Keeping in mind that the time evolution operator satisfies the Schrödinger equation on its own—we may divide out the time-independent part of \( |\psi, t\rangle = \hat{U}(t, t_0)|\psi, t_0\rangle \) from Equation (1.36)—the initial supposition for MCM evolution was such that we might write a Schrödinger equation for \( \hat{U} \rightarrow \hat{\Upsilon} = \hat{U} + \hat{M}^3 \). The \( \hat{\Upsilon} \) operator is treated below.

In Reference [62], some mechanics for modifying the Schrödinger equation were laid out without an appeal to \( \hat{M}^3 \). This work relied on the additional \( \chi^4 \) variables generating new \( \partial_4 \) derivatives which were not available when \( \hat{U} \) and \( \hat{M}^3 \) were initially parsed in terms of \( \partial_x \) and \( \partial_t \). Mainly, it was suggested that the \( \partial_t \) Schrödinger operator might be replaced with \( \partial_4 \) so that integration would yield equations of motion in the \( \chi^4 \) direction. An energy function was introduced as well [62] (Section 68) but it referred to time arrow states rather than the position states most relevant to \( \mathcal{H} \rightarrow \Omega \rightarrow \mathcal{A} \rightarrow \mathcal{H} \). In the way that the Pauli equation is a step up in quantum structure from the Schrödinger equation for a scalar wavefunction, the modified Schrödinger equation for time arrow spinor states proposed in Reference [62] was analogue of the Pauli equation. The more fundamental problem regarding \( \hat{M}^3 \), however, asks what new scalar wave equation might we write. After writing it, the spinor structure developed in Reference [62] would be added in an intuitive way.

If we were going to directly modify the Schrödinger equation to use \( \hat{M}^3 \) without introducing a new equation altogether, which is the likely course since the Schrödinger equation must fall out of it as some simplifying limit, there are at least a few things we might try.
• A time gradient:

\[
\frac{\partial}{\partial t} \rightarrow \nabla_t = \frac{\partial}{\partial t} \mathbb{1} + \frac{\partial}{\partial \chi^4} \hat{\Phi}, \quad \text{where} \quad (\mathbb{1}, \hat{\Phi}) = (\hat{\Phi}^0, \hat{\Phi}^1) \quad (1.37)
\]

• Momentum in the $\chi^4$ direction:

\[
\nabla^2 \rightarrow \nabla^2 = \overline{\nabla^2} + \nabla^2_4 \quad (1.38)
\]

• A separable potential energy:

\[
\hat{H} \rightarrow \hat{H} = \hat{H}_0 + \hat{V}(\hat{x}, t) + \hat{H}_{\text{MCM}} \quad (1.39)
\]

• A non-separable potential energy:

\[
\hat{H} \rightarrow \hat{H} = \hat{H}_0 + \hat{V}(\hat{x}, \chi^4, t) \quad (1.40)
\]

1.6.3 A Time Gradient

The notion of a time gradient directly follows the program developed in Reference [62]: we supplement the Schrödinger equation’s usual derivative with respect to chronological time $x^0$ with another derivative with respect to the chirological time $\chi^4 \in \{\chi^4_+, \chi^4_-, \chi^4_\emptyset\}$. Writing the identity as $\mathbb{1} = \hat{\Phi}^0$ gives the gradient’s vector structure as a tuple whose components pertain to different levels of aleph: $\hat{\Phi}^0$ and $\hat{\Phi}^1$. How such components might be combined deserves further study. The meaning of $\partial_4$ also requires further clarification since $\chi^4$ parameterizing a hypothesized smooth curve through impassable topological obstructions at $\mathcal{H}$ and $\mathcal{S}$. At minimum, the metric signature changes between $\Sigma^\pm$. Waves and/or diffusive solutions can’t propagate smoothly through an interface where the number of timelike and spacelike dimensions changes. Adding a time gradient on the left side of the Schrödinger equations would require a modification of the Hamiltonian on the right side as well.

1.6.4 Momentum in the $\chi^4$ Direction

Adding momentum in the $\chi^4$ direction modifies at least the $\hat{H}_0$ part of the usual Hamiltonian. As with the $\partial_4$ operator in the time gradient, the meaning of $\nabla^2_4$ must be parsed in terms of $\{\chi^4_+, \chi^4_-, \chi^4_\emptyset\}$. Furthermore, KK theory requires that there does not exist any 5D matter-energy in the bulk volume of $\Sigma^\pm$ but classical kinetic energy associated with momentum in the $\chi^4_\pm$ directions is an apparent violation.
1.6.5 A New Potential Energy Function

In some sense, we expect that there should be some downhill energy condition which favors motion toward the right in the unit cell over motion toward the left. This more or less reflects the downhill energy condition by which it is proposed to generate the optical effect known as dark energy (Section 8.) Often one inserts terms like $U_{\text{grav}} = mgx$ into a classical Lagrangian to account for gravitational potential energy, and a similar separable energy term would fit bill for gravitational attraction toward the $\mathcal{H}_\infty$-brane. However, QM is such that there is no energy landscape in the Schrödinger equation’s Hamiltonian operator. Rather, the Schrödinger equation is not time symmetric and only one direction agrees with the experimental constraints. It remains to be determined, then, whether the quantum theory which “doesn’t know about time” in the sense of $x^0$ also doesn’t know about the chirological time $\chi^4$. $\chi^4_\pm$ are expected to be alternately timelike and spacelike in $\Sigma^\pm$ so there is some reason to assume the modified quantum mechanics would know about it, and that it would not. Furthermore, as an abstract dimension, $\chi^4$ may not be smoothly synthesized into the spacetime interval $(\Delta s)^2 = (c\Delta x^0)^2 - (\Delta x^i)^2$. As yet, we have no sense of a transposing parameter $c'$ that would convert $\chi^4$ into meters. Still, the entire concept of a unit cell invokes the concept of some regular, periodic potential energy function. If the modified QM is to know about chirological time, then it is likely that we should simply add a new term into the Hamiltonian. What that term might be when KK theory requires no matter-energy in the bulk between branes deserves further study. Specifically, we have no units for $\chi^4$ yet and the periodic function defining lattice structure may not be quantified in Joules. An example of a non-separable new energy function would be one where a dimensionless piece associated with the unit cell multiplies part (or all) of the existing Hamiltonian that is already written in units of $[\text{kg}] [\text{m}^2] / [\text{s}^2]$.

Since it is not yet decided whether the Schrödinger equation itself requires modification or if a modification of the Hamiltonian will suffice, this problem is predicated upon the resolution of more fundamental questions which remain open, at present. In each case of a potential modification, the modification ought to be associated with $\hat{M}^3$. The time gradient’s $\chi^4$ piece might take the exotic form $\partial_- \partial_\phi \partial_+\chi$, for instance. Since $\chi^4$ is alternately spacelike and timelike in $\Sigma^\pm$ whatever exotic structure we might attribute to the time gradient would also show up in the $\nabla^2_4$ part of the Laplacian. Adding a new potential energy term depending on a third derivative would also house an instance $\hat{M}^3$. Such a third derivative in the potential energy, even without modifying the $\partial_t$ on the LHS of the Schrödinger equation constitutes a significant
departure from the usual quantum theory.

1.6.6 A Cubic Function of the Velocity

Feynman wrote the following about the importance of limiting the action to be quadratic in the velocities [50].

\[ \text{"[T]he contribution } \Phi[x(t)] \text{ from a given path } x(t) \text{ [to the total probability amplitude functional for a path of motion] is proportional to } \exp(i/\hbar S[x(t)]), \]

where the action \( S[x(t)] = \int L(\dot{x}(t), x(t))dt \) is the time integral of the classical Lagrangian \( L(\dot{x}, x) \) taken along the path in question. The Lagrangian, which may be an explicit function of the time, is a function of position and velocity. If we suppose it to be a quadratic function of the velocities, we can show the mathematical equivalence of the postulates here and the more usual formulation of quantum mechanics."

Feynman mentions a requirement for limiting the action to a quadratic function of velocity several times in his spacetime formulation of quantum mechanics [50]. Feynman’s framework is equivalent to Dirac’s in which the cubic function of the velocity is not known to be equivalent to the existing theory. Instead, it would be a new theory whose details need to be worked out. Most generally, it is thought that third derivative terms can’t be quantized but that only pertains to the classical quantization procedure. A new procedure for quantization may be required. The canonical quantization step in doing QM is rightly labeled “this is where the magic happens” so the supposition that another magic trick may exist is neither radical nor unreasonable.

Velocity is a vector so its cube cannot be a scalar and that introduces a monkey wrench into the usual ways of doing physics. Furthermore, the cubic function of the velocity is a product of three first derivatives not equal to the linear function of a third derivative which have presumed to be the equation for \( \ddot{M}^3 \). Still, we might seek to write an equation for the third derivative as a cubic function of the first derivative. Firstly, a review of the known roles for \( \dddot{x} \) terms in physics is required, as in Section 92. The advanced and retarded EM potentials implementing retrocausal and causal effects respectively is so alike to the MCM cosmology, it must be assumed that the physics of one follows from the physics of the other. Once the context for \( \dddot{x} \) established in this research program by a review of very well known material, including Feynman’s own work in that line which he ultimately rejected due to some problems with vacuum polarization, we might then seek to forge a connection between
the third derivative and the cube of the first derivative. For example, Hooke’s law restricted to positive displacements is

\[ m\dddot{x} = kx \implies \dddot{x} = \frac{k}{m} \dot{x}. \] (1.41)

Giving the reduced spring constant \( k/m \) as a quadratic function of \( \dot{x} \) yields a requisite \( \dddot{x} = f(\dot{x}^3) \) term amenable to inclusion in \( L(x, \dot{x}) \). Such possibilities are mentioned only in passing for completeness of this survey of things which ought to be looked at. Although the Lagrangian (or Hamiltonian) is not a function of the \( \dddot{x} \) terms most directly associated with \( \hat{M}^3 \), such terms might be rewritten for inclusion in a new energy function associated with the energy landscape of the unit cell, even if that landscape is ultimately abstract and not dimensionful.

1.6.7 Total Evolution by \( \hat{\Upsilon} \)

The Schrödinger equation is known to be experimentally valid and \( \hat{M}^3 \) is meant to complement the unitary evolution operator rather than to replace it. However, the subsequent formulation of the unit cell [6], not to mention the demonstration of a gap in Bell’s theorem [64], gives us more freedom to formulate dynamics. \( \hat{\Upsilon} = \hat{U} + \hat{M}^3 \) was posed originally as a total evolution operator returning the fine structure constant as some characteristic value [24] (see Appendix A.) The idea was that \( \hat{U} \) is proportional to \( \partial_x \) and the new operator \( \hat{M}^3 \) should be proportional to the time derivative, \( \hat{U} \propto \partial_x \), and \( \hat{M}^3 \propto \partial_t \), (1.42)

because there was no other non-hidden variable which might serve as a new avenue for dynamical independence. The main difference between \( \hat{\Upsilon} \) and \( \hat{\alpha} \) is that the former uses a linear space derivative while the later contains the linear time derivative familiar from the Schrödinger equation. Since the fine structure constant is a dimensionless, bare value that we don’t appearing in a product of dimensionful constants, ’t Hooft writes the following [63].

“Powerful techniques were developed, enabling one to guess the right Schrödinger equation if one knows how things evolve classically, that is, in the old theories where quantum mechanics had not yet been incorporated. It all works magnificently well.”

There is always hope that a new guess will work as well as the old guess about classical quantization, i.e.: \( x \to \hat{x} \) and \( \vec{p} \to -i\hbar \vec{\nabla} \). On that, \( \hat{M}^3 \) is proposed to
complement the time evolution operator $\hat{U}$ [24] as
\[
\hat{\Upsilon} = \hat{U} + \hat{M}^3 .
\] (1.43)

This form is proposed so that a new total evolution equation will supplement the existing Schrödinger equation. Acknowledging such nuance as the implicit logarithm and square root needed to pull $\partial_x$ out of $\hat{U}$, namely
\[
\hat{U} \propto \partial_x \quad \iff \quad \partial_x = \sqrt{\frac{2m}{\imath \hbar (t - t_0)}} \ln \hat{U} ,
\] (1.44)

following from the time independent case of
\[
\hat{U}(t, t_0) = \exp \left\{ - \frac{i \hat{H}(t - t_0)}{\hbar} \right\} , \quad \text{with} \quad \hat{H} = \frac{-\hbar^2}{2m} \partial_x^2 ,
\] (1.45)

the operator $\hat{\Upsilon}$ deserves a more careful treatment. Indeed, the appearance of the $\chi^4$ variable in work subsequent to the first appearance of Equation (1.43) completely alters the road map of potential avenues to new physics. Plugging $\hat{\Upsilon}$ into the Schrödinger equation gives
\[
i \hbar \frac{\partial}{\partial t} (\hat{U} + \hat{M}^3) = \hat{H} \left( \hat{U} + \hat{M}^3 \right) .
\] (1.46)

which is separable as
\[
i \hbar \frac{\partial}{\partial t} \hat{U} = \hat{H} \hat{U} , \quad \text{and} \quad i \hbar \frac{\partial}{\partial t} \hat{M}^3 = \hat{H} \hat{M}^3 ,
\] (1.47)

since $\hat{U}$ is known to satisfy the Schrödinger equation on its own. Such nuance was offloaded in the original treatment without care, which is to say handwavingly, onto the $:= \propto$ relation replaced here with $\propto$. The equation on the right would make $\hat{M}^3$ more or less equal to $\hat{U}$, up to a possible constant of integration. This is not useful and, furthermore, $\hat{M}^3$ does not have an expected $x^0$ dependence resulting in a trivial relationship. Given $\chi^4$, we might phrase $\hat{\Upsilon}$ in the context of a time gradient with
\[
\left( i \hbar \frac{\partial}{\partial t} + \gamma \frac{\partial^3}{\partial \chi^3} \right) |\psi\rangle = \hat{H} |\psi\rangle ,
\] (1.48)

where $\hat{M}^3$ complements to the Schrödinger operator $i \hbar \partial_t$. Coupled with a new Hamiltonian, this form might be sufficient to give new equations of motion but the third derivative $\hat{M}^3$ part is mostly unmotivated. It is not consistent with the idea of a
gradient and would have to appear, pending a better understanding, as a postulate.
The Schrödinger equation itself is postulated, however, so the main problem would
be finding a Hamiltonian such that the equation is something other than nonsensical.

Under the naive operation of $\hat{M}^3$ as a translation operator, as in Equation (1.35),
we have

$$
\hat{\Upsilon} |\psi; t_0; \Phi_0\rangle = \hat{U}(t_1, t_0) |\psi; t_0; \Phi_0\rangle + \hat{M}^3 |\psi; t_0; \Phi_0\rangle
= |\psi; t_1, \Phi_0\rangle + c |\psi; t_0; \Phi_1\rangle .
$$

(1.49)

The MCM explanation for the wave-particle duality observed in the double slit exper-
iment (Subsection 1.8) supposes that wavefunctions in $\Phi^k$ cannot interfere with those
in $\Phi^j$ if $k \neq j$. If such states did interfere, then we would recover one coherent prob-
ability amplitude from which we might extract a component with time $t_1$ in $\mathcal{H}_1$, the
manifold associated with $\Phi_1$. However, an important MCM result requires that such
wavefunctions don’t interfere (Subsection 1.8.) Setting $\hat{M}^3$ as a simple translation
operator still does not work even in combined in $\hat{\Upsilon}$. Writing

$$
\hat{\Upsilon} = \hat{U} \hat{M}^3 ,
$$

(1.50)

gives

$$
\hat{U} \hat{M}^3 |\psi; t_0; \Phi_0\rangle = c |\psi; t_1; \Phi_1\rangle .
$$

(1.51)

This outputs a coherent amplitude with the correct $t$ and $\Phi$ specifiers. However,
$\hat{M}^3$ is still executing some form of equal-time parallel transport, albeit discontinu-
ously interrupted with a (presumably) commuting $\hat{U}$ operator. Due to the presumed
commutativity of the operators, the discontinuous time step can be implemented any-
where during the transit of the unit cell such that there is no difference, for instance,
between landing on $\Omega$ or $\mathcal{A}$ at $t_1$ or $t_2$.

Assuming that $\hat{M}^3$ is an exponential operator, one suitable feature of the $\hat{U} \hat{M}^3$
construction is that

$$
\hat{U} \hat{M}^3 = e^{i\hat{H}t/\hbar} e^{\hat{M}^3} = e^{\hat{\Upsilon}} ,
$$

(1.52)

allows us to take $\hat{M}^3$ as complementary to the generator of time translations $\hat{H}$ in
the form

$$
\hat{\Upsilon} = \ln \hat{U} + \hat{M}^3 \simeq \hat{H} + \hat{M}^3 \equiv \hat{H} + \hat{H}_{\text{MCM}} .
$$

(1.53)

In this case, $\hat{\Upsilon}$ is simply a new Hamiltonian. As a vector quantity, the square of
velocity can be a scalar related to the energy but the cube of the velocity is not
readily identifiable with an energy term. However, one would further consider cases
in which the $\hat{M}^3$ operator is not a third derivative, but is instead the product of three derivatives. As stated above, energy functions cubic in the velocity are an exception to the usual formulation formulation of quantum theory which is limited to terms quadratic in the velocity.

Among the options for modification listed above, the case of the separable Hamiltonian gives

$$\hat{H}_{\text{total}} = \hat{H} + \hat{H}_{\text{MCM}} .$$

(1.54)

This new $\hat{H}_{\text{total}}$ takes the place of what we have called $\hat{\Upsilon}$ previously. Plugging this into the time evolution operator yields

$$\hat{U}_{\text{total}}(t, 0) = e^{i\hat{H}_{\text{total}}t/\hbar} = e^{i\hat{H}_0t/\hbar} e^{i\hat{H}_{\text{MCM}}t/\hbar}.$$  (1.55)

Such a modification to the Hamiltonian lends itself directly to the $\hat{U}\hat{M}^3$ form. Equation (1.51) shows that this form is at least superficially more useful than the $\hat{U} + \hat{M}^3$ for because operation on $\psi$ yields the expected output: the state advanced in time and placed on the $H_2$-brane.

One likely deficiency of Equation (1.55) is that we have some interval of $x^0$ specified as $[0, t]$ but we have not included any interval of $\chi^4$. A more fitting description of the total time evolution operator, the one executing chronological and chirological evolution together, would be

$$\hat{U}_{\text{total}}(t, 0; \chi^4_0, 0) = e^{i\hat{H}_0t/\hbar} e^{i\hat{H}_{\text{MCM}}\chi^4_0/\hbar}.$$  (1.56)

Now that $\hat{M}^3$ does not depend on $x^0$, it is suggested that an alteration to the time derivative part of the Schrödinger equation is required as

$$\hat{\nabla}_t \hat{U}\hat{M}^3 = \hat{H}_{\text{total}} \hat{U}\hat{M}^3 ,$$  (1.57)

for some appropriate operator $\hat{\nabla}_t$. If the cubic term appears in $\hat{H}_{\text{MCM}}$, then we might use the linear gradient for $\hat{\nabla}_t$. This reflects the structure for a modified Schrödinger equation posed in Reference [62] and it keeps the cubic operator dependence by which the MCM is to radically diverge from the existing theory.

In Subsection 1.8, we will discuss a utility for using a fixed interval of abstract time $[0, \chi^4_0]$ between two measurement while the physical time interval $[0, t]$ varies. Complementing the fixed chirological time interval, the notion of a unit cell invokes a periodic energy landscape. While the $\hat{H}$ part of $\hat{H}_{\text{total}}$ shall vary reflecting various physical conditions, it is likely that only one $\hat{H}_{\text{MCM}}$ shall be used. The KK condition
that the Ricci tensor vanish everywhere in the bulk of $\Sigma^\pm$ is a strong constraint on what such an energy landscape might look like.

### 1.6.8 Time and Space Derivatives in Quantum Mechanics

Presently, $\partial_0$ is well defined but we have not fully specified $\partial_4$. The derivative with respect to $\chi^4_-$ is a time derivative and the derivative with respect to $\chi^4_+$ is a space derivative. Usually a time derivative in the classical Hamiltonian shows up in the quantum Hamiltonian operator as a space derivative, \textit{i.e.:} the momentum operator. So, in adding $\partial^3_4$ to the Schrödinger equation, we could add it as a time derivative on the left, modifying the total Schrödinger equation as a modified Schrödinger equation, or we could add it as a third spatial derivative in $\hat{H}$.

CONT:::

In brief consideration of the those details, the operator $\hat{M}^3$ should be compared to the several derivative operators which appear in the usual quantum theory. The $\partial_t$ in the Schrödinger equation gives the time dependence of operators or states, depending on the picture. However, $\partial_t$ also shows up in the classical $H = T + V$ formulation of the Hamiltonian. It is well known that

\[ T = \frac{1}{2} m |\dot{x}|^2 = \frac{1}{2} m (\partial_t \dot{x}) \cdot (\partial_t \dot{x}) = \frac{1}{2m} (m \partial_t \dot{x}) \cdot (m \partial_t \dot{x}) = \frac{1}{2m} \vec{p} \cdot \vec{p} \ , \quad (1.58) \]

is quantized as

\[ \hat{T} = \frac{1}{2m} \hat{\vec{p}} \cdot \hat{\vec{p}} = \frac{1}{2m} (-i\hbar \vec{\nabla}) (-i\hbar \vec{\nabla}) = -\frac{\hbar^2}{2m} \vec{\nabla}^2 \ . \quad (1.59) \]

As it is the intention to support the theory of negative time with the classical electromagnetic theory by which EM waves radiate from sources both forwards and backwards through time, a dynamic which involves the third time derivative of position, the $\hat{M}^3$ operator is likely to manifest itself as the operator-quantized representation of that term. A survey of classical electromagnetism is required to find the Hamiltonian of damped radiation in which such a term appears but for now it suffices to acknowledge that such a Hamiltonian exists. So, following along with the statements $\hat{U} \propto \partial_x$ and $\hat{M}^3 \propto \partial_t$, the $\partial_t$ classical momentum term quantized as $\partial_x$ suggests that the $\partial^3_4$ operator pertaining to the advanced and retarded potentials should be quantized as $\partial^3_4$. Eventually, it is a postulate of QM that the momentum operator takes its given form so there is no strict requirement that the operator representation of observable dependent on $\ddot{x}$ jerk terms would need to be linearly dependent on the momentum operator. Finally, while he have referenced the utility of the timelike variable $\chi^4_+ \in \Sigma^-$
for time evolutions, this operator representation gives some reasonable utility for the spacelike $\chi^+ \in \Sigma^+$.

Overall, one would consider such forms as

$$i\hbar \frac{\partial}{\partial x^0} \psi = (\hat{H} + \hat{H}_\text{MCM}) \psi,$$

(1.60)

where $\hat{H}$ is the usual Hamiltonian and $\hat{H}_\text{MCM}$ is as given above. Spinor structure would be imbued via the time arrow spinor Hamiltonian developed in Reference [62] (see Section 68.) Also, units must to be worked out: Equation (3.20) is not properly dimensionful. Since $\alpha_{\text{QED}}$ already has an interpretation as a ratio of two energies, the required dimensionalization seems like no far stretch of the imagination. (One interpretation for the fine structure constant is as the ratio of (i) the energy needed to close the distance between two electrons separated by distance $d$ to (ii) the energy of a photon with wavelength $2\pi d$.)

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NEED TO MENTION THAT THE AUGMENTED STATE SPACES ADD MORE OR LESS EXTRA STATES TO BE MIXED UP WITH UNDER DIFFUSION BY TIME EVOLUTION

1.7 The Fine Structure Constant

Dirac is quoted as saying the origin of $\alpha$ is, “the most important unsolved problem in physics,” and rightly so. Feynman wrote the following in the same vein [67].

“It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won’t recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to pi [as in $\alpha^{-1} = 2\pi + (\Phi\pi)^3$] or perhaps to the base of natural logarithms? Nobody knows. It’s one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the ‘hand of God’ wrote that number, and ‘we don’t know how He pushed his pencil.’ We know what kind of a dance to do experimentally to measure this number very accurately, but we don’t know what kind of dance to do on the computer to make this number come out,

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without putting it in secretly!"

Appendix A reviews the origin of $\hat{M}^3$ in Reference [24]. Briefly, $\hat{M}^3$ was introduced to generate the cubed term in

$$\alpha_{\text{MCM}}^{-1} = 2\pi + (\Phi\pi)^3,$$  \hfill (1.61)

After this initial algebraic construction of the fine structure constant, certain efforts were made toward finding geometric construction. Particularly, the role of a volume in some lattice of multiplicative periodicity (contrasting to the usual additive lattice periodicity [65]) seems like a promising avenue for future inquiry. Reference [65] briefly shows how one might obtain a power series in $\alpha$ if $\alpha$ can be associated with the volume of the unit cell. This has been supposed but not shown uniquely. Other subsequent proposals for the origin of $\alpha$ in the MCM, however, both in geometry and algebra, have not advanced the mechanism for $\hat{M}^3$ much beyond Equation (1.61). Therefore, we will not review them here or in Section 3 dedicated to the fine structure constant. Since $\hat{M}^3$ proposed for no other reason to generate Equation (1.61), we will quickly review the structure of that equation in the present section. We will propose a new mechanism which has not appeared in previous work: a new physical basis for $\alpha$ in the unit cell.

The best way to find a place for $\hat{\alpha}$ and/or its eigenstate would begin with the roles for the fine structure constant in existing theory, e.g.: the electron $g - 2$, the physics of the Josephson junction, Sommerfeld’s work regarding the fine structure splitting of atomic energy levels, etc. After identifying an established theoretical context, one would extract a catalog of inputs consistent with the output. The most obvious physical manifestation of the fine structure is as the ratio of two energies: the energy needed to close the distance $d$ between two electrons divided by the energy of a photon with wavelength $\lambda = 2\pi d$. In standard units, the ratio is

$$\alpha = \frac{E_{\text{ee}}}{E_\gamma} = \frac{\left(\frac{e^2}{4\pi\varepsilon_0 d}\right)}{\left(\frac{\hbar c}{\lambda}\right)} = \frac{\left(\frac{e^2}{4\pi\varepsilon_0 d}\right)}{\left(\frac{\hbar c}{\lambda}\right)} = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cdot \hfill (1.62)$$

This ratio is well suited to the physics of the unit cell when we take the two electrons and the photon as two $\mathcal{H}$-branes and the unit cell respectively. Following the conventions of the MCM scheme for fundamental particles [8], the $\mathcal{H}$-brane is a spin-1/2 quantum of spacetime spanned by $x^0$ and $x^i$. The spin-1 photon is assembled from
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Figure 7: MCM fundamental matter particles are quanta of spacetime spanned by \( x^i \) (space) and either of \( x^0 \) or \( \chi^4 \) (time). The three generations of matter particles are distinguished by the cases of \( A, \mathcal{H}, \Omega \). MCM fundamental bosons are constructed as connections of matter particles. This figure shows that the objects of the unit cell are easily parsed as two electrons and a photon. An elementary interpretation for \( \alpha \) is that it is the energy two electrons divided by the energy of a photon having a wavelength equal to the electrons’ separation. The arrangement of the unit cell in this figure emphasizes chronological continuity of \( x^0 \) between \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \). In this picture, one may simplify the instances of \( \mathcal{H} \) as two Lorentz frames in one Minkowski space.

Two such quanta arranged vertically so that the \( \mathcal{H}_1 \)-brane comes chronologically after the \( \mathcal{H}_0 \)-brane, as in Figure 7. This establishment of a ratio of energies of MCM objects gives a good starting point for future investigations into \( \alpha_{\text{MCM}} \) (Section 3). Furthermore, we might alter the MCM particle scheme so that the photon extends across the unit cell in the horizontal direction, as in the usual representation. All such avenues should be investigated.

The link between electromagnetism, special relativity, and quantum theory given by the inclusion of \( e, c, \) and \( h \) in Equation (1.62) is a tantalizing hint of some fundamental unification which has thus far escaped detection. That the physics of Equation (1.62) is so easily parsed in terms of the unit cell’s objects, as in Figure (7), is more evidence that the MCM and its theoretical or conceptual underpinnings should achieve the desired unification.
1.8 Wavefunction Collapse

1.8.1 Possibility for Retrocausality

A good and modern overview of the issues related to wavefunction collapse, particularly the interpretive issues regarding retrocausality, is found in Reference [68]. To paraphrase very briefly, Ellerman’s thesis is that Schrödinger’s cat is in an entangled superposition of life eigenstates while the box is closed, and that opening the box does not retrocausally affect the life or death of the cat during the time in which the box was closed. Rather, opening the box forces the collapse of the life/death superposition into one eigenstate or the other by placing a detector outside the box. Detectors are modeled in QM as operators which project quantum systems onto their eigenstates. If the box is opened at time $t$, then wavefunction is is collapsed only for times later than $t$. Ellerman contends, rightly, that the language of QM is not such that we may determine the life or death of the cat prior to the measurement. This writer’s main criticism is the lack of a caveat: Ellerman assumes that QM is the correct description of nature. He discounts the possibility that QM is merely a hack allowing us to predict experiments’ results. He does not contextualize the possibility that such weird effects as retrocausality may be objectively real. What is “real” or not is a matter of semantics but it remains true that there may exist a better description for reality than QM whose interpretation makes an appeal to retrocausality.

Even while agreeing with Ellerman regarding the interpretation of QM, is that it is not known what is inside the closed box. Not knowing what is inside is different that knowing that there is a superposition inside the box. If QM is correct, which we have good reason to believe, then we would know that the cat exists as a superposition until a detector projects the cat into one of its life eigenstates. So, the reader is encouraged to understand that opening the box, in fact, may retrocausally affect the life or death of the cat because ignorance of the cat’s state is not exactly knowledge that the state is an entangled superposition of eigenstates. That implication depends on an assumption that QM is more than just a hack for telling the results of experiments. Obviously, this writer’s opinion is that QM is exactly that: there probably does exist a better description than is given by QM. Whether or not such an alternative would preclude retrocausality is unknown.

The MCM assumes retrocausality, at least in part. The context is limited to the propagation of EM waves backward and forward in time from sources and the unit cell with its theoretical constructions does not immediately answer the question regarding the life or death of the cat prior to measurement. The context of retrocausality in the MCM is that the EM potential $A^\mu$ in $\mathcal{H}$ is a superposition of contributions from $A^\mu_{\pm}$ in
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\[ \Sigma^\pm, \text{ meaning that physics in the present is at least retrocausal from the abstract future } \chi_4^+ > 0. \] How that may or may not relate to objective chronological retrocausality from the Minkowskian future light cone in \( \mathcal{H} \) remains to be determined.

1.9 A Thought Experiment for Retrocausality

Consider a Schrödinger’s cat experiment in the presence of a time machine. A cat is placed inside a box with a radioactive isotope and a detector that will release a poison if the isotope decays. In the present experiment, the isotope is removed after a duration of time such that there is a 50% chance of the cat being poisoned. The isotope is removed automatically from the box at time \( t_0 \) and the box is opened at some time \( t_1 > t_0 \), and the cat observed to be alive or dead. Now, the observer uses a time machine to go back in time to time \( t \), and he opens the box at time \( t_0 < t < t_1 \). The isotope was already removed from the box at \( t_0 \) so the poison was either released or not before time \( t \). If wave function collapse does not have retrocausal effects, there should be a 50% chance of finding the cat either alive or dead despite the cat being found in one state or another at \( t_1 \). The theory of quantum mechanics predicts that the collapse of the cat’s wavefunction to the alive or dead eigenstate at \( t_1 \) should no effect on the probability for observing one state or the other at time \( t \), but the theory alone is not sufficient to determine the outcome of the measurement. It is possible that real time machine experiments would show that if the cat is observed to be alive or dead at \( t_1 \), then opening the box at any earlier time \( t_0 < t < t_1 \) would yield the same result. In that case, wavefunction collapse would have exactly the retrocausal effects denied in Reference [68]. Without doing the experiment, there is no way to know what the result would be.

1.9.1 The Collapse Problem

Aside from the issue regarding retrocausal effects emanating backward in time from measurements, the issue of collapse itself remains mysterious. How exactly does the detector put a superposition quantum state into an eigenstate? It is an axiom of QM, more or less, that observables are represented in the theory my Hermitian operators. Once that is established, mathematical collapse follows directly. However, the detector axiom is unsatisfying for a number of reasons, not the least of which is the instantaneous rate of collapse. An operator operates on a state at time \( t \) and the new state is instantaneously output. While the idea of superluminal wavefunction collapse lends itself to such MCM concepts as the abstract coordinates where the speed of light may not be a barrier as it is in the physical coordinates, it is disappointing that QM
provides no smooth equations of motion such that diffuse, unmeasured states evolve smoothly into sharp, measured states.

1.9.2 The MCM Double Slit Experiment

The MCM proposal to solve wavefunction collapse first appeared in Reference [53]. The mechanism proposes to encode the level of aleph into the wavefunction and then have wavefunctions on different levels of aleph interfere or not interfere in the way that monochromatic beams interfere according to \( \delta_{\lambda'\lambda} \): interference requires \( \lambda = \lambda' \). The ansatz for a modified plane wave solution is

\[
|\psi; \hat{\Phi}^k\rangle = \psi(x^i, x^0, \chi^4; \hat{\Phi}^k) = e^{i(k_ix^i - \omega x^0 + \beta \chi^4)} \hat{\Phi}^k,
\]

(1.63)

where we seek to generate a new interference condition with the \( \beta \chi^4 \) term. The MCM wavefunction is linear in \( \chi^4 \) and it reduces to the usual wavefunction in \( \mathcal{H} \) where \( \chi^4 = 0 \). \( \hat{\Phi}^k = 1 \) on the 0th level of aleph. We have unlimited freedom to change the equations we put states into but the states themselves are experimentally constrained, and Equation (1.63) is the ansatz. This form of \( \psi \) reduces to the usual wavefunction where \( \chi^4 = 0 \) but this value is undefined. \( \mathcal{H} \) acts as a topological obstruction between \( \Sigma^\pm \) principally because the abstract coordinates are not defined in \( \mathcal{H} \). Whether one might add \( \chi^4 = 0 \) later remains to be determined but it does not exist in the present iteration.

The main idea for wavefunction collapse is that the discontinuous process of collapse in \( \psi(x, t) \) can be made continuous in the abstract domain of \( \psi(x, t, \chi^4) \). By writing an analytically smooth evolution on \( \chi^4 \) into the collapsed state, it is likely that consequential inferences could be made regarding the behavior of \( \psi \) between measurements. It is not unreasonable to think that retrocausal effects might be inferred from the propagation to abstract future infinity (\( \Omega \)), and then to abstract past infinity (\( \mathcal{A} \)), before arriving at \( t_1 > t_0 \) in \( \mathcal{H} \). While the exact form of smooth evolutions on \( \chi^4 \) requires a determination of the equation for \( \hat{\mathcal{M}}^3 \), something foremost among the open questions listed in this paper, the general mechanism was given as follows [53].

"The [MCM] is a lattice multiverse theory and physical observations are made at \( \hat{\pi} \) sites[, also called \( \mathcal{H} \)-branes]. [T]here are three temporal regimes \( \{i_k, \hat{\Phi}_k, \pi_k\} \) associated with each moment [labeled] \( k \): the past, present and future. \( \hat{\mathcal{M}}^3 \) moves [states] from the present \( \hat{\Phi}_1 \) to the future \( \pi_1 \), to the past of the next moment \( i_2 \), and finally into the next moment itself \( \hat{\Phi}_2 \). The universe is holographically encoded on some unknown power of \( \pi \) which
functions as a cog in an open cosmological matrix.¹

\[
\hat{M}^3 | \psi; \hat{\Phi}_1 \rangle = \hat{M}_3 \hat{M}_2 \hat{M}_1 | \psi; \hat{\Phi}_1 \rangle \\
= \pi \hat{M}_2 \hat{M}_1 | \psi; \hat{\pi}_1 \rangle \\
= \Phi \pi \hat{M}_1 | \psi; \hat{i}_2 \rangle \\
= i \Phi \pi | \psi; \hat{\Phi}_2 \rangle .
\] (1.64)

"We may use this logic to define modes on the double slit apparatus seen in [Figure 8]. Require that a wavefunction in \( \hat{\Phi}_k \) cannot interfere with another wavefunction in \( \hat{\Phi}_j \) \([\text{when } k \neq j]\). Let \( t_0 \) be the time at the source, \( t_p \) at the plate, and \( t_s \) at the screen. Label the slits \( a \) and \( b \).

\[
\begin{align*}
\text{Waves} \rightarrow & \quad \begin{cases} 
\psi_a(t_0) \hat{\Phi}_1 \rightarrow \psi_a(t_p) \hat{\Phi}_1 \rightarrow \psi_a(t_s) \hat{\Phi}_2 \\
\psi_b(t_0) \hat{\Phi}_1 \rightarrow \psi_b(t_p) \hat{\Phi}_1 \rightarrow \psi_b(t_s) \hat{\Phi}_2
\end{cases} \\
\text{Particles} \rightarrow & \quad \begin{cases} 
\psi_a(t_0) \hat{\Phi}_1 \rightarrow \psi_a(t_p) \hat{\Phi}_2 \rightarrow \psi_a(t_s) \hat{\Phi}_3 \\
\psi_b(t_0) \hat{\Phi}_1 \rightarrow \psi_b(t_p) \hat{\Phi}_1 \rightarrow \psi_b(t_s) \hat{\Phi}_2
\end{cases}.
\end{align*}
\] (1.65)

"When [the observer] observes the particle at the source and screen only, the probability amplitude on the screen is wave-like. When [the observer] also checks the plate to see which slit the particle went through at \( t_p \), the amplitude on the screen is particulate. The usual culprit in this strange behavior is wavefunction collapse dependent on the particle’s choosing one slit or the other in response to [the observer’s measurement]. Here we assume all choices (if there truly are any) are made in the mind of an observer. This implies the particle must always go through both slits and never depend on observers. The interface of [the observer’s] mind with physical reality will define a unitarity-preserving boundary condition.² It is possible to formulate this in a mathematically natural way through reliance on the observer’s dynamical inclusion in the theory. An observation at the plate intersects the particle’s trajectory causing a bifurcation in its worldline [that] marks the intersection of [the observer’s] chronological and

¹Holographic encoding refers to the universe \( \mathcal{H} \) appearing in the MCM as a limiting surface between the \( \Sigma^\pm \) bulk spaces.

²As a non-unitary process, an MCM evolution operator would send a non-unit quantity of probability amplitude forward in time, of which exactly one unit will remain on the observer’s final level of aleph.
chirological worldlines.\textsuperscript{1} Chiros tracks the observer’s attention in some abstract space so worldlines intersect when [the observer’s] attention turns to the physical world to observe something[, thus implementing wavefunction collapse]. When observing a particle [, the observer’s] and the particle’s worldline[s] form a vertex at some $\hat{\Phi}$-site[, also called an $\mathcal{H}$-brane].”

\textbf{1.9.3 MAYBE MAKE A SECTION HERE}

\texttt{================ EDIT HERE =================}

NEED TO REVIEW CONDITIONS FOR BEAM INTERFERENCE!!!!!!!!

NEED TO DETAIL THE FREQUENCY MECHANISM FROM [53]

\texttt{================================}

It is known that out of phase beams and beams with different momenta don’t interfere, so we will attach a wavelength-analogue proportional to the level of the aleph to the wavefunction. This is the $\beta \chi^4$ part of Equation (1.63). Thus, by making an extra measurement at the screen in the double slit apparatus, one introduces an extra level of aleph which can be parlayed (hopefully) into an intuitive difference between the interfering and non-interfering wave and particle cases of wave-particle duality in the double slit experiment. The idea was originally stated as follows [53]. (Certain notation is revised to comply with the present convention.)

The problem which remains to actually develop an evolution operator, or discover the equations of motion, which give this mechanism in a dynamical way rather than

\textsuperscript{1}This is the intersection of $x^0$ and $\chi^4$ in $\mathcal{H}_k$.
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the ad hoc manner in which it is currently stated. If the observer preps a beam at \( \mathcal{H}_1 \) and then does not determine which slit the particle went through, the probability density on the screen will be wavelike because the wavefunction is coming through each slit on the same level of aleph. If the path is determined with a measurement at the screen, the wavefunction of the beam particle is projected into an eigenstate on a certain level of aleph. In this way, the mechanism differs from that described by Ellerman [68]. Ellerman describes the measurement at the screen as a projection operator acting on the superposition state to give a position eigenstate. In the MCM, the measurement operator separates the two superposed path eigenstates onto different levels of aleph.

1.9.4 The Single Slit Problem

When a wavefunction in position space is said to undergo state reduction, what is meant is that the probability amplitude collapses to a very narrowly peaked distribution from its formerly diffuse, unmeasured distribution of probability amplitude. For simplicity, we can say the narrow peak is a \( \delta \) function. Whether or not the eigenstate is a true \( \delta \) function or only the eigenstate of being located somewhere in the width of one slit or another is mostly immaterial because the time-evolved wavefunction observed on the screen will be identical in either case. Certainly, is it important for the MCM whether or not the state actually collapses to a \( \delta \) function because such states are more readily integrated into the physical scheme of general relativity being a theory of points in spacetime but the difference of one case or the other is mostly immaterial regarding what is observed in the double slit experiment.

Since we have introduced Hilbert space precisely to host position eigenstates, however, the double slit experiment is an appropriate place for further exposition. \( \delta \) functions belong to \( \Omega' \) but not the Hilbert space \( \mathcal{A}' \) or its subdomain \( \mathcal{H}' \). Whether or not a measurement is made at the slits, and considering a low particle flux through the screen, the particles show up on the screen as points, which we will assume are \( \delta \) functions. The scintillation mechanism has some finite expanse on the screen and a measurement of the scintillation point’s location will add another layer of uncertainty onto the particle’s final location so that a fuzzy state in Hilbert space is observed in \( \mathcal{H} \). However, since \( \chi^4 \) is an abstract dimension, and the wavefunction is an abstract object, we may suppose that it becomes a \( \delta \) function at some point in the motion. How such a state might arise under Schrödinger evolution is an open question.
1.9.5 An Application for the Theory of Negative Time

The theory of negative time gives at least a hint of a mechanism by which we might observe pointlike states at the end of periods of diffusive Schrödinger-like evolution. The Schrödinger equation exhibits time symmetry only under $t \rightarrow -t$ and $\hat{h} \rightarrow -\hat{h}$, so Schrödinger evolution by negative time will have the effect of collecting rather than diffusing the probability amplitude. If we evolve it to $t = -\infty$, the a position wave-function will invariably become a $\delta$ function. It has been the general understanding that the $\mathcal{H} \rightarrow \Omega$ step is an evolution to timelike infinity (in the abstract time at least), and then the $A \rightarrow \mathcal{H}$ step is like evolution from $t = -\infty$ to the time on $\mathcal{H}_{\text{next}}$. So, keeping in mind that the entire of concept of $\chi_4^-$ depends on it being reverse time with respect to $\chi_4^+$, we can associate some path with infinite reverse time evolution. On the preceding forward evolution to $t = \infty$, the probability density will have spread out to fill all of space and a naive reverse evolution will always result in a $\delta$ function at the origin. To agree with the experiment, however, the $\delta$ functions need to show up on the screen with at various points so that the statistics of large numbers replicates the wavelike or particulate interference pattern. Therefore, one might use the Schrödinger-derived probability density on the screen to probabilistically assign various displacements from the centerline to the screen such that the time reversed evolutionary step results in pointlike states appearing on the screen in the experimentally required distribution. So, now we have given a good reason to see the MCM’s RHS $\{\mathcal{H}, A', \Omega'\}$ as intimately related to the abstract infinities at the outer bounds of the unit cell. The end result of the infinite negative time step is not a state that can exist in $A'$. This mechanism addresses the main issue in the double slit experiment and also proposes to explain the more fundamental issue regarding why low-flux but wavelike beams show up on scintillation screens as quanta whether or not a measurement is made at the screen. Now we have given a good description of the MCM mechanism for wavefunction collapse. Works remains to formalize it.

1.9.6 The Non-Unitary Property of $\hat{M}^3$

Before moving on, the non-unitary character of $\hat{M}^3$ deserves a brief comment. In the original ideation, the operator was non-unitary and the probability amplitude would pass through an aperture of some sort on the way from $\Sigma_1^+$ to $\Sigma_2^-$. By the non-unitarity, the wavefunction passing through the aperture would be normalized to unity. In subsequent work, $\mathcal{O}$ has been implemented as a topological singularity. Any passage through it—ingress to a black hole along $\chi_4^+$ followed by egress from a white hole along $\chi_4^-$—will have all utilities which had been assigned to the aperture.
However, the black hole differs from the aperture in that all infalling probability amplitude is admitted/transmitted. The aperture reflects almost all infalling probability amplitude. Therefore, one would reevaluate the requirement that $\hat{M}^3$ must be non-unitary under the condition where $\emptyset$ transmits all infalling information rather than only an infinitesimal amount of it.

1.10 Quantum Gravity

There are a few equations which can be used to initiate the MCM rout to Einstein’s equation. Firstly, we suppose that the present is a superposition of positive and negative frequency modes, or that there exists some operator which takes a state in the present and decomposes it into contributions from the past and future.

\[
|t_+\rangle = |t_+\rangle + |t_-\rangle , \quad \text{and} \quad \widehat{LQC}|t_+\rangle = |t_+\rangle + |t_-\rangle .
\] (1.66)

The equation on the left reflects the usual superposition postulate of QM. The one on the right is more like the idea presented in previous section such that a measurement operation executed at the screen resolves the two eigenstates onto different levels of aleph. It is implicitly understood that $|t_+\rangle$ is a universe on a certain level of aleph with a given time arrow, and $|t_\pm\rangle$ are universes with forward and backward arrows of time on higher and lower levels of aleph.

The $\widehat{LQC}$ operator is non-trivial. In the lab, it is often observed that one particle decays to two particles, for instance

\[
\psi \rightarrow \chi + \phi ,
\] (1.67)

but there is not any mechanism within the existing quantum theory by which a which the $\psi(x)$ function of one variable might smoothly evolve into the $\psi' = \chi(y) + \phi(z)$ function of two variables. Isham writes the following [51].

“Consider a scattering experiment in which two particles collide and turn into three particles. Ignoring internal and spin quantum numbers, the initial and final states could be described by wavefunctions $\psi(x_1, x_2)$ and $\psi(x_1, x_2, x_3)$. However, it is by no means obvious what type of time-dependent Schrödinger equation could allow a function of two variables to evolve smoothly into a function of three variables.”

Since the coordinates of the $t_\pm$ universes are $x^\mu_\pm$ respectively, and we can expect that they will be found at two different values of $\chi^4_\pm$, the $\widehat{LQC}$ operator is of the sort needed to solve the problem posed by Isham. Something in excess of the Schrödinger
equation is required. In the current theory, QFT gives us tools to determine an
amplitude for a particle with $k_1^\mu$ to be found later as pair of particles with $k_2^\mu$ and $k_2^\mu$ respectively but the question of how we might get from here to there is not answered. Thus, we suppose that $\hat{LQC}$ is really $\hat{M}^3$ which operates as

$$\hat{M}^3|\psi; \hat{\Phi}_1\rangle = i\Phi \pi |\psi; \hat{\Phi}_2\rangle .$$

(1.68)

Using the tensor transformations defined in Subsection 1.3, we may rewrite the state on the right as

$$|\psi; \hat{\Phi}_2\rangle = -i\Phi |\psi; \hat{i}_2\rangle + \Phi \pi |\psi; \hat{\pi}_2\rangle .$$

(1.69)

This says that the physics in $\mathcal{H}_2$ is the sum of a causal contribution from $\mathcal{A}_2$ and a retrocausal contribution from $\Omega_2$. Coupled with Equation (1.68), the time advanced state in $\mathcal{H}_2$ becomes the sum of two things on the higher level of aleph.

The right side of Equation (1.69) is a restatement of $|t_\star\rangle = |t_+\rangle + |t_-\rangle$. This equation is very well motivated by a requirement for conservation of 4-momentum at a big bang. However, the $\hat{LQC}|t_\star\rangle = |t_+\rangle + |t_-\rangle$ which is rewritten in Equation (1.68) is conjured into existence by supposition after assuming the structure of the unit cell is the true structure of reality. To show that Equation (1.68) is Einstein’s equation, we must further suppose that $\hat{M}^3$ is a third derivative operator. Then, Einstein’s equation results from a rather long string of speculative ideas when usually the scientific method is such that one considers only one such idea at a time. Some of the motivations for entertaining such a speculative reach are given in the following Subsection 1.12. The main two reasons for seeing the resulting Einstein’s equation as more than a coincidence are that it is in the analytical form of, and in the same interpretative frame, as the equation for $\hat{LQC}$ around which the earliest MCM constructions were arranged. Secondly, the derivation relies on the $\Phi = 1 + \varphi$ property of the golden ratio which was also in usage before Einstein’s equation was found. Although algebra is such that any constant may be written as $\beta = 1 + (\beta - 1)$, this is property is uniquely golden for the $\Phi$ because we may preserve the necessary relationship across any level of aleph with $\Phi^{k+1} = \Phi^k + \Phi^{k-1}$.

So, by now we have come up with very many things to do with $\hat{M}^3$. It needs to modify the Schrödinger equation, possibly in total structural identity and at least in some new $\hat{H}_{\text{MCM}}$, it must function in some equation to return $\alpha_{\text{MCM}}$ and this may be a dimensionless truncation or modification of Schrödinger’s equation, or it could be another Hermitian operator in the way that observables always appear as the real-valued eigenvalues of Hermitian operators in QM. Now, $\hat{M}^3$ must also appear in
Equation (1.68) is we are to arrive by it at Einstein’s equation. As alluded to above, the many variations of \( \chi^4 \equiv \{ \chi_4^+, \chi_4^-, \chi_4^\pm \} \) give a lot more leeway to define permutations of \( \hat{M}^3 \) than was available when the operator was first supposed. Furthermore, the canonical quantization prescription such that \( m\partial_t x \rightarrow -i\hbar \partial_x \) also adds even more freedom to implement \( \hat{M}^3 \) in different ways. Although \( \chi^4 \pm \) are abstract and may not be properly spacelike and timelike in the sense of spacetime interval, their opposite topology in the \( \{ - + + - + \} \) metric signatures of \( \Sigma^\pm \) suggest that we might use on or the other of \( \chi^4 \) one each side of the quantization rule relating derivatives with respect to time and space.

To arrive at Einstein’s equation (to derive it) we must set \( \hat{M}^3 = \partial_0^3 \). Assuming wavefunctions in the form

\[
\psi(x, t, \chi^4) = \exp\{i(kx - \omega t + \beta \chi^4)\} ,
\]

(1.70)

(which reduce to the usual wavefunction in the \( \mathcal{H} \) limit of \( \chi^4 \rightarrow 0 \)), and using the properties of the golden ratio \( \Phi^{-1} = \varphi \) and \( \Phi = 1 + \varphi \), Equation (1.68) gives

\[
\begin{align*}
\partial_0^3|\psi; \hat{\pi}_1\rangle &= i\Phi \pi|\psi; \hat{\pi}_2\rangle \\
(i\omega)^3|\psi; \hat{\pi}_1\rangle &= i\pi|\psi; \hat{\pi}_2\rangle + i\varphi \pi|\psi; \hat{\pi}_2\rangle \\
-i(2\pi \nu)^3|\psi; \hat{\pi}_1\rangle &= i\varphi \pi^2|\psi; \hat{\Phi}_2\rangle - \varphi \pi^2|\psi; \hat{\pi}_2\rangle \\
8\pi^3 \nu^3|\psi; \hat{\pi}_1\rangle &= -\varphi \pi^2|\psi; \hat{\Phi}_2\rangle - i\varphi \pi^2|\psi; \hat{\pi}_2\rangle \\
8\pi \nu^3|\psi; \hat{\pi}_1\rangle &= -\varphi|\psi; \hat{\Phi}_2\rangle - i\varphi|\psi; \hat{\pi}_2\rangle .
\end{align*}
\]

(1.71)

On the right hand side of Equation (1.71), we must refer to the original MCM idea about how a big bounce has positive and negative time coming into it. This was developed into a requirement that each present moment is the sum of contributions from the past and the future. This is the main gist with taking the metric in \( \mathcal{H} \) as the simultaneous limit of vanishing curvature in \( \Sigma^\pm \). If the present is to receive a signal from the future, that is evocative of the advanced electromagnetic potential, which is in turn approximately the only place in classical physics that the \( \partial_0^3 \) operator appears. So, we rewrite the state on the left as a sum of contributions from the past and present.

Due to the constant \( 8\pi \) appearing at the end of Equation (1.71), and due to that coincidence alone, the resultant expression was recognized to be in the form of Einstein’s equation

\[
8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu} \Lambda .
\]

(1.72)
Introducing
\[ \nu^3 | \psi; \pi_1 \rangle \sim T_{\mu\nu} \]
\[ -\phi | \psi; \Phi_2 \rangle \sim G_{\mu\nu} \]
\[ i\phi | \psi; \pi_2 \rangle \sim g_{\mu\nu} \Lambda \]
(1.73)
immediately yields Einstein’s equation.

It remains to find a better reason to write Equation (1.68) in the first place, and then to replace \( \hat{M}^3 \) with \( \partial_0^3 \). The tensor transformation for changing the hats on MCM states are unitary and the \( \hat{M}^3 \) operator breaks unitarity by injecting new factors of \( i \), \( \Phi \), and \( \pi \) as states are pushed through the \( \hat{\imath} \)-, \( \hat{\Phi} \)-, and \( \hat{\pi} \)-sites corresponding to the \( A \)-, \( H \)-, and \( \Omega \)-branes of the unit cell. In this regard, \( \hat{M}^3 \) has little to nothing to do with \( x^0 \) so now we will suppose a new mechanism by which to arrive at Equation (1.71): one which has not appeared in previous work. Firstly, we will resolve \( \chi^4 \) as a literal time cube in the wavefunction by writing
\[ \psi(x, t, \chi^4) = \exp \{ i(kx - \omega t + \Phi \chi^4_\uparrow + \pi \chi^4_\downarrow + i\chi^4_\emptyset) \} . \]
(1.74)
Then we will suppose that the third derivative with respect to the chronological time is equal to the third derivative with respect to the chirological time \( \partial_0^3 \) written in the form \( \partial_+ \partial_0 \partial_- \). Up to an irrelevant factor of \( i \), this leads back to Equation (1.71).

A further case it to examine when \( \psi \) is not an eigenvector of \( \partial_4 \) is its variants, and that the \( \partial_4 \) operator advances the state from one brane the next with some necessary structure.

It sounds reasonable enough that the third derivatives should be equal. A physical basis for such an equation might invoke the third derivative radiation damping term which was fresh in this writer’s mind many years ago but now needs to be revisited as part of the work outlined in Section 92. Most broadly, gravitational radiation is an \( R_{AB} = 0 \) solution allowed by KK theory in the bulk of \( \Sigma^\pm \) and provides a good channel by which information might be transmitted from \( H_1 \) to \( H_2 \). If we require that the radiation is totally attenuated at \( H_2 \), that makes an appeal to the third-derivative damping of EM radiation whose extended mechanism must be studied for gravitational radiation. Another possible explanation for
\[ \partial_0 | \psi \rangle = i\Phi \pi | \psi \rangle \]
(1.75)
Next Steps and the Way Forward in the Modified Cosmological Model

might be found in the separate forward and backward difference formulas for the derivative:

\[
\begin{align*}
    f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}, \quad \text{and} \quad f'(x) = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x}, \\
\end{align*}
\]  

Referencing that \( \chi^4_{\pm} \) are expected to oppositely spacelike and timelike, one might set a boundary condition such that the third backward derivative with respect to \( \chi^4_{\pm} \) is equal to the third forward derivative with respect to \( \chi^4_{\mp} \) and then build some sufficient context around that. Again, some mechanism of radiation damping would be identified as a likely culprit requiring a match in the third derivative instead of the first derivative.

Overall, much work remains to formalize the MCM mechanism for quantum gravity, not the least of which is identifying the unstated maps relied upon in Equation (1.73). How exactly does \( \hat{M}^3 \) generate quantum gravity? We have shown that it can but have only offered several reasons for why it could without nailing down any formal equation for \( \hat{M}^3 \), or even a precise analytical form for it. The problem that remains, the problem which is the topic of this subsection, is as described by de la Madrid [41].

“Dirac’s bra-ket formalism was introduced by Dirac in his classic monograph. Since its inception, Dirac’s abstract algebraic model of bras and kets (from the bracket notation for the inner product) proved to be of great calculational value, although there were serious difficulties in finding a mathematical justification for the actual calculations within the Hilbert space, as Dirac and von Neumann themselves state in their books.”

A better mathematical justification for the mechanisms presented in this subsection is very much in order.

1.11 Local Gauge Symmetry

To say a little about the scheme for MCM quantum gravity, consider the standard model of particle physics’ group theoretical structure \( SU(3) \times SU(2) \times U(1) \). While there are variations on the standard model which are allowed, this algebraic structure seems to be enforced at the experimental level. The \( U(1) \) part describes, loosely, the oscillations of the EM field. We write \( \vec{E} \) and \( \vec{B} \) with sines and cosines, and they oscillate with \( 2\pi \) radians of freedom. \( U(1) \) is the circle group and, for EM, this circle pertains to the sinusoidal oscillation: at each point in space, the EM field has some \( U(1) \) phase. In quantum mechanics, the \( U(1) \) gauge symmetry allows us to make
changes like $\psi \rightarrow e^{i\lambda} \psi$ as long we make corresponding gauge transformations elsewhere in the theory. In this case, the U(1) circle group describes the $2\pi$ radian periodicity in the function $e^{i\lambda} = e^{i\lambda + 2\pi}$. The SU(2) weak theory is slightly more complicated. It adds a 2-sphere of coordinate freedom to each point in spacetime. The SU(3) strong force adds a 3-sphere to each point. The weak and strong forces are progressively more complicated than the EM force which adds a 1-sphere (a circle) to each point in spacetime. These degrees of freedom assigned to the points of spacetime are called local gauge symmetries. Following the MCM particle scheme in which gravitational manifolds are like elementary particles, one would link the dynamical quantum metric defined with four degrees of freedom to a new local QFT gauge symmetries. In other words, the internal coordinates associated with the gauge symmetries at a point on one level of aleph would be like the coordinates of a gravitational manifold on another level of aleph (see Section 90.) This reflects the different tack of the MCM in taking the quantum scale as higher than the gravitational scale. For instance, it has been the idea in the MCM to solve the problem of baryon symmetry by taking the whole universe as a single baryon. A baryon is an excitation of the baryon field at a point and we would like to associate four new degrees of local gauge freedom at that point with the four diagonal components of the universe’s metric.

Note how the MCM mechanism for quantum gravity starts with $\Phi^1$ and then decomposes it to $\Phi^0$ and $\Phi^{-1}$. In this way, the gravitational theory is resolved on the lower level of aleph. This is also like the idea to associate the metric with the internal coordinates of some scheme at least qualitatively akin to methods of local gauge symmetry. Also note that he we have the objects on two levels of aleph. In Reference [6], we wrote the metric in $H$ as a shared limit of two other metrics, but a mechanism for metric superposition was also supposed. More about this in Section 90.

INCREASING INFINITUDE— THE CHANGING LEVEL OF ALEPH IS GOOD FOR HAVING THE METRIC RELATED TO THE INTERNAL DEGREES OF FREEDOM AT A POINT. THEY ARE SMALL (BIG) TO BEGIN WITH, BUT THEN THEY ARE RESCALED.

1.12 A Large Enough Number of Coincidences

It is hoped that all of the material regarding $\hat{M}^3$ will pan out as planned. There is much evidence but strong signals sometimes fade as more data comes in. Even then, KK theory shone in the early 20th century, faded, and is not reformulated more or
less exactly as posed in the first place. There remains much hope but if the proof is in
the pudding, we have only presented a nice looking basket of ingredient for $\hat{M}^3$. If
$\hat{M}^3$ should never pan out, however, other results regarding the Riemann hypothesis,
classical electrogravity, and the fundamental problem of QFT will stand on their own.
The Riemann hypothesis pudding is done. It is superb, magnificent, and tasty. The
electrogravity pudding is done enough and it smells very good. Experimental data
to published eventually, unless the USA is able to permanently abort the ordinary
scientific proceedings, will confirm that the spectrum of MCM lattice vibrations is the
ture particle spectrum, or it won’t. To treat the facts rightly, however, it is possible
that the $\hat{M}^3$ issues developed in the present may be nothing but a high-sigma string
of coincidences. We call the string high-sigma because it is uncommon for a wrong
idea to produce so many positive leads. Detractors of this work will claim that any
nonsense can be cobbled together by dime-a-dozen crackpots to yield a similar catalog
of false leads. To the extent that the proof is in the pudding, such work with numerical
results, substantiated physical corollaries, and falsifiable yet unfalsified predictions
does not exist. Still, to be very objective in the telling, if $\hat{M}^3$ works out as planned,
it will come as the end product of a few different guesses. Long strings of guesses are
rare in physics and this contributes to what may be interpreted negatively, should one
choose to take such a position. Another position suggests that since the guesses have
led to such good results as a solution to the Riemann hypothesis, these particular
guesses ought to be held in higher esteem than ordinary guesses. In the hope and
belief that $\hat{M}^3$ will pan out, the purpose of this subsection is to review an unusually
large number of positive—yet inconclusive—results which have followed from $\hat{M}^3$.

Before proceeding, it must be emphasized that the original discovery of Einstein’s
equation in the MCM was not goal-seeked in any way. It was not sought. There was
no intention to find it. It was not recognized until it had already been written. After
discovering $2\pi + (\Phi \pi)^3 \approx 137$, the operator $\hat{M}^3$ was goal-seeked as a way to find a
reason to compute the desired number. When the dimensionless constant $8\pi$ from

\[ 8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda \]  \hspace{1cm} (1.77)

first appeared in 2012, the context had nothing to do with GR. Considering the
equation proposed for $\hat{M}^3$ to turn the crank on the unit cell

\[ \hat{M}^3|\psi; \Phi_1\rangle = i\Phi \pi|\psi; \Phi_2\rangle \]  \hspace{1cm} (1.78)

it was asked what would happen if $\hat{M}$ as a time derivative. The result followed the
form of Equation (1.71) leading to Equation (1.77): Einstein’s equation for general
relativity. With that now very clearly stated, it is acknowledged that the number of hypothesized and/or supposed inputs required to construct the mechanism is large enough to generate the superficial appearance that any sufficiently long string of suppositions can be used to output any desired result. The quantum gravity result was not desired, however. It fell out on its own from unrelated thinking.

The requisite number theoretical assignments for the $\{\hat{e}_A, \hat{e}_H, \hat{e}_\Omega\}$ set of basis vectors and the proposal to use the $\partial^3_t$ operator were independently entertained [24], and then found only later to output Einstein’s equation in synthesis [3]. When the dimensionless coefficient $8\pi$ familiar from $8\pi T_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda$ popped out of the assumed form for $\{\hat{e}_A, \hat{e}_H, \hat{e}_\Omega\}$, it popped out closely on the heels of another famous dimensionless constant: $\alpha_{\text{MCM}}$ [3, 24]. Furthermore, the emergence of Einstein’s equation as a formal statement of the bouncing relationship at the heart of the MCM was too much to be assigned as mere coincidence in the eyes of this writer. Writing

$$\hat{LQC}|\text{bounce}\rangle = |t_+\rangle + |t_-\rangle \ , \text{ and } 8\pi v^3|\psi; \hat{\pi}_1\rangle = -\varphi|\psi; \hat{\Phi}_2\rangle - i\varphi|\psi; \hat{i}_2\rangle ,$$

makes the tantalizing suggestion that MCM requirement for global cosmological momentum conservation is a restatement of the law already recorded as Einstein’s equation. The dimensionless $8\pi$ following so closely after the dimensionless $\alpha_{\text{MCM}}$ may be written of by third parties as mere coincidence but, as the personal project of this writer, the coincidence hypothesis is rejected on the ground of too much coincidence. To argue that the reader should find $\hat{M}^3$ deserving of further study, we will briefly summarize the evolution of the underlying ideas.

- The first idea in the MCM was that the bounce should decay to a superposition of positive and negative time modes.

$$\hat{LQC}|\text{bounce}\rangle = |t_+\rangle + |t_-\rangle .$$

There is no time arrow in the bounce itself. The absence of a time arrow is not a superposition of time arrows so the $\hat{LQC}$ operator augments what would otherwise be the usual additive superposition relationship $|\text{bounce}\rangle = |t_+\rangle + |t_-\rangle$. In QFT, often one constructs an arbitrary momentum state $|k\rangle$ by acting on $|0\rangle$ with a boost operator. We might give $\hat{LQC}$ that form, generally, or we might make an appeal to new physics. The current theory allows us only to compute the amplitude for finding a particle already decayed at some later time but there is no evolution equation by which the bounce state might evolve smoothly into two universes. In that regard, the bounce state is likely to exist only at one time,
and that evolution by an arbitrarily small time step with always result in the $t_{\pm}$ output states and forbid the notion of “smooth” evolution from one state to the other.

- Parallel transport of the Lorentz frame though the bounce requires that the present moment should also be resolved positive and negative time modes. If such a decomposition is possible at the bounce, then it should be possible at all other times. In this case, we arrive at
  \[ \mathcal{LQC}(t_*) = |t_+\rangle + |t_-\rangle, \]
  which makes an appeal to a continuous evolution from one to the other because the present is usually thought not to be a single moment. Even then, however, we have associated the MCM present $\mathcal{H}$ only with the times at which measurement forces wavefunction collapse. For the purposes detailing the evolution of ideas which led to the current framework of thinking, fixation on the bounce as a novel moment in time was supplanted by the present being taken as the novel moment of greatest interest.

- A labeling basis
  \[ \hat{\Phi} \equiv \hat{e}_\mathcal{H}, \quad \hat{i} \equiv \hat{e}_A, \quad \hat{\pi} \equiv \hat{e}_\Omega, \quad \text{and} \quad \hat{2} \equiv \hat{e}_\emptyset, \]
  was introduced to associate the usual $|\psi\rangle$ analytical formalism with the $\{t_*, t_+, t_-, \text{bounce}\}$ language:
  \[
  |\psi; \hat{\Phi}\rangle = \psi(x^i, x^0)
  \\
  |\psi; \hat{i}\rangle = \psi(x^-_i, x^0)
  \\
  |\psi; \hat{\pi}\rangle = \psi(x^+_i, x^0)
  \\
  |\psi; \hat{2}\rangle = \psi(x^i, x^0) .
  \]
  Originally, $\hat{\pi}$ was associated with $\hat{e}_\mathcal{H}$ but it was changed to $\hat{\Phi}$. By putting the hat on $\Phi^k$ we have $|\psi; \hat{\Phi}\rangle = |\psi\rangle \hat{\Phi}$. Here, $\hat{\Phi}^k$ marks the $k$th level of aleph, it does not appear in the analytical part of $|\psi\rangle$ where factors of $2$, $\pi$, and $i$ are ordinary but factors of $\Phi$ are not, and it reduces to the identity operator $\hat{\Phi}^0 = 1$ on the 0th level of aleph. This form of the $\hat{e}_\mu$ basis was called the ontological basis in reference to an intention to explain certain fundamental qualities of nature with the number theoretical assignments. In particular, by attaching exponents
to mark successive unit cells in a bulk lattice, we might address the hierarchy problem which asks where very large and small numbers come from in physics. Usually the linear independence of a set of basis vectors is all that matters but in the ontological case we have taken the independence and the given norms as important, specified properties. Future work may explore a further change $\hat{2} \rightarrow \hat{\tau}$ with $\tau = 2\pi$.

- It was determined that the numbers in the chosen basis can be used to construct the *dimensionless quantum electrodynamic coupling constant* $\alpha_{\text{MCM}}$ which disagrees with $\alpha_{\text{QED}}$ by about 0.4%.

$$2\pi + \left(\Phi\pi\right)^3 \approx 137.$$  

While some will cite the notion that $\alpha_{\text{QED}}$ is known to an accuracy far exceeding the 0.4% discrepancy with $\alpha_{\text{MCM}}$, Section 3 discusses the conditions by which 0.4% cannot rule out $\alpha_{\text{MCM}}^{-1} \approx 137$ from its given context.

- The classical theory of advanced and retarded electromagnetic potentials is such that electromagnetic radiation comes into the present causally from the past and retrocausally from the future. Also, this is theory is of of very few places in physics where the third time derivative of position contributes to the equations of motion (see Section 92.) Therefore, the supposition of a third power quantum operator $\hat{M}^3$ is well reasoned in the given context: physics in the present is determined as a limit of positive and negative time modes coming from the past and future. The past may dominate over the future due to non-unitary level-of-aleph effects while leaving non-zero retrocausal contributions to explain why we can’t put a coffee mug back together once it has been broken.

- Following the novel result regarding $\alpha_{\text{MCM}}$, the proposed operation for $\hat{M}^3$ yielded a second novel numerical result: the *dimensionless constant* $8\pi$ well known from Einstein’s equation

$$8\pi T_{\mu \nu} = G_{\mu \nu} + g_{\mu \nu}\Lambda.$$  

The third derivative had already been under consideration, as had the number theoretical basis $\{\hat{\pi}, \hat{i}, \hat{\Phi}\}$ by which this equation was derived in Subsection 1.10. There was no intention beforehand to show anything related to GR. These numerical results are interpreted as strong evidence that $\hat{\Phi}$ is useful for physics. Einstein’s equation was the second time a famous dimensionless coupling con-
stant popped out of the ontological numbers. The appearance of one such number is easily written off as meaningless coincidence, but two such numbers are written off less easily.

- Following the initial derivation of Einstein’s equation, a third famous dimensionless coupling constant popped out with the addition of $\hat{\sigma} = \hat{\Phi} = \hat{\pi} = \hat{\phi} = \hat{2}$.

$$\hat{1} = \frac{1}{4\pi} \hat{\pi} - \frac{\phi}{4} \hat{\Phi} + \frac{1}{8} \hat{\phi} - \frac{i}{4} \hat{\pi}.$$

In certain natural units, $4\pi$ is the dimensionless constant attached to the Poisson equations for Newtonian gravity and classical electromagnetism.

$$\rho = \frac{1}{4\pi} \nabla^2 \phi, \quad \text{and} \quad J^\mu = \frac{1}{4\pi} \eta^\mu_{\nu\lambda} \partial_\nu \partial_\lambda A^\lambda.$$

It is hoped that this equation will have vast applications toward unifying disparate areas of physics in one total ontological resolution of the identity. Here, ontology refers to the number theoretical properties of $\{\hat{\pi}, \hat{\phi}, \hat{\Phi}, \hat{2}\}$ pertaining to the fundamental qualities of nature.

### 1.13 A New Equation

The real work in GR was coming up with the equivalence principle.

The real work in developing general relativity was Einstein coming up with the equivalence principle. After he got the idea worked out, it took him several years to get it reduced to an equation. He had to work with a lot of collaborators to get it too. I understand Grossmann was extremely helpful for that. In my own work, I have principle: the three-fold process. The nitty-gritty of getting it reduced to an equation is not something I have devoted much time to. However, as with Einstein, the real "work" of the physics was thinking about how experiments are done and then formulating a new principle.

Due to the single time derivative in the Schrödinger equation, the dynamics are not time symmetric. Probability amplitude diffuses like heat in forward time but not in reverse time. To the contrary, waves can move to the left or right in time given positive or negative $k^0$ because the wave equation uses two time derivatives. Somehow, this must be reconciled with the reversal of the time direction at $\emptyset$. Noting that

$$\partial_0 e^{ik_\mu x^\mu} \propto i, \quad \text{and} \quad \partial_0^3 e^{ik_\mu x^\mu} \propto -i,$$

(1.80)
one imagines a place for the third derivative in the modification of the Schrödinger equation needed for total MCM evolution. Since the Schrödinger equation is not symmetric under time reversal, time reversal in QM requires $t \rightarrow -t$ and the complex conjugation operation $i \rightarrow -i$. So, one imagines that adding the third time derivative into the differential equation of motion would balance something out. Determining what that something might be is part of the work outlined here. All of this generally relates to

"The fundamental idea is that the ‘negative energy’ states represent the states of electrons moving backward in time [sic] reversing the direction of proper time $s$ amounts to the same as reversing the sign of the charge so that the electron moving backward in time would look like a positron moving forward in time."

This is already the idea we are using for the reverse time universe. If an electron moving backward in time looks like a positron moving forward in time, then an electron moving forward in time in the $t_+$ universe is like a positron moving forward in the $t_-$ universe???????????? NEED TO CLEAR THIS UP.

The problem in EM is that electromagnetic waves can leave sources in either direction through time. If the wavefunction of a particle is collapse with a measurement in some localized region of space, there is no probability for finding the particle at another point in space at earlier time... or is there?

MAX ACTION CARRIES A LINEAR SCHRODINGER SYSTEM IN A CLASSICAL QUADRATIC SYSTEM —¿ MAKES THREE DERIVATIVES

MAX ACTION

While state reduction is a property of all state vectors, those written in continuous and discrete eigenbases, the problem of propagation from $\mathcal{H}_1$ to $\mathcal{H}_2$ is geometric and refers to a representation by continuous position eigenstates. At the end of $\hat{M}^3$, the wavefunction needs to collapse onto $\mathcal{H}_2$ before we can begin to speak of collapsing to some narrowly peaked spike in the lab frame, as is usually the case for the collapse of a wavefunction. One consideration in that regard is that the path from $\mathcal{H}_1$ to $\mathcal{H}_2$ is expected to be the path which maximizes the action. Any motion within the universe $\mathcal{H}$ is associated with some finite action, so it follows that path that leaves $\mathcal{H}$ to transit the unit cell before returning to it is the path of maximum action. The correspondence principle tells us that quantum equations of motion should reduce to their classical counterparts when the action is large compared to $\hbar$. Sometimes this is stated as the limit where $\hbar \rightarrow 0$. In either case, we should expect that the maximum
action path across the unit cell is effectively classical and that, as such, there is no issue getting the wavefunction to collapse onto $H_2$. The issue that remains pertains to the diffuse wavefunction that has undergone classical transport.

In closing this section, one further possible interpretation for a third derivative operator is briefly discussed. Since classical equations of motion are second order in time, and since quantum motions are first order, one might seek to implement the classical motion of a quantum system across the abstract bulk of the unit cell. In this picture, $\dot{\psi}$ replaces the classical position $x$.

2 Elliptic Curves

The MCM equations of motions, whatever they are, should be third order functions of two variables: chronos $x^0$ and chiros $\chi^4$. Elliptic curves are also third order functions of two variables. The elliptic curves in Figure 9 are of the form

$$y^2 = x^3 + ax + b$$

but the most general form is slightly more complicated. Since we have supposed that the equation for $\hat{M}^3$ is a third order differential equation in two variables, we might suppose that its auxiliary equation is an elliptic curve and search for solutions accordingly. One would attempt to reverse engineer an equation for $\hat{M}^3$ by looking for differential equations whose auxiliary equations are elliptic curves.

Figure 9 shows a remarkable semblance between the behavior of elliptic curves and two charts of the behavior of the Riemann $\zeta$ function (RZF) near the north pole (coordinate singularity) of the Riemann sphere. These plots of $\zeta$ specifically show the behavior of $\zeta$ in the neighborhood of infinity. The existence of such numbers are motivated by defining the positive branch of the real number line on a Euclidean line segment $AB$ and then invoking the invariance of the line segment under the permutations of its end points [58]. The number $n \in \mathbb{N} \subset \mathbb{R}$ is implicitly given as a cut in the real number line $n$ basis units to the right of the origin. We can state this explicitly as $n = \hat{0} + n$. Numbers in the neighborhood of infinity must exist because we may cut the line $n$ unit away from either endpoint $A$ or $B$. If the origin is attached to $A$ and infinity is attached to $B$, the permutation of the labels of the endpoints send the number $\hat{0} + n$ in the neighborhood of the origin to the number $\hat{\infty} + n$ in the neighborhood of infinity. Likewise, the coordinate axes mark the origin $\hat{0}$ in Figures 9(a) and 9(b), but the crossed axes in Figures 9(c) and 9(d) mark the
Figure 9: Two elliptic curves and two figures showing the behavior of the Riemann \( \zeta \) function around the north pole of the Riemann sphere. The left-right asymmetry of the given curves are qualitatively very similar to the given figures. (a) The curve \( y^2 = x^3 - x \). (b) The curve \( y^2 = x^3 - x + 1 \). (c) Taken from Reference [57] wherein the negation of the Riemann hypothesis was laid out in principle. (d) Taken from Reference [58], one of a few papers in which independent formal negations of the Riemann hypothesis are given.

one-point compactified infinity \( \infty \).

In Reference [61], a structure for extended complex analysis was developed by which successively larger neighborhood of fractional distance are resolved around the coordinate singularity of the Riemann sphere. Each successive neighborhood of fractional distance is to be associated with what was level of aleph in earlier work.\(^1\) As in Section 37, it was suggested that this scheme of complex analysis might provide new insight into the issue of the Yang–Mills mass gap. Specifically, Reference [61] sought to develop a method for connecting adjacent levels of aleph by separating the origins of the real and imaginary axes of the complex plane which is not the coordinate system shown in any of Figure 9’s subfigures. However, since the general idea is that the infinity of one coordinate system is like the zero of another, it is intriguing that

\(^1\)The reliance on the Hebrew character \( \aleph \) in Reference motivated the change of notation for what is now labeled \( \mathcal{A} \) in the unit cell.
behavior of elliptic curves near $\hat{0}$ is at least qualitatively similar to the behavior of $\zeta$ near $\infty$.

To emphasize that which is most intriguing, and to work toward dispelling a suggestion that the similar quality is meaningless in the way that certain qualia pertaining to $\hat{M}^3$ are said to be meaningless, consider the following statement of the Birch and Swinnerton-Dyer conjecture [69].

“Mathematicians have always been fascinated by the problem of describing all solutions in whole numbers $x, y, z$ to algebraic equations like

$$x^2 + y^2 = z^2.$$ 

Euclid gave the complete solution for that equation, but for more complicated equations this becomes extremely difficult. Indeed, in 1970 Yu. V. Matiyasevich showed that Hilbert’s tenth problem is unsolvable, i.e., there is no general method for determining when such equations have a solution in whole numbers. But in special cases one can hope to say something. When the solutions are the points of an abelian variety, the Birch and Swinnerton-Dyer conjecture asserts that the size of the group of rational points is related to the behavior of an associated zeta function $\zeta(s)$ near the point $s = 1$. In particular this amazing conjecture asserts that if $\zeta(1)$ is equal to 0, then there are an infinite number of rational points (solutions), and conversely, if $\zeta(1)$ is not equal to 0, then there is only a finite number of such points.”

So, it is known that there exists at least one famous problem of interest relating elliptic curves to $\zeta$ functions, of which the RZF is one. Therefore, a condition of maximal irrelevance which detractors might cite for the likeness in Figure 9 is not the true condition. As to what the true condition might be, the formal statement of the Birch and Swinnerton-Dyer conjecture certainly exceeds this writer’s ability to understand [70]. In that regard, Johnson writes the following [71].

“There is no doubt that elliptic curves are amongst the most closely and widely studied objects in mathematics today. The arithmetic complexity of these particular curves is absolutely astonishing, so it isn’t surprising the Birch and Swinnerton-Dyer conjecture has been honored with a place amongst the Clay Mathematics Institute’s famous Millennium Prize Problems. Although some great unsolved problems carry the benefit of simplicity in statement, this conjecture is not one of them. There even seems to be an aura of ‘hardness’ over the problem that keeps many from discovering the
true beauty of the conjecture. [sic] The Birch and Swinnerton-Dyer conjecture today remains, of course, unsolved and most mathematicians agree that new ideas will need to be developed to tackle the great problem. A proof will take a great deal of work and mathematical power.”

The present problem regarding the elliptic curve application as the auxiliary equation for $M^3$’s equation requires a survey of some large volume of number theory. The work would far exceed the ordinary scope of a PhD problem. This writer may get around to such a study eventually as it seems so keenly poignant but readers who are already well acquainted with number theory at the requisite level might undertake the inquiry suggested here.

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STACK QUESTION
NEEDED: General Method of Characteristics for Third-Order Equations
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3 The Fine Structure Constant

The most obvious physical manifestation of the fine structure is as the ratio of two energies: the energy needed to close the distance $d$ between two electrons divided by the energy of a photon with wavelength $\lambda = 2\pi d$. In standard units, the ratio is

$$\alpha = \frac{E_{ee}}{E_{\gamma}} = \left(\frac{\frac{e^2}{4\pi\varepsilon_0 d}}{\frac{hc}{\lambda}}\right) = \left(\frac{\frac{e^2}{4\pi\varepsilon_0 d}}{\frac{hc}{\lambda}}\right) = \frac{e^2}{4\pi\varepsilon_0 \hbar c}.$$  \hspace{1cm} (3.1)

In either case, we have a time independent Schrödinger equation

$$\hat{H}\left|\psi_{\gamma}\right\rangle = E_{\gamma}\left|\psi_{\gamma}\right\rangle, \quad \text{and} \quad \hat{H}\left|\psi_{ee}\right\rangle = E_{ee}\left|\psi_{ee}\right\rangle.$$  \hspace{1cm} (3.2)

The electrons are multiparticle function of two spatial variables and the photon is a single particle state of one spatial variable. We have the intention for the MCM operator to resolve one type of state into another. Now we will consider the electrons and photon in terms of the MCM particle scheme. An electron is a $x^0x^i$ spacetime, and the photon is the union of two such quanta of spacetime. Letting $M^3$ be the
operator such that
\[
\hat{M}^3 |x^0 x^0\rangle = \frac{1}{\sqrt{2}} \left( |x^0 x^3\rangle_1 + |x^0 x^i\rangle_2 \right) ,
\] (3.3)

CHECK IF SYMMETRIC OR ANTISYMMETRIC FOR IDENTICAL PARTICLES
then we just add some $2\pi$ associated with a branch cut and a changing level of aleph, and then we get the fine structure constant.

In Leighton’s discussion [72] of the de Vries formula [73] for the fine structure constant (FSC), the low Kolmogorov complexity is cited as a “critical characteristic” lending support to the formula as a result which may be of interest. Low Kolmogorov complexity is mathematical elegance and this writer is in total concordance. Mathematical elegance coupled to philosophical validity is a bright flashing sign pointing the way to physical truth, most of the time. $\alpha_{\text{MCM}}$ has much lower Kolmogorov complexity than does the de Vries formula. The de Vries formula agrees with $\alpha_{\text{QED}}$ far better than does the MCM formula—that discrepancy is treated below—but it offers no physical mechanism for the meaning of the calculation. On the other hand, the MCM offers a pretty good explanation for the origin of the $\alpha_{\text{MCM}}$ which agrees fairly well with $\alpha_{\text{QED}}$. While many suppositions for a geometrical place for $\alpha$ have appeared in the MCM, here will focus on the original structure reliant on $\hat{M}^3$ [3,24]. In Section 1 it was mentioned that $\alpha_{\text{MCM}}$ remains hard to motivate but the material in Section 2 makes it somewhat easier.

Following the method of characteristics for first-order PDEs given in Section 2, we found that
\[
y = x + A , \quad \text{is characteristic of} \quad - \frac{\partial}{\partial y} u(x,y) = \frac{\partial}{\partial x} u(x,y) .
\] (3.4)

It seems reasonable to suppose that
\[
y^2 = x_3 + x ,
\] (3.5)
is a characteristic curve of the third-order equation (up to some constants)

\[ -\frac{\partial^2}{\partial y^2} u(x, y) = \frac{\partial^3}{\partial x^3} u(x, y) + \frac{\partial}{\partial x} u(x, y) \]  

This is almost exactly the form of the PDE

\[ \hat{\Upsilon} |\psi\rangle = \hat{M}^3 |\psi\rangle + \hat{U} |\psi\rangle , \]

supposed to generate \( \alpha_{\text{MCM}} \) in Reference [24]. A little more added complexity is needed since \( \psi(x^i; x^0, \chi^4) \) is function of three variables, but one is presented with the quality of everything seeming like it should work out exactly as originally stated. The second derivative is evocative of a Hamiltonian operator one might expect a PDE in the form

\[ -\frac{\partial^2}{\partial x_i^2} \psi = \frac{\partial^3}{\partial x_i \partial x_j \partial x_k} \psi + \frac{\partial}{\partial x_0} \psi , \]

or similar. The statement of a problem regarding the FSC is that such details ought to be worked out. Perhaps instead of being a value returned by an operator as in

\[ \hat{M}^3 |\psi\rangle = (\Phi \pi)^3 \] \( \hat{U} |\psi\rangle = 2\pi \]  \[ \Longrightarrow \hat{\Upsilon} |\psi\rangle \approx 137 , \]

\( \alpha_{\text{MCM}} \) would arise as a some characteristic value of the PDE rather than coming from \( \psi \) itself. In that case, the first guess cooked up for Reference [24] would need to be changed slightly. Additionally, we might implement a change of notation \( \hat{\Upsilon} = \hat{\Xi}^2 \) for consistency in the differential operator convention. Feynman wrote the following emphasizing that \( \alpha \) is really the square of a more elementary number [50].

"It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won’t recognize this number, because they like to remember it as the inverse of its square: about 137.03597[.)"]

Since we need to write the wavefunction of a universe to implement the quantum gravity condition, we can think about writing a simultaneous equation for the wave function of the universe and a state within the universe. It is needed to put both parts into an equation to promote the metric of space from an undynamical background and such an equation would work well.

If \( \hat{\Upsilon}^2 = \hat{M}^3 + \hat{U} \) is the equation for \( \hat{M}^3 \), then what is the other equation that we
have attached to Einstein’s equation? We may distinguish between such equations as
\[ -i\hbar \partial_t \psi = \hat{H} \psi, \quad \text{and} \quad \hat{H} \psi = E \psi. \] (3.10)

Also, we have a direct method for altering the Schrödinger equation. If the Schrödinger equation is written as

\[ \hat{\Upsilon} = \hat{\chi}^4 \]

then \( \hat{\Upsilon} \) is the sum of the usual Hamiltonian and the Hamiltonian related to \( \chi^4 \). We can separate that part out as

\[ \hat{\Upsilon} = 2\pi \]

However, if \( \hat{U} \) is the ordinary Schrödinger equation, we need some reason for it to return \( 2\pi \).

In addition to questions about \( \hat{\Upsilon} \) (or \( \hat{\Xi}^2 \)) raised above, there exists the issue of the discrepancy between \( \alpha_{\text{MCM}} \) and \( \alpha_{\text{QED}} \).

"Not only the complex structure of unconventional order parameters have an impact on the Josephson effects, but also may profoundly alter the quasi-particle excitation spectrum near a junction."

"A microscopic mechanism that induces the time-reversal symmetry broken state is elusive and should be investigated theoretically."

From: Evidence for time-reversal symmetry breaking of the superconducting state near twin-boundary interfaces in FeSe

Schwinger’s famous calculation will shed light on the place for \( \hat{M}^3 \).

Electron \( g = 2 \)

NON SI UNITS

\[ \hat{M}^3 \] was introduced to generate the \( (\Phi \pi)^3 \) part of \( \alpha_{\text{MCM}} \). All of the context for \( \hat{M}^3 \) was reverse engineered from this requirement. To avoid any ambiguity related to the
cases for $\hat{\Upsilon} = \hat{U} + \hat{M}^3$ discussed in Section 1, here we will consider
\[ \hat{\alpha}^{-1} = \partial_a + \partial_b^3, \] (3.11)
where the $a$ and $b$ derivatives generate the requisite numerical factors as required. In the original treatment predating the $\chi$ variables [24], we had $a = x$ and $b = t$. Prominent issues with the original formulation were as follows.

- The well known state $\psi$ of a quantum particle in a 2D infinite square well (see Appendix A) was employed as
\[ \hat{\alpha}^{-1}|\psi\rangle = (\partial_x + \partial_t^3)|\psi\rangle = \alpha_{\text{MCM}}^{-1}|\psi\rangle. \] (3.12)
The tick mark showing that $\psi$ is not an eigenstate of $\hat{\alpha}^{-1}$ was omitted in Reference [24]. However, the state of a 2D particle in a box spanned by $x$ and $t$ in respective dimensions $L$ and $D$ (length and duration) is manifestly not an eigenstate of $\hat{\alpha}$. The state is
\[ \psi_{nm} = \frac{2}{\sqrt{LD}} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi t}{D}\right). \] (3.13)
Since $\alpha$ is observable, the general axiomatic framework of QM suggests that a Hermitian operator $\hat{\alpha}$ should return $\alpha^{-1}$ when acting on its eigenvector with eigenvalue $\alpha^{-1}$. The given $\psi_{nm}$ fails to satisfy an eigenvalue equation with $\hat{\alpha}^{-1}$ as defined.

- $\alpha^{-1}$ is returned only upon choosing $L = 1/2$ and $D = 2\varphi L$. The fixed dimensions of the box are associated easily enough with the fixed abstract dimensions of the unit cell, in theory, but no explanation for this ratio has been proposed.

- $\alpha^{-1}$ is returned only as the $nm = 11$ eigenvalue of $\psi_{nm}$ but there is no ready interpretation for the other $nm$ eigenvalues. $\alpha^{-1}$ should be returned by some ontological, or unique, eigenstate without an associated spectrum of other values for $n$ and $m$.

- While the single space derivative in $\hat{\alpha}^{-1} = \partial_x + \partial_t^3$ was natural to the 2D box model given in Reference [24], the full theory would have three space derivatives. With the fine structure constant being rooted historically in 3D atomic physics, the context of one spatial dimension must be generalized in the full theory. It seems likely that this generalization would alter the $2\pi$ which is uniquely associated with $\alpha_{\text{MCM}}$. 

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In the statement \( \hat{\Upsilon} = \hat{U} + \hat{M}^3 \), the relationship between \( \hat{U} \) and \( \partial_x \) given by Equation (1.44) reflects only the simplest case of a time independent Hamiltonian. This \( \partial_x = f(\hat{U}) \) already seems too complicated and the generalization to a time-dependent Hamiltonian would be an analytical mess. So, even while it is reasonable to think that \( \alpha_{\text{MCM}} \) would be associated with a \( V=0 \) free-particle Hamiltonian, it may not. If not, it does not seem likely that the relevant Hamiltonian would be time independent, at least in the sense of chirological time, and is questionable whether or not the Hamiltonian would commute at unequal times. Backing \( \partial_x \) out of the Dyson series representation of a Hamiltonian such that \( [\hat{H}(t_0), \hat{H}(t_1)] \neq 0 \) is probably not possible.

The two electrons are like two \( \mathcal{H} \)-branes and the unit cell is like a photon. What is the identity operator \( \mathbb{1} \)? It is simply the number one, it is the butterfly operator in its discrete and continuous incarnations

\[
\mathbb{1} = \sum_k |a_k\rangle\langle a_k|, \quad \text{and} \quad \mathbb{1} = \int_{-\infty}^{\infty} dx |x\rangle\langle x|,
\]

it is the identity matrix in arbitrary dimensions, \textit{etc}. If we want to return the dimensionless fine structure constant, dimensional analysis of

\[
i\hbar \frac{\partial}{\partial x} \mathbb{1} = \hat{H} \mathbb{1},
\]

suggest we should look at the inverse unit of energy.

Regarding the \( 2\pi \) factor, notice that

\[
\hat{p} \mathbb{1} = \hat{p}^x = -i\hbar \partial_x (e^{2\pi i} x/\hbar) = 2\pi (e^{2\pi i} x/\hbar) = 2\pi \mathbb{1}.
\]

The momentum operator acting on the identity returns the required value \( 2\pi \). Here, the identity \( \mathbb{1} \) has functioned as the momentum eigenstate corresponding to \( p = 2\pi \). While \( p = 2\pi \) casts the momentum eigenstate as the identity operator, any momentum eigenstate can be rewritten as the identity operator [54]. This appears as

\[
e^{ipx/\hbar} = e^{ipx/\hbar(2\pi/2\pi)} = (e^{2\pi i})^{xp/2\pi \hbar} = \mathbb{1}^{xp/2\pi \hbar} = \mathbb{1}.
\]

This curious artifact of the theory is a little remarked upon feature inherited when it is taken as a postulate that the momentum operator is \( \hat{p} = -i\hbar \partial_x \).
So where we have proposed that \( \hat{\Upsilon} \) should operate with \( \partial_x \) and \( \partial_t \) before splitting \( t \) into \( x^0 \) and \( \chi^4 \), we might also do this with the Schrödinger operator

\[
i\hbar \partial_t \mathbb{1} = -2\pi \left( e^{2\pi i/\hbar} \right)^t = -2\pi . \tag{3.18}
\]

Therefore, one might write the equation for \( \alpha_{\text{MCM}} \) as the Schrödinger equation for the identity operator. This is a robust philosophical context for the origin of the most important dimensionless constant in quantum theory. Specifically, consider

\[
i\hbar \partial_t \mathbb{1} = \hat{H}_{\text{MCM}} \mathbb{1} = (\hat{M}^3 - \alpha_{\text{MCM}}) \mathbb{1} . \tag{3.19}
\]

In Reference [24] (see Appendix A), the functioning of \( \hat{\Upsilon} \) pertained to a 2D box with sides \( L \) and \( D \) in proportion \( 2L = \Phi D \). Letting \( \hbar = 1 \) and \( \hat{M}^3 = (ih)^3 \partial^3 \), a simple extension to dimensional transposing between \( x^0 \) and \( \chi^4 \) in the form \( 2x^0 = \Phi \chi^4 \) gives

\[
i \left( \partial_{x^0} \right) = \left( i^3 \frac{\partial^3}{\partial \chi^4} - \alpha_{\text{MCM}} \right) \mathbb{1}
\]

\[
i \left( \frac{\partial}{\partial x^0} \right) e^{2\pi i x^0} = \left( -i \frac{\partial^3}{\partial \chi^4} - \alpha_{\text{MCM}} \right) (e^{2\pi i} \Phi \chi^4 / 2)
\]

\[
i (2\pi i) e^{2\pi i x^0} = \left[ -i \left( -i \Phi^3 \pi^3 \right) - \alpha_{\text{MCM}} \right] (e^{2\pi i} \Phi \chi^4 / 2)
\]

\[
-2\pi \mathbb{1} = \left( (\Phi \pi)^3 - \alpha_{\text{MCM}} \right) \mathbb{1}
\]

\[
2\pi + (\Phi \pi)^3 = \alpha_{\text{MCM}} .
\]

Mention the problem of time from Wikipedia

This is an equivalent statement of \( \hat{\Upsilon} = \partial_x + \partial^3 \) written as the Schrödinger equation for the identity operator, and in different variables. The dimensional transposing \( 2x^0 = \Phi \chi^4 \) makes the derivative on the right equivalent to \( \partial^3 \) and the transposing \( 2x^i = \Phi \chi^4 \) would set the equation more normally in the form of the Schrödinger equation. Here, \( \alpha \) is a dimensionless zero point energy and the effective Hamiltonian is is third order in \( \partial_x \). This is more good evidence that the MCM is contains new ideas and is not a simple rehashing of the existing material in search of mode of arbitrage.

Make distinction between an operator that returns an eigenvalue and some an evolution operator that has some characteristic value in some "ground state" limit

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4 The Planck Law

This problem regards one of the most exciting features of the MCM mechanism for quantum gravity. As stated in Section 1, we introduce some transformations

\[
\nu^3 |\psi; \pi\rangle \sim T_{\mu\nu} \\
i |\psi; \Phi\rangle \sim G_{\mu\nu} \\
|\psi; \hat{\pi}\rangle \sim g_{\mu\nu} \Lambda,
\]

and the equation for $\dot{M}_3$ becomes Einstein’s equation. In this prescription, the stress-energy tensor $T_{\mu\nu}$ contains an exotic $\nu^3$ frequency dependence. Physics’ foremost setting for $\nu^3$ is the Planck law

\[
B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\frac{\nu}{ekT} - 1},
\]

for blackbody radiation. $B$ is called the spectral radiance and it is a measure of how much energy is carried by each wavelength or frequency of black body photons. Written it terms of the frequency, we have

\[
B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\frac{hc}{\lambda kT} - 1}.
\]

So, Planck’s law gives the energy density at any slice of the EM frequency spectrum and $\nu^3 |\psi; \pi\rangle$ is taken as the energy at some slice of $\chi^4$ written relativistic language as the stress-energy tensor $T_{\mu\nu}$. Also, the state $\psi$ itself must depend on frequency and the Planck law contains one more instance of $\nu$ in the denominator of its statistical part. Following on the tails of of the previous insistences that something is definitely going regarding a very large number of hints, and a stark absence of any hints to the contrary, this problem is more well-suited to immediate analysis. If $|\psi\rangle$ is a Dirac bispinor, then $|\psi, \pi\rangle$ is most likely a 4×4 array (Section sec:REPS) and the operation casting it as a stress-energy should not exceed left and right multiplication by some other 4×4 arrays.

The main guidance for this problem is that a connection of $\nu^3 |\psi; \pi\rangle$ to the Planck law must also be such that the appropriate form is obtained when $\psi$ is written in terms of wavelength. Following Equations (1.63) and (??), and naively assuming the massless dispersion relation $\omega(\lambda) = 2\pi c\lambda^{-1}$, we have

\[
|\psi\rangle = \exp\{i (k_i x^i - 2\pi c\lambda^{-1} x^0 + \beta \chi^4)\},
\]
and
\[ \partial_0^3 |\psi; \hat{\pi}_1\rangle = 8i\pi^3 c^3 \lambda^{-3} |\psi; \hat{\pi}_1\rangle . \]  
(4.5)

This does not agree with Equation (4.3.) Evidently, a dispersion relation in the form
\[ \omega(\lambda) \propto \lambda^{-5} \]  
(4.6)
is required to force the equation for $\hat{M}^3$ into compliance with the Planck law. One notes that the dispersion relation of a massless photon is probably not the correct relation for the wavefunction of a massive universe.

Since the Planck law describes photons, there has to be some difference. Particularly, $\omega \propto \lambda^{-\frac{5}{3}}$ will be such that it preserves the $\lambda^{-5}$ but it will not preserve $\lambda^{-1}$ in the statistical part. Furthermore, the requirement for $\lambda^{-5}$ follows from the photonic dispersion relation. Fundamentally, energy is attached to frequency, however. We often speak of the energy of a quantum oscillator but rarely do we speak of the energy of a standing wave. For this reason, the likeness to $B(\nu, T)$ would be said to force a change in the $B(\lambda, T)$ analogue rather than vice versa. Measurements of time, especially on the quantum scale, are for more precise than measurements of length.

It seems likely that the requirement for a dispersion relation proportional to $\lambda^{-\frac{5}{3}}$ could be parlayed in to some observable prediction. The Planck law also depends on the temperature $T$ and it is not unreasonable to think the requisite dispersion relation should depend on $T$ or its MCM analogue.

An integral starting at $\lambda = 0$m begins where the energy diverges but an integral starting at $\nu = 0$hz is such that the energy vanishes at the origin.

PLANCK LAW FOR GRAVITONS????????

5 Vacuum Energy

In practice, the Planck law must be integrated over some spectral band because mathematically exact frequency or wavelength is not observable. For this reason, Planck’s law are given in terms of $\nu$ cubed but the fifth inverse power of $\lambda$. Th antiderivatives $\nu^4$ and $\lambda^{-4}$ obey the on-shell dispersion relation for photons $\lambda \nu = c$.

In quantum field theory, the energy density of the vacuum is infinite but in the Planck law, the energy contained in each wavelength is an infinitesimal amount rep-
resented by a differential form. We would like to combine the two get a finite vacuum energy density.

Quantum field theory states that all fundamental fields, such as the electromagnetic field, must be quantized at each and every point in space. A field in physics may be envisioned as if space were filled with interconnected vibrating balls and springs, and the strength of the field is like the displacement of a ball from its rest position. The theory requires "vibrations" in, or more accurately changes in the strength of, such a field to propagate as per the appropriate wave equation for the particular field in question. The second quantization of quantum field theory requires that each such ball–spring combination be quantized, that is, that the strength of the field be quantized at each point in space. Canonically, if the field at each point in space is a simple harmonic oscillator, its quantization places a quantum harmonic oscillator at each point. Excitations of the field correspond to the elementary particles of particle physics. Thus, according to the theory, even the vacuum has a vastly complex structure and all calculations of quantum field theory must be made in relation to this model of the vacuum.

The theory considers vacuum to implicitly have the same properties as a particle, such as spin or polarization in the case of light, energy, and so on. According to the theory, most of these properties cancel out on average leaving the vacuum empty in the literal sense of the word. One important exception, however, is the vacuum energy or the vacuum expectation value of the energy. The quantization of a simple harmonic oscillator requires the lowest possible energy, or zero-point energy of such an oscillator to be

\[ E = \frac{1}{2} \hbar \nu . \]

Summing over all possible oscillators at all points in space gives an infinite quantity. To remove this infinity, one may argue that only differences in energy are physically measurable, much as the concept of potential energy has been treated in classical mechanics for centuries. This argument is the underpinning of the theory of renormalization. In all practical calculations, this is how the infinity is handled.

Vacuum energy can also be thought of in terms of virtual particles (also known as vacuum fluctuations) which are created and destroyed out of the vacuum. These particles are always created out of the vacuum in particle–antiparticle pairs, which in most cases shortly annihilate each other and disappear. However, these particles and antiparticles may interact with others before disappearing, a process which can be mapped using Feynman diagrams. Note that this method of computing vacuum
energy is mathematically equivalent to having a quantum harmonic oscillator at each point and, therefore, suffers the same renormalization problems.

Additional contributions to the vacuum energy come from spontaneous symmetry breaking in quantum field theory.

6 Field Line Breaking

Classical electromagnetism does not allow the formation of electromagnetic waves which is unfortunate because such waves are thought to exist. EM field lines are the level sets of the $\vec{E}$ and $\vec{B}$ fields, and fields which satisfy Maxwell’s equations cannot have cusps in their level sets. However, for loops of flux to break off from their source to become propagating waves, the level sets must acquire a cusp at some point. The cusp can be smoothed over with quantum mechanics but it is desirable to formulate classical electromagnetism in a language which does allow field lines to cross and break at cusps. This is a great place to apply the fractional distance standard of real analysis developed in Reference [2].

The level sets of a vector field are the surfaces of constant electric potential. If there was a cusp in the field lines, then we would say that the field points in two different directions at that point which is not allowed. This can be avoided by adding bulk terms such that the field is equal to zero where they cross. This is like when a charge near a metal surface causes the charge on the surface to rearrange in order to enforce the $\vec{E} = 0$ boundary condition.

If field lines crossed, the strength of the field at that point would be infinite.

When magnetic field lines break, they supposedly do it in a region of low rather than high energy.

I anticipate a nice use case for infinity hat in ODEs. Due to the identity $1/x=0$ for any number in the neighborhood of infinity, certain ODEs involving, for instance, terms like $1/x$ which might have only trivial solutions in the neighborhood of the origin might have non-trivial solutions in the neighborhood of infinity. One problem on mind in this regard is the breaking off of loops of EM flux to form propagating waves disconnected from charges. We know this happens but it is not possible to find solutions to Maxwell’s equations which achieve the disconnection of electric field lines from charges. I hope infinity will give a new way to treat this problem. For instance, we could use the normal procedure of normalization to set the scale of
fractional distance as the macroscopic scale of an electromagnetic system, and then we could use the scale of natural numbers to do something tricky where field lines have to cross and form cusps before they can separate from sources. The process of $\hat{M}^3$ that I have come up with would suggest, actually, that we start with the natural scale as the EM scale and renormalize to the scale of fractional distance at each time step, then iterative keep setting the scale of fractional distance back to the scale of natural numbers with iterative normalizations. In this way, I hope we could achieve truncation effects that allow electric field lines to disconnect from their sources.

ELLiptic Curves

Period Doubling Cascade In Classical Chaos
Field Lines Can’t Cross Because The Field Would Point In Two Directions There–Intersecting Universes Can Explain This

Elliptic Curve Pinching
de Sitter Hyperboloids

7 Curvature in the Neighborhood of Infinity

The de Sitter parameter in maximally symmetric spacetime is a simple linear function of the Ricci scalar.

Need To Divide By Zero????????

= = = = = = = = = = = = = = = = = = = = =

Allows structure.
Perhaps measure from separate origins as in [61]. Original motivation was for gravity to go to zero on finite distance.
EM also goes to zero on finite distance.
Curvature in the neighborhood of infinity plays nice with coordinates in the neighborhood of infinity.
Numbers don’t have square roots as needed for metrics but the modification not nearly a step so far as the introduction of Grassman numbers when forcing the path integral formulation of QFT to work for fermions.

Gravitational Interaction Going to Zero

= = = = = = = = = = = = = = = = = = = = =

If a $\delta$-valued state is obtained from an observable state by negative time evolution suggests that we might associate this step with the $\hat{M}^2$ step Future→Past. In that case, this trajectory goes through $\emptyset$ and some other parameter would be defined. In that regard, one recalls that Euler was prone to consider negative numbers as
greater than infinity and that is exactly what would be required presently. The \( \delta \) state appearing at infinite negative time certainly suggests that it should land on \( A \) at this step. If the states attached to the \( A \)-brane are the \( \delta \) functions then the RHS is \( \{ \mathcal{H}', \Omega', \mathcal{A}' \} \). Certainly, the diffusion of the wavefunction did not increase in this step but experiment requires that the wavefunction in \( \mathcal{H}_2 \) is more diffuse than it was in \( \mathcal{H}_1 \). Forward evolution again across an infinite amount of time will result in plane waves filling all of space. We can use numbers in the neighborhood of infinity to make it stop short of being totally washed out. More about this in Section 7. The main point here is that theory of negative time provides a mechanism by which one would dynamically collapse the wavefunction at some point in \( \hat{M}^3 \).

To that end, there are a few possible cases for the curvature properties of the \( A \)- and \( \Omega \)-branes.

The curvature can be finite where the de Sitter parameter is the golden ratio or its inverse.

CITE GOLDEN RATIO IN BLACK HOLE.

Another possibility is that they can be branes of infinite negative and positive curvature respectively. As singularities, that would reduce the problem of how one might connected to the other. We would simply call one a black hole and another a white hole, and put them in the same place. Another fascinating possibility (see Section 7) is that these branes mark the boundary between curvature (de Sitter parameter) in the neighborhood of the origin and curvature in the neighborhood of infinity. If it is later determined that \( A \) and \( \Omega \) should have infinite negative and positive curvatures respectively, then \( \emptyset \) would describe their union or superposition. Another possibility discussed in Section 7 is that \( A \) and \( \Omega \) bound the domain of curvature in the neighborhood of infinity.

The relative placement of the \( A \)- and \( \Omega \)-branes around the \( \mathcal{H} \)-brane spawned inquiry which resulted in the fractional distance framework of real analysis in which numbers in the neighborhood of infinity are defined. The unit cell requires that the gravitational interaction between the labeled branes go zero while still being a finite distance. However, we have to ask if \( A \) and \( \Omega \) are physical locations which would gravitate. The original idea was that \( A \) and \( \Omega \) should be places that quantum particles tunnel to between measurements. As in Section 8, however, the gravitational interaction between successive \( \mathcal{H} \)-branes very nearly reproduces the dark energy effect which was supposed for a periodic succession of \( \mathcal{H} \)'s in the vertical direction.
In big bang cosmology, and not regarding the ADM positive-definiteness theorem, the universe can have positive, zero, or negative total energy. If it is positive, the universe will continue to expand forever. If it is zero, the universe asymptotically stop expanding but never collapse. If the energy is negative, expansion will eventually stop and the universe will recondense into a big crunch singularity. Dark energy is a cosmological anomaly because even the positive energy solution which permits eternal expansion predicts that the rate of expansion should decrease with time, but certain cosmological redshift effects show that the rate is increasing with time. The MCM solution to dark energy avoids this by supposing that time is being rarefied at an accelerating rate instead of the universe expanding at an accelerating rate. A preliminary metrical analysis appears in Reference [75].

Although photons are quantum mechanical in nature, dark energy is a classical optical effect. The MCM solution to this Nobel-prize winning cosmological anomaly [25] does not require a fifth dimension. However, the proposed solution describes something quite like the horizontal arrangement of the unit cell. Since the bounce at the end has a reverse time universe on the other side of it, as is required for conservation of 4-momentum in the singular bounce frame, we may think of the horizontal direction across the unit cell as $x^0$ in this case, as in Figure 10.

To implement the effect, it is assumed that the universe does eventually contract down to a big crunch in the distant future, and that the crunch is actually a bounce as per ordinary models of cyclic cosmology. In that case, there exists the mass of
another universe at some point in the future. Since the present is closer to that
future mass than is the past, the present should accelerate toward it at a greater
rate than do objects in the past. The optics of such an effect are predicted to be
exactly like the optics of dark energy in which cosmological distant objects, those
very far removed from the present along the past light cone, appear to recede at an
accelerating rate due accelerating spatial expansion. Although the solution is posed
as a problem in Newtonian gravity, Newtonian gravitation is usually only felt across $x^i$
and not $x^0$. If objects gravitate toward objects in the distant future, then they should
also gravitate toward object in the near future. For instance, an asteroid at time $t$
would experience gravitation toward itself at time $t+\varepsilon$. To solve this problem, we may
engage in hand waving or cite “back reaction.” Perhaps the reversed direction of time
in the past causes a canceling anti-gravitation with the asteroid at $t−\varepsilon$. Certainly,
the asteroid cannot move backward through time so there is some condition which
favors gravitation toward the future. Regarding the MCM dark energy effect which
favors attraction toward the future over attraction toward the past, one might use
numbers in the neighborhood of infinity to describe distance to the past singularity,
and numbers in the neighborhood of the origin for the future singularity. Then the
gravitational interaction with the past singularity goes to zero while that with the
future singularity does not. Clearly,

$$V(\infty − b_−) = \frac{GM}{\infty − b_−} = 0 \quad \text{and} \quad V(b_+) = \frac{GM}{b_+} \neq 0 ,$$

allow two different regimes of gravitation. One might favor causality over retrocausal-
ity for some reason along these lines. (Indeed, the requirement for a finite length across
which the gravitational interaction must vanish was the original motivation behind
the line of inquiry which resulted in the analysis of numbers in the neighborhood of
infinity.) The formalization and refinement of the total mechanism should proceed as
follows.

The presence of mass-energy is associated with positive curvature, and only $\Sigma^+$
has positive curvature in the unit cell. Therefore, the existence of past and future
singularities cannot be treated equally in the MCM. Equal treatment would require
positive curvature for $\Sigma^−$ as well.

One must obtain a gravitational potential energy function such that objects in the
present experience gravitation toward the future as an acceleration of the passage of
time. Furthermore, the energy landscape must be such that decent into the gravi-
tational well of the bounce is continued as continued descent beyond the well. The
ordinary $V(r) = GM/r$ is not sufficient for this purpose. To construct the requisite
analogue of a Newtonian potential, we should use the Gauss law. In classical planetary gravitation, the Gauss allows us, for example, to treat the Earth and the Moon as if they were point masses located the centers of mass of the respective bodies. In the context for dark energy, we will treat the mass of the universe in the future as a point mass located on the $\emptyset$-brane. Perhaps a full analysis of the Gauss flux problem will show that gravitation toward the future requires the reversal of the time at the bounce and the effect is not present between $t$ and $t \pm \varepsilon$. The full treatment of the problem described here requires some formal treatment for this issue. However, the first step will be to write the Newtonian gravitational potential energy as if there was a point mass having the mass of the entire universe located at some point in the distant future which we call $\emptyset$ in analogy with the horizontal arrangement of the unit cell.

Upon first glance, the Newtonian well surrounding $\emptyset$ would have its minimum at $\emptyset$ and matter would become trapped there without propagating forward through $\Sigma^-$. To avoid this, the negative sign of the reverse time in $\Sigma^-$ must be invoked so that approach to a black hole in $\Sigma^+$ is carried forward as egress from a white hole. The specific form of the Newtonian gravitational potential must be determined, and then it must be rewritten in terms of a curved metric and compared to existing models of black hole/white hole wormhole physics.

After delving further into the Newtonian case, this problem requires that the Newtonian potential should be rewritten as a metric in curved space.

One idea which has not yet been followed up on is that we might work in the Newtonian picture to send plane waves into the gravity well a $\emptyset$ where a specular reflection boundary condition would be imposed. Assuming that the black brane $\emptyset$ has a higher index of refraction than the empty 5D bulk of $\Sigma^+$, reflection would cause the reflected wave to be phase shifted by $\pi$ radians. This phase shift is exactly the acquired minus sign expected for forward propagation of a plane wave in $\Sigma^+$ to continue as reverse propagation in $\Sigma^-$. Therefore, one might take this Newtonian solution as the basis for a generalization to curved spacetime without being forced to reconstruct the physics of black holes and white holes in the highly complicated 5D analogue of the usual 4D general relativity. Once the requisite metric is constructed, the null intervals should be computed to find the paths of photons. One might use the observed dark energy effect to find the normalization of the 5-velocity which is not yet known or supposed.

All of relativity revolves around the normalization of the 4-velocity squared to $c^2$ but there is as yet no proposed normalization of the 5-velocity by which observables
in 5D are relative to 4D motions. However, the MCM proposes that all observations take place in $\mathcal{H}$ so there may be no 5D MCM motions.

One would attempt to correlate the observed rate of dark energy acceleration with some dimensionless ratio characterizing the unit cell.

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WHY DARK ENERGY DELAY FOR PHOTONS:

To close this section before moving on to the promised theses, we will frame the prediction for delay correlations in the context of the modified particle model. Firstly, cosmological photons are observed in telescopes after having been emitted in events in the very distant past. If delay correlations were present, it is expected that a difference would have been noticed between the physics of intergalactic, solar, and lab photons. None have been observed. To see why that might be, notice the $x^0x^0$ placement of $\gamma$ in the unit cell. This connection can never experience any effects related to the horizontal direction across the unit cell. On the other hand, the $B^0$ meson system in which time reversal symmetry violation was observed to confirm the prediction for delay correlations [26], uses quarks. All mesons are formed from pairs of quarks. Therefore, with quarks being fundamentally different than the photons and leptons of QED in their $\chi^4$ structure, we have reason to expect that delay correlations would be observed for quark matter even while no such effects would be observed in the $\gamma+e$ lab systems, and similar. The issue of the onset of de facto delay correlations for dark energy photons is revisited in Section 8.

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THE EFFECT IN [1] SHOWED SMALLER IN THE FUTURE. THERE WAS SOMETHING I WANTED TO CONNECT TO THIS BUT FORGOT.

9 The Greatest Action Principle

Bohr’s correspondence principle: the motion is classical when the action per cycle is large compared to $h$

Feynman’s thesis CITE leaves some room for extra physics in this department.

The action principle is satisfied by any extrema of the action. In the previous problem regarding dark energy, light follows the minimum of the action in the unit cell’s vertical direction. However, the MCM supposes that quantum motions differ from classical motions because horizontal trajectories across the unit cell satisfy the maximum of the action. Historically, maximum action solutions have been discarded as unphysical due to the requirement for infinite energies. However, the arithmetic of numbers in the neighborhood of infinity is well-suited to the computation of such
solutions.

Lay out some energy landscape and solve it in 1D. Going outside the universe should require infinite energy. This is like a ball rolling directly over the top of a hill. Compute the classical action of such problems. Then put it in 2D and use radial diffusion of the wavefunction.

Talk about Feynman’s program of smearing out the classical trajectory.

Quantum fuzziness should come in through phase decoherence when sines’ and cosines’ arguments are rescaled as $n\pi x \to n\pi \Phi x'$, or some such mechanism which remains to be determined (see Subsection 1.8.)

The least number in the neighborhood of infinity $F_0$ may be useful in that the new arithmetic of fractional distance should satisfy the action principle?

MENTION UTILITY FOR $F(1)$ IN THE ACTION PRINCIPLE:

Regarding the fractional distance application toward the action principle,

So, as it relates to the present material, we might take $S = \min(F)$ to satisfy the action principle, treating it as $S = \infty$ for the purposes of variational analysis (the calculus of variations) while conceding that it is in some sense still a finite quantity, if required.

10 Rigged Hilbert Space

After laying out the energy landscape from the previous problems of dark energy and maximum action, we will have assembled the requirements need to construct the MCM state spaces.

We say the problem of quantum gravity is hard because general relativity is a theory of points in spacetime while quantum mechanics is a theory of states in Hilbert space, and position eigenstates do not exist in Hilbert space. How can the theories be united when the objects of one do not appear in the other? The MCM’s first step in this regard, one taken by others many times, is to introduce rigged Hilbert space which is also called a Gelfand triple $[41, 51]$. Such a space contains position eigenstates well suited to interpretation in terms of points in space, if not spacetime directly.

Everything is in the unallowed region?

Problem similar to the Dirac Comb
Need to construct the Hamiltonian. This is related to the dark energy solution of
the energy well.

Transmission and reflection problem solutions don’t belong to \(L^2\)

\text{COMPARE DISCRETE vs CONTINUOUS FROM MCM}

\text{QUOTE ISHAM p113}

\text{RIGGED HILBERT SPACE}

The triple space is such that \(\Omega'\) is an superspace of \(\mathcal{H}'\) including position eigenstates. These states have representations as bras and
kets, so there is actually a fourth state space associated to rigged Hilbert space. If
\(\Omega'\) is the dual space to \(\mathcal{A}'\), it contains position eigenbras. While the eigenstates of
operators with discrete spectra return numbers through a simple scalar product as in

\[ \hat{A}|a_k\rangle = a_k|a_k\rangle \quad \longleftrightarrow \quad \langle a_m|a_n\rangle = \delta_{mn} \quad , \tag{10.1}\]

the case is slightly different for the eigenstates of an operator with a continuous
spectrum, call it \(\hat{x}\). Hilbert space is its own dual space so in the eigenbras of an
operator with a discrete spectrum are just functions. Hilbert space is formally a linear
space equipped with an inner product and it is a postulate of QM that one obtains
probability amplitudes by the inner product of bras and kets. In the continuous case,
the \(\mathcal{A}\) contains what are called test functions and the dual space contains functionals.
The generalization of Equation (10.1) to continuous eigenbases, using the example of
the position operator, is

\[ \hat{x}|x\rangle = x|x\rangle \quad \longleftrightarrow \quad \langle x_1|x_2\rangle = \delta(x_1 - x_2) \quad . \tag{10.2}\]

Here we have the Dirac \(\delta\) instead of the Kronecker \(\delta\) of Equation (10.1.) The
structure of quantum mechanics is predicated upon the existence of various resolutions
of the identity. The expansion of \(|\phi\rangle\) in the eigenbases of \(\hat{A}\) and \(\hat{x}\) are

\[ |\phi\rangle = \sum_k |a_k\rangle\langle a_k|\phi\rangle \quad , \quad \text{and} \quad |\phi\rangle = \int |x'|\langle x'|\phi\rangle dx' \quad . \tag{10.3}\]

\(|a_k\rangle\) and \(\langle a_k|\) both live in \(\mathcal{H}\) (because \(\psi\) and \(\psi^*\) are just two functions) but \(|x'\rangle\) and
⟨x′| live in Ω′ and Ω× respectively. Neither of them lives in H. The ak states also live in Ω′ and Ω× due to H ⊂ Ω′ and H ⊂ Ω×. Once spin in introduced, it is no longer exactly true that ψ and ψ* belong to the same space. ψ lives in a space of column arrays and ψ* lives in a space of transposed column arrays, or row arrays.

Now, the spaces are nested, we need to separate them. If we only talk about a state in A in the usual sense, then one would assume that the state must also be in H′ and Ω′ (or Ω×) due to the nested subset relationship. We want to avoid this. Moving forward in this review, we will work with {A, H, Ω} and an implicit understanding that there exists a fourth manifold to associate with Ø. To take away the nesting relationship, we say that the domain of the functions in each space belong to the manifold with the corresponding name. If ψ(x) ∈ A′, then x is a position in anti-de Sitter space, etc. To associate states with distinct, non-nested spaces, we introduce the so-called ontological basis \{eA, eH, eΩ\} as a labeling scheme.

\[
\begin{align*}
\psi \in A' & \implies |\psi; e_A\rangle = \psi(x^-_i) \quad (10.4) \\
\psi \in H' & \implies |\psi; e_H\rangle = \psi(x^+_i) \quad (10.5) \\
\psi \in \Omega' & \implies |\psi; e_\Omega\rangle = \psi(x^i_+) \quad (10.6)
\end{align*}
\]

Particularly in the case of Ω′, we add the capacity for new behaviors because states in a state space may be written as linear combinations of all of the states in the space, and Ω′ has position eigenstates in it. These are δ functions and may prove useful in the process of wavefunction collapse which reduces wavefunctions to sharply peaked δ functions in an ideal measurement devoid of Heisenberg uncertainty.

So, \(\hat{U}\), the time evolution operator, operates on states in Hilbert space and returns states in Hilbert space. However, to implement collapse of the position space wave function, we need to end up with a position eigenstate which does not exist in Hilbert space. Since the Schrödinger equation is a heat equation, the position eigenfunction will spread out by the time it can be observed again, and the observable states live in A′. So, we put all the ingredients into \(\hat{M}^3\). In the way we have things set up right now, the first step H → Ω would be the collapse. However, we need to figure out the mechanism mo’ betta.

The first thing that happens is that a physical state becomes a position eigenstate. This happens in Ω which is not a lattice site where observations can be made. Due to Heisenberg uncertainty, such states are not observed in nature. However, the requirement to move a state from one place to another suggest that we should look at the translation operator.

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11 The Pauli Algebra

In Reference [76], we demonstrated the utility for physics in replacing the imaginary number with a quaternion. Later, this channel for parallel phase activity came to prominence once a Hamiltonian for the MCM was finally constructed [62]. The property \( i^2 = 1 \) of the imaginary is replicated by any of the quaternions individually, meaning \( i^2 = 1, j^2 = 1, \) and \( k^2 = 1 \). The quaternions have the additional property that \( ijk = 1 \). This part of the algebra of \( \mathbb{H} \) is not found in \( \mathbb{C} \). However, it must be true that in the absence of at least a third embedding dimension, a 2D plane spanned by the unit real number 1 and any of \( \{i, j, k\} \) is indistinguishable from the complex plane. The entirety of the theory of functions of complex variables applies equally to functions of variables with real and quaternion parts as long as there is only a single quaternion. To finally belabor the point before moving on, a plane spanned by 1 and a second orthogonal basis unit \( \gamma \) having the property \( \gamma^2 = 1 \) cannot be distinguished from \( \mathbb{C} \) in the absence of a quaternion-valued embedding dimension.

Sometimes, the Lorentzian signature of Minkowski space is taken as equivalent to the form of the Minkowski metric

\[
\eta_{\mu\nu} = \begin{pmatrix}
-c^2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}.
\] (11.1)

Then, differential element along a geodesic is given by

\[
ds^2 = \eta_{\mu\nu} \, dx^\mu \, dx^\nu.
\] (11.2)

However, in the underlying theory of differential geometry pioneered by Riemann, we have a differential line element \( ds \) as the fundamental descriptor of curvature on manifolds. For physics, the quantity \( ds^2 \) is more useful but for the mathematical construction of 4D manifolds with Riemannian curvature, everything is encoded on

\[
ds = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3.
\] (11.3)

The only way to generate Lorentzian structure at from this level, specifically the one in Equation (11.1), is to set \( a_0 = \pm ic \) and \( a_i = 1 \). As we have demonstrated,
however, we may obtain the correct matrix representation of the $\eta_{\mu\nu}$ tensor if we use a quaternion such as $a_0 = kc$. Therefore, is we make a distinction between $\Sigma^\pm$ that the Lorentzian structure is given by one quaternion in one and an unequal quaternion in the other, the limit of small $\chi^4$ as $\Sigma^\pm$ approach a shared boundary at $\mathcal{H}$ is also the limit where the individual complex plane analogues will come into contact with a third embedding dimension. Since the algebra of the Pauli matrices is exactly the algebra of the quaternions $\mathbb{H}$, one would seek to extract the Pauli algebra as a consequence of the slices of $\Sigma^\pm$ approaching a shared limit at $\mathcal{H}$.

Fundamentally, while integer spin states in quantum mechanics have classical counterparts, the half-integer spin states have no classical counterpart. The operator algebra of observable (and anomalous!) half-integer spin states is known to be identically the quaternion algebra. Since we have taken the $\mathcal{H}$-brane as the domain of quantum mechanics, we are well motivated to attempt to the half-integer spin algebra as the limiting algebra of the coming together of the two halves of the MCM unit cell at $\mathcal{H}$.

12 Spin Angular Momentum

Torsion is required to preserve spin in general relativity. The computation for the spinning wheel in Reference [3] used “leftovers.” Spin angular momentum should be a non-classical property because it is the increment of the leftover.

Winding of one worldline around the other implies torsion.

13 Representations of Quantum Algebras

Matrices form a vector space. It is possible to quantum mechanics in the vector way instead of the matrix way.

WRITE STATES WITH $\hat{\sigma}^\mu$

The Dirac algebra is basically two copies of the Pauli algebra, and the 16 dyads function as the spanning basis of the Clifford space.

Quatennion and Clifford

14 Wavepackets Extending to Infinity

A standing problem in physics is the infinite spatial extent of wavepackets. We would like to construct analytical wavepackets localized in space but this is not possible. One way to summarize the problem is that the exponential function $e^x$ has no zeros on
the real line. However, one of the main results to come of the study of real numbers in the neighborhood of infinity is that $e^x$ does have an infinite number of zeros in $\mathbb{R}$. This result should be extended to define a new framework for the analysis of wavepackets. Hypothetically, one would model the lab scale across some spectrum of neighborhood of fractional distance while the wavepackets themselves would be defined to vanish outside of their local (comoving) neighborhoods. In other words, one might implement finite wavepacket structures in theory by setting the lab scale to be define some characteristic spectrum of “big part” while the mechanics of the wavepackets themselves would be defined only in terms of “small part” and in terms of vanishing ratios of small part of real numbers to big parts of real numbers. The language of small and big parts of real numbers in developed in Reference [2].

15 Antisymmetric Fermion Wavefunctions

16 A Clopen Universe

The division of the time domain, an interval $[t_{\min}, t_{\max}]$, explicitly imposes the closed topology on time. While it is an open question whether or not the observable physical universe is topologically flat, all data to date indicates that the deviation from topological flatness on cosmological scales is very small if it exists. However, the 5D de Sitter and Anti-de Sitter spaces of the MCM unit cell have positive and negative curvature respectively corresponding to topological openness and closedness. In 4D, de Sitter, Minkowski, and Anti-de Sitter spaces are known as spacetimes of maximal symmetry. 4D Minkowski space has zero Ricci curvature in 3-space slices. Anti-de Sitter spacetimes have topologically closed 3-spaces at each value of the time coordinate. de Sitter spaces slices are uniformly positive curved. The condition of maximal symmetry means that the curvature everywhere in spacetime can be determined from a single scalar number. This number is called the de Sitter parameter. 5D (Anti-) de Sitter space are constructed from 4D ones by plugging in the de Sitter parameter as the fifth diagonal entry in 5D metric. Because this parameter is scalar, sometimes we try to associate a “scalar field” with the object in the fifth diagonal position. This is done in Kaluza–Klein theory. The de Sitter parameter in Minkowski space is zero and the abstract $\chi^4$ dimension added to 4D spacetime directly measures de Sitter parameter across the unit cell.

As mentioned by Joshi, however, there are two periodicities in the MCM: uniform curvature increase across vanishing curvature in $\mathcal{H}$, and also curvature increasing to infinity at $\emptyset$ and then continuing to increase as negative quantity. Infinity curvature is
Next Steps and the Way Forward in the Modified Cosmological Model

a classical black hole in general relativity so Joshi’s words are clear: “there is a wormhole or a conduit through which one can funnel arithmetic/geometric information in the K-universe to the L-universe through the choice of an isomorphism of Galois groups $G_k \simeq GL$, which serves as a wormhole.” As the information goes through $\varnothing$ from a space of metric signature $-++++$ into of $-+++-$, as is the difference between de Sitter and Anti-de Sitter metrics, then we see why it should pick up a minus sign.

Also, specular reflection picks up a phase shift. Therefore, in the sense of “quantum bouncing,” one might simplify the problem of evolution through $\varnothing$ as classical reflection with a phase shift instead of quantum tunneling into a different spacetime topology. An eccentric topology beyond closed and open is called topological clopen-ness, and one would conduct a survey of the properties of clopen spaces with the goal of finding an existence set of topological properties.

17 FLRW Cosmology

$\Lambda$, is a positive constant which is related to the cosmological constant by

The curvature, represented by the scalar $R$ (see (1)), is four times the cosmological constant.

18 Kaluza–Klein Theory

Kaluza–Klein theories are very well studied. However, the MCM has taken them for granted without going into great detail regarding the theories’ facets or the equations of motion which come of them. Particularly, care should be taken to distinguish Kaluza theory from the Kaluza–Klein theory with a compact fifth dimension. Furthermore, one would study exotic variants such as Kaluza–Klein analogue theory in which the fifth dimension has a clopen topology.

CYLINDER CONDITION

EQs (27) and (28) from [6]: the part about the potentials being multiplied by the $\chi_4^\pm$ needs to get worked out.

[74] 5D Conformal space
Campbell-Magaard MAYBE CITE:
Schrödinger 1956 showed $dS$ in flat 5D space
19 Electrogravity

REVIEW OF [6]
Mixed up de Sitter and Ricci params?

Equation (??) is the foremost hard equation for new MCM physics.

This is why the branes of the unit cell are vertical slices taken at constant values of the fifth embedding dimension. Following Reference [7], the Kaluza-Klein (KK) metric is

\[ g_{AB} = \begin{pmatrix} g_{\mu\nu} + \kappa^2 \phi^2 A_\mu A_\nu & \kappa \phi^2 A_\mu \\ \kappa \phi^2 A_\nu & \phi^2 \end{pmatrix} . \]  \hspace{1cm} (19.1)

\( g_{\mu\nu} \) is the metric of 4-space and \( A_\mu \) is an electromagnetic potential 4-vector. This is implemented in the unit cell with \( g^{\pm}_{AB} \) as the respective metrics in \( \Sigma^\pm \).

\[ g^{\pm}_{AB} = \begin{pmatrix} g^{\pm}_{\mu\nu} + \kappa^2 f(\chi^4_{\pm}) A^\pm_\mu A^\pm_\nu & \kappa f(\chi^4_{\pm}) A^\pm_\mu \\ \kappa f(\chi^4_{\pm}) A^\pm_\nu & f(\chi^4_{\pm}) \end{pmatrix} . \]  \hspace{1cm} (19.2)

The Ricci scalar is equal to zero in Minkowski space. For \( d+1=4 \) spacetime dimensions, the Ricci scalar in maximally symmetric spacetime is

\[ R = \frac{d(d-1)}{\pm l^2} = \frac{6}{\pm l^2} . \]

The radius \( l \) is the radius of curvature, also called the de Sitter parameter. It defines maximally symmetric 4D spacetime embedded in 5D Minkowski space as

\[ -(x^0)^2 + \sum_{j=1}^{4} (x^j)^2 = \pm l^2 . \]

The unit cell’s continuum of smoothly varying AdS4, M4, and dS4 branes is generated most simply by

\[ \pm l^2 = \frac{1}{\chi^4_{\pm}} \implies \lim_{\chi^4_{\pm} \to 0^\pm} R = 0 . \]

Slices of \( \chi^4_{-} \in \Sigma^- \) have a constant negative Ricci scalar at every point in spacetime: it is AdS4. Likewise, every slice of \( \chi^4_{+} \in \Sigma^+ \) is dS4. The notion of \( \mathcal{H} \) as a topological obstruction between \( \Sigma^\pm \) is enforced when division by zero is not allowed. Indeed,
Next Steps and the Way Forward in the Modified Cosmological Model

\( \chi_4^\pm = 0 \) does not exist in \( \Sigma^\pm \) so the obstruction is inherent in the unit cell without reference to any division by zero. It was the earliest convention in the MCM that \( A \) and \( \Omega \) should have some finite curvature in the physical coordinates but now it seems most useful to define them with Ricci scalars \( \pm \infty \) respectively. Namely, \( \chi^- \) and \( \chi^+ \) are timelike and spacelike respectively so there is some issue regarding the implementation of a smooth affine parameter on a path from \( \Sigma_1^+ \) into \( \Sigma_2^- \). An infinite Ricci scalar on \( \Omega \) and \( A \) should provide a method for joining the disparate topologies of \( \Sigma^\pm \) on a topological singularity. In that case, one would set \( \pm t^2 \) as an appropriate function of \( \chi_4^\pm \) giving \( R_A = -\infty \) and \( R_\Omega = \infty \).

\[ \text{May be combine with next section?} \]

20 Holographic Duality

The keen new insight of the MCM, exceeding the direct utilization of Perelman’s first-ever solution to a Millennium Problem, is concisely expressed in terms of holographic duality. This principle, sometimes called the AdS/CFT correspondence because the first and foremost application of “holographic duality” outside of classical optics was a result of Maldacena showing that physics in Anti-de Sitter space can be totally encoded in a conformal field theory (CFT) whose domain is only the boundary of the AdS space. In this way, one takes a slice of \( \Sigma^- \) as a model for the real universe. Then, the topological closedness is said to guarantee the existence of a boundary on which to encode the CFT. Since a boundary has one less dimension than its bulk in many cases, the boundary of a 3D ball is a 2D sphere, for instance, this method for simplifying is desirable in physics. Because physics is a Hamiltonian system, once any two degrees of information are known about a physical system, the whole thing is said to be determined. This is why Hamiltonian mechanics is formulated in terms of pairs of what are called “conjugate variables.” Once momentum and position are known, classical motion is determined. Dirac’s great inroads in quantum theory are summarized as the development of conjugate observable operators subject to Hamiltonian constraints. By reducing the problem in a bulk to another problem on its surface, we remove one degree of freedom from the physical theory. This is usually sufficient to totally solve a problem and this is the reason why the AdS/CFT is so celebrated without it every actually being taken forward to final solution of a theory of everything.

Usually, the holographic surface is taken in the outside of one bulk but the MCM idea was to put the holographic surface squished between two bulks. In a phrase,
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this is the sharpest new idea in the MCM and it is seen as $\mathcal{H}$ is squished between $\Sigma^\pm$ in the unit cell. Therefore, one would make a survey of the primary applications of the extensively treated AdS/CFT problem posed by Maldacena with the goal to project from $\Sigma^{-}$ into $\mathcal{H}$ but then disrupt the duality so that solution to the reduced problem in $\mathcal{H}$ goes into $\Sigma^{+}$. Usually, there is a problem in the AdS/CFT because the surface and the bulk are perfectly dual. There is no way to do physics in one without it being mirrored exactly in the other. In the MCM, the bulk geometry in $\Sigma^{-}$ is dual to an initial state $|q_I\rangle$ in a CFT, that state is sent to $|q_F\rangle$ across the unit cell, and then that final state is dual to geometry in $\Sigma^{+}$. In the AdS/CFT that only has $\Sigma^{-}$ and $\mathcal{H}$, which is the most well-studied one, evolution in one is always countered by counter-evolution in the other causing a dead end in what at first to be an amazingly promising result.

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21 Randall–Sundrum Models

Randall-Sundrum models are a survey of the kinds of tricks that can be done with AdS/CFT methods. There are two RS models, one with a surface at infinity and another with a surface at the origin. Sundrum and Randall did not discover the third model given by the MCM unit cell. Rather than one surface near or far, we can consider one surface squished between two surfaces, and then that can be the $\mathcal{H}$ brane at the origin or the $\emptyset$ brane at infinity. Since the MCM unit cell is totally conforming to what are usually called Randall–Sundrum models, a survey the RS literature in is order. One would use the jargon of RS theory to cast the MCM unit cell formally as a third type, or third and fourth type, of RS model.

22 Brans–Dicke Theory

23 Numerics

24 Boundary Terms at Infinity

If the CFT in the AdS/CFT can be written onto a surface at infinity, and the utility of the method is to reduce degree of freedom by one, we should revisit what physicists mean when it is said, “And ignoring boundary terms at infinity.”

Few phrases are repeated more often in QFT than, “Integrating by parts and
assuming that boundary terms at infinity go to zero...” Such boundary terms are an ideal place to explore avenues for physics. Where does the idea come from that these terms should vanish? In non-relativistic quantum quantum mechanics, one usually restricts the Hilbert space of states to be such that the $L^2$ condition is satisfied. If $|\psi\rangle \in L^2$, then

$$\psi(\infty) = 0.$$ 

In the most general space of position states, which is preferred over other operator eigenstates by our intention to connect the quantum theory to classical theory of motions, this reflects the idea that the probability for observing a particle at an infinite distance is zero\(^1\). Consider the free field Lagrangian

$$\mathcal{L}(\varphi) = \frac{1}{2} \left[ (\partial \varphi)^2 - m^2 \varphi^2 \right].$$

The generator of the free field theory (with a source $J$) is

$$Z = \int D\varphi e^{i\int d^4x \left\{ \frac{1}{2} (\partial \varphi)^2 - m^2 \varphi^2 + J \varphi \right\}} .$$

The exponent’s integral is solved analytically by integration by parts. We have

$$\mathcal{I} = \int d^4x (\partial \varphi)^2 = uv\bigg|_{-\infty}^{\infty} - \int v \, du ,$$

with

$$u = \partial \varphi, \quad v = \varphi,$$

$$du = \partial^2 \varphi \, d^4x, \quad dv = \partial \varphi \, d^4x ,$$

so that

$$\mathcal{I} = \varphi \partial \varphi\bigg|_{-\infty}^{\infty} - \int d^4x \, \varphi \partial^2 \varphi .$$

Here, the most-repeated phrase in quantum field theory sets the first term $\varphi \partial \varphi|_{-\infty}^{\infty}$ equal to zero. The anti-derivative evaluated is constrained to vanish at its upper and lower bounds, and the integral’s “boundary term” is said to vanish when the difference

\(^1\)While it is common to make explicit formulations that the probability amplitude should go to zero at infinity, there is also a local identical zero which is often overlooked. The probability for observing a particle at the observer’s own location is also zero. It is equally impossible to observe something at one’s own location as it is to observe it at infinity. This condition should be added to the $L^2$ boundary condition on quantum state spaces. Often new physics follows directly from new physics. Quantum eigenstates are usually written in terms of sines and cosines which have zeros hard-coded into their analytical representations, but their superpositions (which always go to zero at infinity) might in certain instances be such that the local zeros are removed. These states are unphysical and a condition about a zero at the observer’s own location should be a formal constraint on Hilbert space in the same way that they famous $L^2$ condition is a constraint.
of two zeros vanishes. We integrate by parts accordingly and plug the integral back into the exponential in the the generator to obtain

\[ Z = \int D\varphi e^{i \int d^4x \left( \frac{i}{2} \left[ -\varphi \partial^2 \varphi - m^2 \varphi^2 \right] + J \varphi \right)} = \int D\varphi e^{i \int d^4x \left( -\frac{i}{2} \left[ \varphi \left( \partial^2 + m^2 \right) \varphi \right] + J \varphi \right)} . \]

About nine out of ten things in the entire discipline of quantum field theory are the permutations of this integral, we should very closely examine why we have set the first term in \( \mathcal{I} \) as an identical zero. Most generally, we have assumed that the field \( \varphi \) trails off to zero at infinity. However, the MCM arithmetic axioms are such that we need only assume that physical fields go to zero at the outskirts of some local neighborhood of fractional distance. Now, we will review the relevant facets of the fractional distance analysis from Reference [2].

The equation of motion generated by \( Z \) is the Klein–Gordon equation whose solutions are such that

\[ \varphi(\vec{x}, t) = e^{i(\omega t - \vec{k} \cdot \vec{x})} . \]

Obviously, therefore we will need to re-examine the difference between the big exponential function \( E^x \) and the usual exponential function \( e^x \). Furthermore, in the definition of \( Z \) given by Equation (24.2) is such that \( d^4x \) is over all of space, and not just all of a local neighborhood of fractional distance, and this has tremendous implications for physics. Therefore, we must make explicit notation such that

\[ \int_{-\infty}^{\infty} dx = \int_{\mathbb{R}} dx \quad \rightarrow \quad \int_{\mathbb{R}} dx = \int_{a}^{b} dx . \]

As \( a \) and \( b \) appear in Reference [2], these would be non-arithmatic numbers from \( \mathbb{F} = \{ \mathcal{F}_X \} \) such that

\[ \int_{\mathbb{R}} dx = \int_{\mathcal{F}_X} dx . \]

Furthermore, these would need to be sequential non-arithmatic numbers such that

\[ \int_{\mathbb{R}} dx = \int_{\mathbb{R}}^{\mathcal{F}(n)} dx = \int_{\mathcal{F}(n-1)}^{\mathcal{F}(n)} dx . \]

We have shown problems related to rigor when using the \( (n) \) enumeration scheme but those problems are indeed quite tiny compared to the non-rigor in the \( \int D\varphi \) path
integral measure notation. So, then therefore, arrive at the next problem: arithmetic is not defined among non-arithmatic numbers:

\[ \int_{\mathcal{F}(n-1)}^{\mathcal{F}(n)} dx = x \left|_{\mathcal{F}(n-1)}^{\mathcal{F}(n)} \right. = \mathcal{F}(n) - \mathcal{F}(n-1) = \text{undefined} \, . \]

We have already discussed this issue in Reference [2] as well. The resolution is that here we must treat the non-arithmatic numbers on the \( n \)th level of aleph as the natural numbers on the next higher level of aleph. In general, there is a normalization factor that we have neglected to write in Equation (24.1.) If we include something in that factor’s denominator to normalize the difference \( \mathcal{F}(n) - \mathcal{F}(n - 1) \), then that factor would go to zero. Something more complicated is called for. However, we can just go ahead and say that something shifts the level of aleph so that

\[ \int_{\mathbb{R}_x} dx = \int_{\mathbb{R}(n)} dx \simeq \int_{n-1}^{n} dx = 1 \, . \] (24.4)

Maybe we should use a \( \delta \) function of some kind? Without going into the details, there are a few points that must be raised. In the analysis of Reference [2], the metric was a constant function on the number line. In the MCM, however, we have the intention that each such successive level of aleph grows with respect to the previous level. This is desirable for the dark energy effect of References [25,29], whose dimensional analysis was briefly treated in Reference [77]. With the specific case of \( dx \to dx^0 \) required for dark energy, we would also hope to generate the thermodynamic arrow of time from first principles. An energy density on the \( n \)th level of aleph should expand into the larger “volume” of the higher level of aleph. In this way, we generated a free fall condition of metastability that is actually stable. Due to some nuance associate with the Higgs, it is suspected the universe occupies a metastable false which might spontaneously decay at any moment, but with metastability due to thermodynamic expansion, one would not expect spontaneous decay because the number line whose metric expands should go on forever (up to possible heat death effects.) Furthermore, the best reason that we should expect expansion from one neighborhood to the next is that it is required to generate the Cauchy–Riemann gravity mechanism of Reference [78].

For Cauchy–Riemann gravity, we have some general sense that we want to combine the \( \partial_0^2 \) operator of Newtonian mechanics with \( \partial_0 \) operator of the Schrödinger equation. These ideas need to be developed.

Another idea in the MCM is that the whole universe is just a fundamental particle
on a higher level of aleph and the present treatment of boundary terms at infinity is
highly germane to that interest. If we replace the integral over “all of spacetime” with
another integral over “all of one level aleph” whose bounds are set by non-arithmetic
numbers instead of infinity, the outside of the level of aleph, the bigger universe exists
where the level of aleph is like a particle. In particular,

If the integral over “all of spacetime” becomes normalized in the sense of Equation
(24.4), then we start to generate objects can be used as quantum numbers. If “all of
spacetime” is centered on \( \mathbb{N} X \), then we have a similar kind of integral centered on \( F_X \)
which should generate two one half quantum numbers. Indeed, this is more or less the
scheme of integer and half integers spin developed in Reference [8]. The fundamental
particles represented as 2D branes in the MCM unit cell are actually 4D spacetimes
due to the 3D span of \( x^i \) in the \( x^0 - x^i \) and \( \chi^5 - x^i \) automata.

Let \( f \) be some energy density function in 5D hyperspacetime. The MCM has the
goal to cure the problem of QFT’s infinite vacuum energy by dividing infinite energy
by vanishing hypervolume.

\[
E_{H(n)} = \int d\chi^5 \int dx^0 \int d^3x \, \delta(\chi^5 - X(n)) \, f(x^\mu, \chi^5)
\]

In Equation (24.3) we generate a boundary term

\[
I_{\text{boundary}} = \varphi \partial \varphi \bigg|_{-\infty}^{\infty}.
\]

In terms of fractional distance, all measurements must take place on a single level of
aleph so we arrived at

\[
I_{\text{boundary}} = \varphi \partial \varphi \bigg|_{F(n)}^{F(n-1)}.
\]

Even if we go to another level of aleph, what do we do with the analytic content of
\( \varphi \) and \( \partial \varphi \)? If \( \varphi(\vec{x}, t) = e^{i(\omega t - \vec{k} \cdot \vec{x})} \), what must we make of difference between the \( \vec{x} \) and
the \( t \)? We get a hint from Feynman in Reference [50] when he points out that the
time integral needs to be arbitrarily restricted to finite time even though the spatial
integral does not diverge over all of space. Might we, then, simply use infinity in the
usually for space but restrict time in the fashion of fractional distance? Indeed, the
MCM propagation is only in the “chiros” direction of \( \chi^5 \), so the branes of the unit
Next Steps and the Way Forward in the Modified Cosmological Model

cell only require strict localization in some temporal sense. What if we write

$$I_{\text{boundary}} = \varphi \partial \varphi \bigg|_{x^0 = F(n)}^{x^1 = \infty} \bigg|_{x^1 = -\infty}^{x^2 = \infty} \bigg|_{x^2 = -\infty}^{x^3 = \infty} \bigg|_{x^3 = -\infty}^?$$

We obtain

$$I_{\text{boundary}} = \varphi(\infty, F(n)) \partial \varphi(-\infty, F(n - 1)) ,$$

where

$$\varphi(\infty, F(n)) = e^{i(\omega F(n) - \infty)} , \quad \text{and} \quad \partial \mathbf{BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH BLAH 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BLAH BLAH BLA
a general algorithm for computing sequences of prime numbers and this problem deserves at least a glance. Before that, a review of the prime number theorem and survey of its corollaries are in order, along with a catalog of the relevant extensions in the neighborhood of infinity.

26 The Riemann Zeta Function in Quantum Theory

We have solved the Riemann hypothesis in Reference [2] and elsewhere [58–60]. However, we have not gone on to treat the problem which made Riemann’s hypothesis interesting: the prime counting problem, as in Section sec:Primes. Furthermore, certain work has relied on the RZF’s well known holomorphism to state that it must be holomorphic in the neighborhood of infinity. This holomorphism should be proven explicitly and a workaround should be devised if it does not. In the modern context, we have treated neither applications in cryptography or Hamiltonian operators in quantum mechanics proportional to $\zeta$. All of this work remains to be done and the latter is the topic of this section. The connection of RH to the prime numbers if well known and concisely stated in many places but the connection to quantum theory seems to be more like an intuition shared by a great number of well respected mathematicians and physicists. A survey of the evidence for this connection is in order, and particularly a survey of the Hilbert–Pólya program for tackling RH. Burnol writes the following regarding that program [80].

“[We are convicted] that the Riemann Hypothesis has a lot to do with (suitably envisioned) Quantum Fields. The belief in a possible link between the Riemann Hypothesis and Quantum Mechanics seems to be widespread and is a modern formulation of the Hilbert–Pólya operator approach. I believe that techniques and philosophy more organic to Quantum Fields will be most relevant. [T]his point of view has not so far led to success[.]”

If it was thought that studies in quantum theory might shed light on RH, then it is reasonable to expect that a solution to RH would shed light in the other direction. What can be extracted from the negation in the arena of quantum theory? Borwein, Bradley, and Crandall write the following [81].

“It is intriguing that any of the various new expansions and associated observations relevant to the critical zeros arise from the field of quantum theory, feeding back, as it were, into the study of the Riemann zeta function. But the feedback of which we speak can move in the other direction,
as techniques attendant on the Riemann zeta function apply to quantum studies.”

Consider an anecdote relayed by Cvitanović to an undergraduate course of this writer on nonlinear dynamics and chaos. Cvitanović explained that he had become stuck in his research for a long time and then he eventually discovered that the problem he was thinking about was equivalent to RH. He then advised, hardly in jest, that if any students should ever run into a problem where they find themselves trying to prove the Riemann hypothesis, then a change of research direction should be made. Now that RH is negated, one would search for the application which depended on it. Most likely, the application can be found in his book *Chaos: Quantum and Classical* [82]. While the exact mechanism by which the RZF is connected to quantum chaos is not yet fully known to this writer, the following respective words from Berry and Keating [83], and Brown [84] suggest that it is worth looking into. If so much association is seen by experts in the field, then it seems likely that the negation of RH would generate fruitful follow-on studies.

“Our purpose is to report on the development of an analogy, in which three areas of mathematics and physics, usually regarded as separate, are intimately connected. The analogy is tentative and tantalizing, but nevertheless fruitful. The three areas are eigenvalue asymptotics in wave (and particularly quantum) physics, dynamical chaos, and prime number theory. At the heart of the analogy is a speculation concerning the zeros of the Riemann zeta function (an infinite sequence of number encoding the primes): the Riemann zeros are related to the eigenvalues (vibration frequencies or quantum energies) of some wave system, underlying which is a dynamical system whose rays or trajectories are chaotic. Identification of this dynamical system would lead directly to a proof of the celebrated Riemann hypothesis. We do not know what the system is, but we do know many of its properties...”

“If you choose a number $n$ and ask how many prime numbers there are less than $n$ it turns out that the answer closely approximates the formula: $n/\log n$. The formula is not exact, though: sometimes it is a little high and sometimes it is a little low. Riemann looked at these deviations and saw that they contained periodicities. Berry likens these to musical harmonics: ‘The question is what are the harmonics in the music of the primes? Amazingly, these harmonics or magic numbers behave exactly like the energy levels in
quantum systems that classically would be chaotic.’ This correspondence emerges from statistical correlations between the spacing of the Riemann numbers and the spacing of the energy levels. Berry and his collaborator Jon Keating used them to show how techniques in number theory can be applied to problems in quantum chaos and vice versa. In itself such a connection is very tantalizing. Although sometimes described as the Queen of mathematics, number theory is often thought of as pretty useless, so this deep connection with physics is quite astonishing. [emphasis added] Berry is also convinced that there must be a particular chaotic system which when quantised would have energy levels that exactly duplicate the Riemann numbers. ‘Finding this system could be the discovery of the century,’ he says. It would become a model system for describing chaotic systems in the same way that the simple harmonic oscillator is used as a model for all kinds of complicated oscillators. It could play a fundamental role in describing all kinds of chaos. The search for this model system could be the holy grail of chaos... [We] cannot be sure of its properties, but Berry believes the system is likely to be rather simple, and expects it to lead to totally new physics. It is a tantalizing thought.”

27 The Hodge Theater and Anabelomorphy

In the MCM, $H_n$ a 4D Minkowski space devoid of matter energy. Since it is an identical Minkowski space, it is also a Lorentz frame. We assign to it a Hilbert space $H'_n$. The quantum mechanical state space of position states does not contain position eigenstates. It contains “wave functions” which are functions of three spatial variables. Although the Schrödinger equation is a time evolution equation, we say “time does not exist in quantum mechanics” because wavefunctions are functions of $x^i$ but never $x^0$. However, each is domain space spanned by $x^i$ is uniquely associated to a specific value of $x^0$ because 4D spacetime is constructible as an uncountably infinite set of 3D metrics at scalar values of $x^0$. Scholze has refused to acknowledge that Mochizuki is valid in his ABC proof to take two isomorphic objects as unequal. To the extent that Mochizuki’s “Hodge Theater” is only the MCM unit cell dressed in inaccessible jargon, now we will motivate the existence of two isomorphic objects which are not then same object in the sense abstract algebra. Although the domain of the wavefunctions $\psi(x^i)$ in each $H'_n$ is just a Euclidean 3-space, meaning the 3D part of the 4D metric is perfectly flat Euclidean space, we may take each Euclidean 3-space to the one at a given slice of constant $x^0$. Therefore, although all infinite
Euclidean 3-spaces are isomorphic, we may distinguish among them by labeling them with the affine parameter $x_0$. This can likely be parlayed into a rebuttal of Scholze’s claim the Mochizuki’s pseudo-theft of the MCM is pseudo-theft of worthless property.

Now we will briefly review the Mochizuki theory and demonstrate it’s similitude to the MCM as described in Reference [85].

“One could think of anabelomorphy in the following picturesque way: One has two parallel universes (in the sense of physics) of geometry/arithmetic over p-adic fields K and L respectively. If $K, L$ are anabelomorphic (i.e. $K \neq L$) then there is a worm-hole or a conduit through which one can funnel arithmetic/geometric information in the $K$-universe to the $L$-universe through the choice of an isomorphism of Galois groups $GK \simeq GL$, which serves as a wormhole. Information is transferred by means of amphoric quantities, properties and alg. structures. The $K$ and $L$ universes themselves follow different laws (of algebra) as addition has different meaning in the two anabelomorphic fields $K, L$ (in general.) As one might expect, some information appears unscathed on the other side, while some is altered by its passage through the wormhole. Readers will find ample evidence of this information funneling throughout this paper (and also in [Mochizuki’s papers] which lay the foundations to it.)

“I hope that these results will convince the readers that Mochizuki’s idea of anabelomorphy is a useful new tool in number theory with many potential applications (one of which is Mochizuki’s work on the abc-conjecture.) Especially it should be clear to the readers, after reading this paper, that assimilation of this idea (and the idea of anabelomorphic connectivity) into the theory of Galois representations should have interesting consequences for number theory. Here I have considered anabelomorphy for number fields but interpolating between the number field case and my observation that perfectoid algebraic geometry is a form of anabelomorphy, it seems reasonable to imagine that anabelomorphy of higher dimensional fields will have applications to higher dimensional algebraic geometry as well.”

Mochizuki has intuitively taken the “L” and ”R” universes abutted to each instance of $\mathcal{H}$ and constructed a “Hodge theater” of “L” and “K” universes, where the similarity of the Latin characters $K$ and $R$ is a credit to Mochizuki’s pseudo- forthrightness about using the MCM unit cell without citing it. Here, the reader’s attention is called to what Joshi hopes will “be clear to readers.”

HODGE STAR OPERATOR AND EM THEORY

118
28 Regularity Structures

ADD DESCRIPTION OF $\hat{M}^3$

“Martin Hairer takes $3m$ Breakthrough prize for work a colleague said must have been done by aliens.” [39]

Hairer’s work in regularity structures is said to have been by aliens by his colleague because his colleague knows very well that Hairer’s regularity structure is the MCM unit cell and the $\hat{M}^3$ operator. Consider the following from Reference [38].

“Remark 1.1 In the language of quantum field theory (QFT), equations that are subcritical in the way just described give rise to ‘superrenormalisable’ theories. One major difference between the results presented in this article and most of the literature on quantum field theory is that the approach explored here is truly non-perturbative and therefore allows one to deal also with some non-polynomial equations like (PAMg) or (KPZ) below. We furthermore consider parabolic problems, where we need to deal with the problem of initial conditions and local (rather than global) solutions. Nevertheless, the mathematical analysis of QFT was one of the main inspirations in the development of the techniques and notations presented in Sections 8 and 10.

Conceptually, the approach developed in this article for formulating and solving problems of the type (1.1) consists of three steps.

1. In an algebraic step, one first builds a ‘regularity structure’, which is sufficiently rich to be able to describe the fixed point problem associated to (1.1.) Essentially, a regularity structure is a vector space that allows to describe the coefficients in a kind of ‘Taylor expansion’ of the solution around any point in space-time. The twist is that the ‘model’ for the Taylor expansion does not only consist of polynomials, but can in general contain other functions and/or distributions built from multilinear expressions involving $\xi$.

2. In an analytical step, one solves the fixed point problem formulated in the algebraic step. This allows to build an ‘abstract’ solution map to (1.1.) In a way, this is a closure procedure: the abstract solution map essentially describes all ‘reasonable’ limits that can be obtained when solving (1.1) for sequences of regular driving noises that converge to something very rough.
“3. In a final probabilistic step, one builds a ‘model’ corresponding to the Gaussian process $\xi$ we are really interested in. In this step, one typically has to choose a renormalisation procedure allowing to make sense of finitely many products of distributions that have no classical meaning. Although there is some freedom involved, there usually is a canonical model, which is ‘almost unique’ in the sense that it is naturally parametrized by elements in some finite-dimensional Lie group, which has an interpretation as a ‘renormalisation group’ for (1.1.)

“We will see that there is a very general theory that allows to build a “black box”, which performs the first two steps for a very large class of stochastic PDEs. For the last step, we do not have a completely general theory at the moment, but we have a general methodology, as well as a general toolbox, which seem to be very useful in practice.”

Here, we see that Hairer’s regularity structure is built to accommodate the logical structure of the $\dot{M}^3$ operator. Consider the following from Reference [24].

“There are varying philosophies on quantum experimentation so let us define a process thoroughly. Two measurements must be made, $A$ and $B$. The boundary condition set by $A$ will be used to predict the state at $B$. The observer applies physical theory to trace a trajectory [from $A$] into the future and [to] predict what the state will be at that time. Before the observer can verify the theory, sufficient time must pass that the future event occurs. Once this happens a signal from the event reaches the observer in the present and a second measurement $B$ becomes possible. From the present we predict into the future. In time that becomes the past. When the signal from that event reaches the observer a theory can be tested. A three-fold process.

\[
\text{Present} \leftrightarrow \text{Future} \leftrightarrow \text{Past} \leftrightarrow \text{Present}.
\]

Therefore, since Hairer seems to have sufficiently developed the mathematical foundations of the issues we have posed in Section 1, one would conduct of survey of Hairer’s main results regarding “regularity structures.”

The reader is encourage to carefully note the 2013 publication of Reference [38] comes chronological later in the literature than does the 2012 publication date of Reference [3].
Is Hairer’s work really that great, or is the entire purpose of the Israeli funded Millennium Prize to recruit and fund people interested in stealing this writer’s due accolades? Does Hairer’s program contain anything which wasn’t already in the MCM, or is perhaps a pure rewrite?

29 **Gold Nuggets**

30 **Time Crystals**

31 **Amplituhedron**

Shortly after CITE, Arkani-Hamed published a famously received paper describing an “amplituhedron.” The purpose the research was to

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QUOTE.
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Obviously, this follows on the MCM work seeking to geometrize everything. One would conduct a survey of work pertaining to the amplituhedron and reformat the jargon in terms of the objects of the MCM.

32 **The Ehrenfest Paradox**

33 **Quantum Set Algebra**

When donning his anonymous reviewer mask at IJTPD to detract from this writer’s theory without having to allow responses or rebuttals, Finkelstein remarked that that this writer, “Doesn’t know the ADM theorem from a hole in the ground,” in slightly more polite language. While this writer had never heard of the ADM theorem at that time, this writer studied it and immediately identified a gap.

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DAVID
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“the author clearly has no concept of the ADM mass-energy”

the author is what physicists hate most: someone who writes as if they know what they are talking about but does not

34 **The Poincaré Conjecture**

and the Sphere Theorem
The fundamental idea of the MCM that smooth deformations of spacetime are allowed between Anti-de Sitter and de Sitter spaces follows from a famous result of Perelman.

**I THINK I AM MIXING UP IDEAS IN THE ABOVE PARAGRAPH**

Turn the sphere inside out, then shrink the old sphere to zero by going to a new “level of aleph” which is now quantified as taking a discrete step toward a new neighborhood of fractional distance along a time interval \([t_{\text{min}}, t_{\text{max}}]\).

Perelman’s result describes what is called Ricci flow. This is the problem of the flow of the metric across slices of spacetime referred to in Section XXX. Although physics often concerns itself with the limit of dynamical 3-metrics stepped from one Lorentz frame to the next, Perelman was able to find an analytical solution for the specific case of maximally symmetry. It is due to the spherical symmetry used by Perelman that he was able to obtain and analytical expression for the deformation of a negatively curved manifold into a positively curved one. Before Perelman, it was not known if it could be guaranteed that such deformations are possible. In the MCM, we take it for granted that such deformations guarantee the existence of solutions by which \(\Sigma^{\pm}\) can be connected. In this way, the MCM’s idea was cutting edge because it relied on an abstract mathematical result which was not available when other people made their own earlier inquiries into quantum cosmology. For this reason, one would undertake a survey of the sphere theorem and other applications of the

### 35 Bifurcations

Usually, we can only approach the X-point. The arithmetic of numbers in the neighborhood of infinity allows us to put \(\infty\) at the X-point and then study the immediate neighborhood of the bifurcation. In other words, without \(\infty\) the parameter where the bifurcation happens can only be identified within some range but \(\infty\) may afford us the opportunity to determine exact X-points.

**NEED TO CHECK IF THAT’S RIGHT THAT WE DON’T KNOW THE X-POINT**

CHECK PERIOD DOUBLING CASCADE

Reference [65]

### 36 Breakdown of Navier–Stokes Solutions

This problem, again, is similar to the problems about electric field lines and the EF coords. The problem asks if there exist solutions which do not blow up in finite time.
We would like to apply the principles of fractional distance analysis to show that before a solution could blow up, the time would have to enter the neighborhood of infinity. Since “finite time” means “a natural number of discrete time steps,” the can prove that the time can never enter the neighborhood of infinity.

CITE PROBLEM

37 The Yang–Mills Mass Gap

A solution to this problem was proposed in Reference [78] and it remains to be developed. In Reference [78], we show that the successive application of $\hat{M}^3$ should be such that the a Cauchy line integral around the entire complex plane (on a given level of aleph, or in a given neighborhood of fractional distance) should contain three poles, and not two. In quantum field theory, the properties of fields are often linked to poles on the complex plane. While the usual two poles can be expected to cancel, a third pole should generate the non-vanishing mass gap. It remains to identify the exact details of both the problem and solution but the non-zero condition should follow from the Cauchy formula. If a closed path of integration enclosed three poles, two of which have residues summing to zero, the residue of the third pole must have a manifestation in the physics. A formal statement of the problem must be obtained because this writer has never seen it.

POLE MASS

38 The Banach–Tarski Paradox

In 1924 CITE, Banch and Tarski published a decomposition of the unit sphere. They showed that pointwise operations on a sphere’s points may be executed such that the recombined points constitute two spheres. While some claim that there is no paradox because Banach and Tarski were correct to show that one sphere’s points can be used to construct a second exactly equal sphere, the paradox is that one does not equal two. For instance, in set theory, one might have a set with one sphere in it which would be expected to remain as a set with one sphere in it under operations of the form considered by Banach and Tarski. To avoid this paradox, one would invoke the infinite point density of points suggested in Reference [2]. Usually, one assumes that the point density of points is unity, meaning there exists one point per point. We have argued to the contrary that points, being singular, must contain an infinite number of points due to certain reasons which could make for an entire thesis problem on their own. When the point density of points is infinite rather than unity,
the Banach–Tarski paradox goes away immediately. The points left over at the end have appeared miraculously but instead they have been take from the infinite well of points created in the initial pointwise decomposition of the sphere.

39 Topology of the Real Line

In Reference [2], we have to some great length to define a topology for $\mathbb{R}$. In the opinion of this writer, it is likely that the work can be extended to show that the topology of $\mathbb{R}$ is $S^0$: the zero sphere. The argument proceeds as follows.

Firstly, Reference [2] goes very far to define the downward representation of geometric objects in algebraic language but questions remain about the representations of algebraic objects with geometric language. We know it is possible to put an infinite number of algebraic points into a geometric point, but the reverse relationship is not yet determined. However, by the infinite point density of points, meaning that each geometric point contains infinite points as opposed to the single point per point usually presumed, it should be provable that $\mathbb{R}$ is equivalent to two points. The
topology of two points is $S^0$.

However, if the idea were to be implemented, then that would have further application in the MCM resolving $\chi_0^4$ as an interval rather than a single point. In general, the protocols of local gauge symmetries in QFT require the resolution of internal coordinate systems at ordinary spacetime points.

40 The Erdős–Strauss Conjecture

Kyle boiled this down to some 1D problem relating addition to multiplication. Try to impose the constraint in the exponent with

$$\Phi^2 = \Phi + 1.$$ 

41 Twin Primes Conjecture

This is closely related to RH question about the distribution of the primes.

REITERATE THE PREVIOUS

42 The Limits of Sine and Cosine at Infinity

The results

$$\lim_{x \to \infty} \sin(x) = 0 \, , \, \text{ and } \lim_{x \to \infty} \cos(x) = 1 \, ,$$

(42.1)
derived in Reference [86] rely on $\infty$ having multiplicative absorption but not additive absorption. This has several undesirable results for basic arithmetic. Given subsequent work and study, the framework in which the result was derived no longer seems general enough to call the result a general result. Therefore, the result should revisited under the arithmetic axioms posed in Reference [2]. This framework of analysis is sufficiently general that it does not diverge from what is usually called real mathematical analysis, or standard analysis. It is expected that the result will hold up when the immeasurable, or non-arithmetic numbers $x \in \mathbb{F}$ serve as some regularized boundary condition along $\mathbb{R}$ such that the behavior of sine and cosine on approach to $\infty$ is the mirror image of the behavior on egress from the origin, along $\mathbb{R}^+$. However, further analysis is required to determine whether the result will hold up under the general arithmetic axioms.
Vacuum Polarization

The Arnowitt–Deser–Misner Theorem

The Borde–Guth–Vilenkin Theorem

This is a very nice theorem they have here. I will discuss the simple case of the null geodesic, pic related. Since the affine parameter is clearly finite for monotonic increasing $a(t)$, we can assume that there is a "quantum nucleation event" back there somewhere. For that event to conserve 4-momentum, there has to be a universe on the other side of it where time has a minus sign on it. Namely, the ADM theorem proves that the universe must have positive-definite $p^0 = E$. If there’s not another universe on the other side of the quantum nucleation event, then it didn’t conserve 4-momentum. (There is some wiggle room in the ADM theorem to allow either positive-definite or negative-definite $P^0$. The main result is that is it non-zero.) If the energy of the other universe is negative-definite, then binding energy is positive there and eventually we get a minus sign attached to "d tau" on the other side of the nucleation event. I think you can use that other "d tau" with the minus sign on it to smoothly continue the geodesic through the nucleation event, thus restoring past completeness for null geodesics. Since the whole BGV theorem is a generalization of pic related, the completeness I suggest should also generalize.

At the end of the BGV paper, they criticize the cyclic cosmology of Steinhardt and Turok as not avoiding the past incompleteness they have demonstrated. However, Steinhardt and Turok have conjured an anomalous increment of 4-momentum from somewhere. Putting two universes at each bounce fixes this momentum problem. Steinhardt and Turok also make some nonsense remark about expansion "diluting" the entropy accumulated in one bounce so that the next bounce resumes at the low entropy state. The issue that fixes the problem with 4-momentum also fixes the entropy problem the increase in the entropy of the forward time universe is exactly offset by the decrease of entropy in the universe whose "d tau" has a minus sign on it.

Rather than forcing past incompleteness, the BGV theorem forces piecewise structure onto the affine parameterization of geodesics longer than some scale.

The Higgs Boson

CITE GURALNIK
47 The $L^2$ Condition in Quantum Theory

The usual Hilbert space of position space wavefunctions is $L^2$: the space of square integrable functions.

$$L^2(\mathbb{R}) \ni \psi : \mathbb{R} \rightarrow \mathbb{C} \iff \int_{-\infty}^{\infty} dx |\psi(x)|^2 < \infty . \tag{47.1}$$

The physics of the $L^2$ condition is that the wavefunction must support the probability interpretation. Being less than infinity, the integral of the absolute square of the wavefunction $|\psi|^2 = \psi^* \psi$ over all of space can be normalized to unity. Recalling that $c_1 |\psi\rangle$ and $c_2 |\psi\rangle$ are the same state in Hilbert space, meaning they are not linearly independent vectors, for any $\int dx |\psi|^2 = A^{-2}$ we simply introduce $\psi' = A\psi$ as a properly normalized wavefunction such that $\int dx |\psi'|^2 = 1$. This tells us that the probability of finding the particle somewhere in the universe is 100%. The $L^2$ condition also tells us that the probability of finding the particle at infinity is 0%. Sometimes, the $L^2$ condition is informally stated as a requirement that $\psi(\infty) = 0$. When one comes across the single most common phrase in physics, “Assuming boundary terms at infinity go to zero,” this is a reference to the assumed $\psi(\infty) = 0$ condition, as in Section 24. Very often, the Cauchy residue theorem is applied with part of a closed integration path at infinity where the integral vanishes due to $\psi(\infty) = 0$.

Very much of quantum of theory is constructed around this boundary condition. New boundary conditions are always a first thought in the search for new physics and there exists an one other place where the probability of finding the particle vanishes, one that has been little considered, if at all: the location of the observer. As a matter of practice, the MCM convention is to place the observer at the origin. This is related to the Lorentz frame but it is also an independent convention for tidiness. Therefore, one would construct a state space of wavefunctions which go to zero at infinity, and at the origin. One would examine the case for taking the subdomain of $L^2$ supplemented with the boundary condition $\psi(0) = 0$ such that it is the $S_1$ space in a $\{S_1, S_2, S_3\}$ RHS.

In addition to new boundary conditions, new symmetries are also highly regarded in the search for new physics. In fact, symmetries are a type of boundary condition. Using the one point compactification of $\mathbb{R}$, the $L^2$ condition may be approximated as

$$\psi(x) \in L^2 \implies \begin{cases} \psi : S^1 \setminus \{\infty\} \rightarrow \mathbb{C} \\
\lim_{x \rightarrow \infty} \psi(x) = 0 . \end{cases} \tag{47.2}$$
Calling the proposed subdomain $L^2_0$, we may write

$$
\psi(x) \in L^2_0 \implies \begin{cases} 
\psi : S^1 \setminus S^0 \to \mathbb{C} \\
\lim_{x \to 0} \psi(x) = 0 \\
\lim_{x \to \infty} \psi(x) = 0
\end{cases} \quad (47.3)
$$

$S^0$ is two points and we have removed $x = 0, \infty$ from the domain of functions in $L^2_0$. This represents a radical change in the topological structure of quantum theory and it may provide powerful new tools for doing quantum mechanics. Furthermore, the Lorentzian structure of spacetime is such that we may treat $\psi(x, t)$ as if it were a function $\psi(z)$ of a single complex variable. (See Section 11.) Denoting the Riemann sphere $S^R = S^2 \setminus \{\infty\}$ and employing the one-point compactification of $\mathbb{C}$, we have

$$
\psi : S^R \to \mathbb{C} \implies \psi : S^2 \setminus S^0 \to \mathbb{C} \quad (47.4)
$$

While the time dependence requires to look beyond the bounds of Hilbert space, the removal of the origin from the domain of $\psi$ generates a new topology with more symmetry. The old domain was a sphere missing a point, a classically vexing object in analysis, while the new domain is the topological difference of two spheres. In the full $L^2(\mathbb{R}^3)$ theory of three spatial dimensions, the domain of $L^2_0(\mathbb{R}^3)$ wavefunctions is still $S^3 \setminus S^0$. With such problems in physics as the axial current anomaly seeming amenable to improved symmetry conditions, the possible applications of the $L^2_0$ condition are very many. Section 39 proposes to prove that real line has topology $S^0$, and that would exotic, rich flavor to the state space proposed here. A statement that $\mathbb{R} = S^0$, that would require some nuance to avoid an obvious contradiction with $\mathbb{R} \setminus \{0, \infty\} = S^1 \setminus S^0$ but such nuance is well suited to description in the fractional distance framework of real analysis [2].

### 48 Intrinsic Periodicity

Dolce has a large

**REVIEW PAPERS**

- Compact time, extra dimensions, etc...
- CFTs can be rescaled by a radius parameter. Rescaling the radius to zero or infinity should be like going to a different level of aleph.

https://arxiv.org/abs/1006.5648

49 The Cauchy Residue Theorem

Compare the coefficient to the $\gamma^5$ in the ontological scheme.

50 Eddington–Finkelstein Coordinates

This is quite similar in nature to the previous remarks regarding electric field line breaking. We have now way to smoothly evolve a solution through the event horizon of a black hole and yet this is required for $\tilde{M}^3$ to push through the black brane. (OR IS IT DUE TO DISCRETE TRANSLATION???)

The Schwarzschild metric is

$$ds^2 = - \left( 1 - \frac{2GM}{r} \right) dt^2 + \left( 1 - \frac{2GM}{r} \right)^{-1} dr^2 + r^2 d\Omega^2 ,$$

with

$$d\Omega^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2 .$$

$$r^* = r + 2GM \ln \left| \frac{r}{2GM} - 1 \right| \implies \frac{dr^*}{dr} = \left( 1 - \frac{2GM}{r} \right)^{-1} .$$

Then, when such coordinates as Eddington–Finkelstein (EF) coordinates necessarily reach infinity on approach to singularities such as $\emptyset$, the arithmetic of numbers in the neighborhood of infinity provides new tools for the transfinite continuation. For instance, we might take $\Omega$ and $A$ as the event horizons about the singularity $\emptyset$, which are exactly where EF coordinates diverge (see Section 50.)
51 Propagator

What happens when it goes to another level of aleph?

52 Mechanical Precession

While some people say that the problem of classical mechanical precession is solved with Newtonian mechanics, this author has never seen a convincing treatment to that effect. This author agrees with Laithwaite and others that the apparent antigravity effects of classical precession are an unsolved anomaly.

For instance, rotations do not commute but they are implicitly assumed to commute when the angular momentum vectors in the Newtonian force diagrams are assumed to have the usual commutative properties of vectors. The non-commutativity of the rotations shows up in the $\mathcal{O}(d\theta^2)$ terms which are ignored classically. However, we now have reason to re-examine these terms with fractional distance analysis.

Nima’s paper about new dimensions at sub-millimeter scale is striking because we have computed the characteristic scale of the effect at $10^{-4}$ meters. We now physics allows weird effects at this scale and this is the scale generated by the MCM.

While it remains to motivate the place of the electrodynamic constant in a statement of pure mechanics, the equation

$$F_{\text{net}} \hat{\Phi} \hat{\theta} := \sum_{n=1}^{\infty} \alpha^n (\hat{F}\hat{\Phi}^n - \hat{F}\hat{\Phi}^{-n}) , \quad \text{with} \quad \hat{F} := m\omega^3 r , \quad (52.1)$$

the implicit reliance on what would be called levels of aleph is obvious. The classical prediction for a the mechanical system in question, Laithwaite’s spinning wheel, is that the net force in the vertical direction should be zero.

53 Quantum Gravity

54 Models of Topology Change

55 Gravitational Waves

The main bulk of the MCM unit cell is constrained, presently, by the stipulation of Kaluza–Klein theory that all 5D matter energy mush vanish. We have implemented 4D matter-energy on $\mathcal{H}$, and there may be matter-energy in $\varnothing$, but $\Sigma^\pm$ are essentially
empty. The formal statement of this emptiness is

$$R_{AB} = 0 .$$

(55.1)

The 5D Ricci tensor vanishes everywhere. While this is innate in Kaluza’s original theory, Wesson points give high praise [74] to a theorem of Campbell and Magaard which proves that 4D general relativity governed by

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

(55.2)

can always be embedded in Equation (55.1.) $T_{\mu\nu}$ is the 4D stress-energy, $R$ is the Ricci scalar, $R_{\mu\nu}$ is the 4D Ricci tensor, and $g_{\mu\nu}$ is the 4D metric. However, 4D gravitational radiation satisfies $R_{\mu\nu} = 0$ and it must follow that 5D gravitational radiation does as well. Therefore, gravitational waves provide a ready mechanism for the propagation of information across the unit cell which is mostly constrained to be empty, but not totally so. A most prominent feature of gravitational radiation is that there is no dipolar gravitational radiation. The lowest multipole moment in gravitational radiation is the quadrupole. The MCM solution for Dark energy supposes that there exist singularities at past and future timelike infinity $\mathscr{I}^\pm$, and the $A$- and $\Omega$-branes are most likely to have infinite curvature as well. Under lattice vibrations, these four singularities may act as a source for gravitational waves.

The MCM is such that the distance between adjacent named branes exceeds the range of the gravitational interaction.

56 Scalar Waves

Scalar waves are a non-trivial solution to Maxwell’s equations in which $\vec{E} = \vec{B} = 0$ everywhere. Therefore, scalar electromagnetic waves are a

Putting the potential into the wavefunction is kind of like achieving $\hat{M}^3$ by putting $F = ma$ into the Schrödinger equation, or vice versa.

PIC ON DESKTOP

57 5D Wave Mechanics

Another paper by Overduin and Wesson.
Figure 12: In the convention where the Ricci scalar $R$ goes $\pm \infty$ at $A$ and $\Omega$, the MCM unit cell arranges four singularities which might function as a source for gravitational radiation. The singularities at $\mathcal{I}^\pm$ follow from the MCM mechanism for dark energy.

58 The Cosmological Constant

Weinberg paper.

59 Wavefunction Collapse

Huygens principle.

Cite excerpt from Isham [51].

Theory needs some new thing for collapse. The two extra steps in $\dot{M}^3$ should be sufficient to get it done. Whatever those steps end up being, two should be enough to implement in a natural way what is currently implemented as a single ad hoc step.

60 Functions of Changing Numbers of Variables

ISHAM

61 Stimulated Emission from the Vacuum

FROM [34]: Summarizing, we have shown that a RI amplifies and scatters light to higher frequencies. Likewise, if the probe pulse were to be reduced to the level of quantum fluctuations, we may expect to see the RI excite the vacuum states.
Everyone is familiar with the idea that the universe might just be a fundamental quantum particle in a larger universe. I think one of the coolest things about my fractional distance framework of analysis is that allows one to quantify this idea. For instance, in the current scheme of analysis, inverse square force laws only go to zero at infinity. In fractional distance, inverse square laws go to zero at the end of a given local neighborhood of constant fractional distance. Furthermore, the whole width of any given neighborhood of fractional distance has only zero percent fractional distance with respect to infinity. This allows us to construct an entire universe with an "effectively infinite internal radius" which will show up as a particle in a larger universe with zero linear scale, like an electron. If you replace the Euclidean metric on R (which should measure some affine radial parameter of linear scale) with "the dark energy expanding metric" on R such that each neighborhood of fractional distance is infinitely large
with respect to it left-adjacent one, then you can can create eternally self-similar
universe-is-a-particle-in-a-universe-which-is-a-particle-in-a-universe-which-is...

72 Intermediate Numerical Scale

as in Reference [87].  
Survey of hyperreals and surreals. 
  WHEELS?

73 Cellular Automata

74 Bell’s Theorem

“No physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics;”

75 The Ford Paradox

The Schrödinger equation for the identity operator is a good way to solve this one. 
Quantum fuzziness should come in through phase decoherence when sines’ and cosines’ arguments are rescaled as $n\pi x \rightarrow n\pi \Phi^n x'$, or some such mechanism which remains to be determined (see Subsection 1.8.)

76 Period Doubling

Reverse Zeno’s paradox

77 String Theory

78 Local Gauge Symmetry

Gauge bosons are called gauge bosons because
Maldacena talk 
Internal coordinates at every point in space.
Regarding the spacelikeness and timelikeness of $\chi^4_{\pm}$, one would also undertake considerations regarding the time and the imaginary time of statistical mechanics. Seemingly, the spacetime interval of imaginary time would be spacelike even while translation by an increment of imaginary time would have to be thought of as time translation. While it would make for another problem deserving its own section on this paper, we cite a survey of use cases for the well known real time and imaginary times, as well as a survey of two-time models such as the those described by Bars. CITE Certainly the gist of the physics surrounding the imaginary time, i.e.: one thing is just like another thing if $t$ is multiplied by $\sqrt{-1}$, is well suited to the theme of the MCM.

83 Reverse Time in QFT

The reverse time application in QFT

Diagrams are like amplitudes. Once a pair has been created, the amplitude for it to annihilate is 1.

84 The Cauchy Residue Formula

85 The Ontological Basis

86 The Wheeler Model of Quantum Cosmology

Is this the DeWitt model?
Next Steps and the Way Forward in the Modified Cosmological Model

87 Geons

88 The MCM Lattice

An early requirement in the MCM was for a finite distance separating the branes in the MCM unit cell across which the gravitational interaction would go to zero.

89 The Ontological Resolution of the Identity

90 Levels of Aleph

91 Rydberg States

Rydberg states are very high energy atomic bounds states near the ionization energy. One of the least understood areas of atomic physics regards the structure of the Hamiltonian near the ionization energy. Below the ionization energy, the Hamiltonian is represented as a diagonal matrix with a countable infinity of energy eigenvalues written as its diagonal entries. Beyond that countable infinity, however, the infinite discrete bound states give way to a continuum of free particle states. Plainly, the Hamiltonian cannot be no longer be represented a matrix where the energy eigenstates form a continuum. However, this is the usual interpretation.

Numbers in the neighborhood of infinity are well suited to the study of the transition from bound states to free states at the ionization level. Using natural numbers, there is no way to even approach energy is near the ionization energy in terms of quantum numbers. For an $n \in \mathbb{N}$, there are an infinite number of higher of higher energy bound states. Obviously one might limit the number of states at the Planck scale but the mathematical structure of the theory is such that for any $n \in \mathbb{N}$, there an infinite number of $E_m < E_{\text{ion}}$ with $m > m$. So, the study of states near $E_{\text{ion}}$ (in terms of the quantum number) can be implemented with the lowest energy free particle state labeled $E_{\infty}$, the highest energy bound state labeled $E_{\infty-1}$, etc. Such a program should be carried out to determine whether or not anything new and useful might be obtained.

92 The Advanced Electromagnetic Potential

The changing level of aleph gives a preference for causality over retro causality.

If it’s getting smaller, that is like gravitational collapse. If it’s getting bigger, that is like expansion into vacuum.
One slightly different take on the issue of retrocausality in the MCM is that causal and retrocausal signals may be associated with different levels of aleph so that one dominates over the other.

93 Stability of the Vacuum

It is said to follow as a result of the observed mass of the Higgslike particle that the vacuum of our universe is only metastable, and that the vacuum could spontaneously collapse to a lower energy state at any moment, destroying the universe in the process. One would conduct of survey of the details of this mechanism, and then study the consequences following from the MCM dark energy solution. In that solution, the present falls into the future as if in atmospheric free fall. Classical terminal velocity is not precisely a stable state, but neither is it unstable. However, there is no risk that an object in free fall might suddenly change its behavior short of hitting the ground. So, when the hypersurface of the present is in gravitational freefall toward the mass of a singularity in the distant future, we might cast the alleged metastability of the vacuum without the possibility of collapse to a lower energy state. Instead, the “metastability” of the vacuum does not imply any lower energy states for the quantum vacuum. Instead, we implement metastability as an analogue of free fall.

94 The Anharmonic Oscillator Lagrangian

The MCM answers the fundamental problem of QFT with the spectrum of cosmological lattice modes [8] but what we might call the fundamental applied problem of QFT remains open. Namely, there is no analytical solution to

$$Z = \int D\varphi \exp\left\{ i \int d^4x \left\{ \frac{1}{2} \left( \partial^2 \varphi^2 - m^2 \varphi^2 \right) - \frac{\lambda}{4!} \varphi^4 + J\varphi \right\} \right\}. \quad (94.1)$$

Here, $\varphi = \varphi(x^\mu)$, $J = J(x^\mu)$, and $D\varphi$ is the Feynman path integral measure. There are some mathematical problems, in the sense of rigor, with Feynman’s notation for the infinite dimensional integral over $D\varphi$ but the present problem bears only on the integral in the exponential. The issue with $D\varphi$ doesn’t impede our ability to make predictions but the lack of any general solution to the exponentiated integral is the major outstanding bottleneck on QFT’s predictive capacity. In the absence of the anharmonic term $\frac{\lambda}{4!} \varphi^4$, the exponentiated integrand reduces to the Lagrangian density of the harmonic oscillator $\mathcal{L}_{\text{HO}}$ added to a source term $J\varphi$. Integration by parts yields a well known analytical solution to that integral. In the anharmonic case, the best
we can do is the truncation of one or another infinite series to arbitrary order. The higher order terms become very hard to calculate and quantum field theorists would prefer an analytical solution in closed form. Zee writes the following [55].

"Doing quantum field theory is no sweat, you say, it just amounts to doing the functional integral (\([94.1]\)). But the integral is not easy! If you could do it, it would be big news."

One method for the approximation of Equation (94.1) is to separate the anharmonic part as

\[
Z = \int D\varphi \exp \left\{ i \int d^4x \left[ \tfrac{1}{2} ((\partial \varphi)^2 - m^2 \varphi^2) + J_\varphi \right] \exp \left\{ -i \frac{\lambda}{4!} \int d^4w \varphi^4 \right\} \right. . \quad (94.2)
\]

Here we have introduced \(w^\mu\) as a dummy integration variable. One applies a trick, sometimes called the Feynman method of integration, to write

\[
Z = \exp \left\{ -i \frac{\lambda}{4!} \int d^4w \left[ \frac{\delta}{\delta (i J(w))} \right]^4 \right\} \int D\varphi \exp \left\{ i \int d^4x \mathcal{L}_{\text{HO}} + J_\varphi \right\} , \quad (94.3)
\]

where \(\delta\) is the functional derivative such that

\[
\frac{\delta}{\delta J(w)} \int d^4x J(x) \varphi(x) = \varphi(w) . \quad (94.4)
\]

This derivative hits the exponential of \(J\) in the usual way. The trick is that the fourth derivative with respect to \(i J(w)\) will pull four powers of \(\varphi\) out of the source term \(J_\varphi\). After taking the derivative, the first term in Equation (94.3) reduces to the second term in Equation (94.2). Written as a derivative, the anharmonic term is no longer has \(\varphi\) in it so we may pull it outside of the path integral over \(D\varphi\), as in Equation (94.3.). The remaining integral on the right may be solved exactly and the exponential on the left is expanded as an infinite series. Thus, the anharmonic oscillator problem in the presence of a source is approximated to arbitrary order in that exponential series. However,

\[
\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots , \quad (94.5)
\]

and each \(x\) in this case has a fourth functional derivative with respect to \(J\) in it, and an integral, so the series gets messy very quickly. The other usual method for tackling Equation (94.1) is to separate the \(J_\varphi\) term at first instead of the \(\varphi^4\) term. Then one writes the exponential of the integral over \(J_\varphi\) as a infinite series and approximates \(Z\) to arbitrary order in that way. This also get messy very quickly.
95 Fast Radio Bursts

This problem is an application in the MCM’s early venue: cosmological phenomenology. It is suggested that fast radio bursts [88,89] should be modeled as black hole lightning. Petroff, Hessels, and Lorimer write the following [89].

“[P]ulsar surveys have led to the serendipitous discovery of fast radio bursts (FRBs.) While FRBs appear similar to the individual pulses from pulsars, their large dispersive delays suggest that they originate from far outside the Milky Way and hence are many orders-of-magnitude more luminous. While most FRBs appear to be one-off, perhaps cataclysmic events, two sources are now known to repeat and thus clearly have a longer-lived central engine. [sic] With peak flux densities of approximately 1 Jy, this implied an isotropic energy of 1032 J (1039 erg) in a few milliseconds or a total power of 1035 J s$^{-1}$ (1042 erg s$^{-1}$.) The implied energies of these new FRBs were within a few orders of magnitude of those estimated for prompt emission from GRBs and supernova explosions, thereby leading to theories of cataclysmic and extreme progenitor mechanisms. [sic] Currently, the research community has no strict and standard formalism for defining an FRB, although attempts to formalize FRB classification are ongoing [sic]. In practice, we identify a signal as an FRB if it matches a set of loosely defined criteria. These criteria include the pulse duration, brightness, and broadbandedness, and in particular whether the [dispersion measure] is larger than expected for a Galactic source.”

The dynamical origin of large-scale charge distributions leading to terrestrial lightning are not understood. EM theory suggests that large-scale charge formations should not appear in the atmosphere because they would seem to neutralize themselves at small-scale. However, lightning is known to occur on a scale which is only possible given unexplained large charge distributions. It is also known that lightning is a radio source. Therefore, one might suppose that the mechanism for the anomalous assembly of large-scale charge distributions between a planet and its atmosphere is also in play between a black hole and its accretion matter. The famous no-hair theorem (which is a conjecture) permits black holes to have only three observable parameters, one of which is electric charge. In the absence of a dense atmosphere, the
amount of charge needed to induce dielectric breakdown in the local neighborhood of a black hole is expected to be very large. Therefore, it is reasonable to suppose that “cataclysmic” and “one-off” FRB events are black hole lightning. Dielectric breakdown of accretion matter is one possible mechanism, and dielectric breakdown of the vacuum is an exotic mechanism which might be investigated.

As a work in phenomenology, one would assemble known radio models of terrestrial lightning and then compute the characteristics of the black hole lightning needed to produce the observed radio flux at cosmological distances. A few known repeating FRB sources are understood as black hole lightning storms. Planetary storm clouds are known not to totally discharge in single lightning bolts so some constraint mechanism must be introduced to explain the possible incomplete electric discharge of a black hole upon a single FRB event.
Appendix A: The Origin of $\hat{\mathcal{M}}^3$

The original motivation for $\hat{\mathcal{M}}^3$ was only a requirement for some third order operator needed to generate the $(\Phi \pi)^3$ term appearing in $\alpha_{\text{MCN}}^{-1} = (\Phi \pi)^3 + 2\pi$. However, the third order operator became independently useful, as in Section 1. For breadth in this appendix, we will fully review the original development noting that, in fact, the $\hat{\mathcal{M}}^3$ operator was first conceived only as a way to force a cubed term into a theory where cubed terms usually don’t appear. The first statement of $\hat{\mathcal{M}}^3$ appeared in Reference [24], hereafter called FSC. This was published before the construction of the unit cell [6] whose structure provides the best framework for understanding $\hat{\mathcal{M}}^3$. While the current program for $\hat{\mathcal{M}}^3$ is focused on the spatial translation across the the unit cell as

$$\hat{\mathcal{M}}^3 : \mathcal{H}_1 \rightarrow \Omega \rightarrow \mathcal{A} \rightarrow \mathcal{H}_2 .$$  \hspace{1cm} (A.1)

the original program [3,24] described abstract algebraic translation across a the state spaces of a rigged Hilbert space (Gelfand triple) as

$$\hat{\mathcal{M}}^3 : \mathcal{H}'_1 \rightarrow \Omega' \rightarrow \mathcal{A}' \rightarrow \mathcal{H}'_2 .$$  \hspace{1cm} (A.2)

For consistency in the following, the original symbol $\aleph$ is replaced with the current convention $\mathcal{A}$. What follows is the first statement of requirements for $\hat{\mathcal{M}}^3$ [24].

“If the observer’s proper time is $t_0[\cdot]$ we can write the following with certainty.

Past := $[t_{\text{min}}, t_0]$
Present := $[t_0]$
Future := $(t_0, t_{\text{max}}]$ .

“In General Relativity there is no inertial frame but one is assumed and $L^2$ is the vector space of this approximation. Unitary evolution in this space is characterized by orders of $\alpha$. This number should be a direct prediction of a complete Quantum Theory. A finely structured theory is needed, one which does not reside in the Hilbert space $\mathcal{H}$ alone. To be precise, define a
Gelfand triple \( \mathcal{A}, \mathcal{H}, \Omega \) where each set contains a Minkowski picture \( S \).

\[
\begin{align*}
\mathcal{A} &= \{ x^\mu \in S | t_{\min} < t < t_0 \} \\
\mathcal{H} &= \{ x^\mu \in S | t = t_0 \} \\
\Omega &= \{ x^\mu_+ \in S | t_0 < t \leq t_{\max} \}
\end{align*}
\]

"The Minkowski diagram gives a clear illustration. The past and future light cones define the half spaces \( \aleph \) and \( \Omega \) and the hypersurface of the present is a delta function \( \delta(t - t_0) \). The present is defined according to the observer so it is an axiom of this interpretation that the observer is isomorphic to the \( \delta \) function. Our task is how to reconcile calculations in \( \mathcal{H} \) with the actual dynamics of Nature proceeding around us and through us in \( \aleph \) and \( \Omega \). To this end define an operator \( \hat{M}^3 \) that is non-unitary and complimentary to the unitary evolution operator \( \hat{U} \).

\[
\begin{align*}
\hat{U} : \mathcal{H} &\rightarrow \mathcal{H} \\
\hat{U} := &\partial_x \\
\hat{M}^3 : \mathcal{H} &\rightarrow \Omega \rightarrow \mathcal{A} \rightarrow \mathcal{H} \\
\hat{M}^3 := &\partial_t .
\end{align*}
\]

The above appeared in FSC only as a segue into the main result of the short paper titled “Derivation of the Fine Structure Constant.” While terse brevity is a hallmark of an academic writing style, the brevity of the segue into the main result of FSC has been cited as rendering the entire work nonsensical. The purpose of this appendix is in some part to refute such claims. To that end, the reader is encouraged to understand that these few words excerpted from the beginning of the paper were written only to introduce the main result that \( \alpha_{\text{MCM}} = (\Phi \pi)^3 + 2\pi \approx 137 \) very nearly replicates the accepted value of the fine structure constant \( \alpha_{\text{QED}} \). This does not hinge on any of the material quoted above yet detractors cite the abrupt progression through the introduction as if it nullifies the fact that \( (\Phi \pi)^3 + 2\pi \approx 137 \): the main result of the paper whose title is as stated.

The first sentence of the above excerpt means that although flat space does not exist, it is assumed to exist. The \( L^2 \) space of position space wavefunctions is usually such that flat space is assumed. In the relativistic extension of quantum mechanics, \( \alpha \) (the subject of the paper) characterizes the interaction between photons and electrons. Various generating functionals for amplitudes of processes involving charged particles
may be decomposed as power series in \( \alpha \), and the unitary evolution of such particles is foremost among those things which are described with quantum theory. These details matter very little since we are only making a segue into what would otherwise be a single line reporting the paper’s main result that \( (\Phi \pi)^3 + 2\pi \approx 137 \) may be of interest to those who wonder about where the fine structure constant comes from. Dirac said finding the origin of this number is, “the most important unsolved problem in physics,” and Feynman wrote the following [67].

“It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won’t recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to \( \pi \) [emphasis added] or perhaps to the base of natural logarithms? Nobody knows. It’s one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the ‘hand of God’ wrote that number, and ‘we don’t know how He pushed his pencil.’ We know what kind of a dance to do experimentally to measure this number very accurately, but we don’t know what kind of dance to do on the computer to make this number come out, without putting it in secretly!”

While all good physicists worry about this number whose origin might be related to \( \pi \), many readers of FSC expressed no interest in it, preferring instead to fixate on a few of the very brief introductory remarks. The main purpose of FSC was to demonstrate that a certain dance with a third order operator does the trick quite nicely. At the end of the paper, it is suggested that if that dance is real not arbitrarily contrived, then there should exist observables correlated with delay. The main point of the paper, however, was to report that there exists a previously unconsidered type of dance which outputs the requisite number to within an accuracy that can probably reconciled via theoretical restructuring. The deviation of the reported \( \alpha_{MCM}^{-1} \) from the accepted value \( \alpha_{QED}^{-1} \) is about 0.4%. So, a segue giving some contextual setting was given, as is usual in physics. As is not usual in physics, detractors cite the segue in FSC as if it were something other than a few brief words given for context. Furthermore, the experiment which was suggested to verify the context returned an affirmative confirmation [26]. If it hadn’t, the paper’s main result that \( (\Phi \pi)^3 + 2\pi \)
might be of interest to those interested in the most important unsolved problem in physics would have continued to stand on its own. In that case, one would have looked for other dances ending in the same way.

The next item in the excerpt regards the introduction of rigged Hilbert space. The reasons for doing quantum theory in rigged Hilbert space are well known. De la Madrid writes the following [41], for instance.

“Noadays, there is a growing consensus that the [rigged Hilbert space], rather than the Hilbert space alone, is the natural mathematical setting of Quantum Mechanics.”

Nothing new about rigged Hilbert space is introduced in FSC. Position eigenstates don’t exist in Hilbert space but location at points in Minkowski space $S$ can only be represented in quantum theory as position eigenstates. Since one uses such eigenstates (and similar) very often in the course of doing quantum mechanics, one would adopt a state space which doesn’t preclude their existence.

One valid criticism of FSC regards a notational deficiency. We fail to distinguish with separate labels the state spaces in the RHS from the subsets of $S$ identified with the Past, Present, and Future. This deficiency has been remedied in subsequent work with the addition of a tick mark to distinguish $\{A', H', \Omega'\}$ from the manifolds $A, H, \Omega$. In FSC, it is clear that $\{A, H, \Omega\}$ is a Gelfand triple and it is clear that the given $A, H, \Omega$ are subsets of Minkowski space. However, the non-tick marked notation relies on the reader’s ability to differentiate between state spaces and geometric manifolds. The statement that each set (a state space is a set of vectors equipped with an inner product) contains a Minkowski picture is imprecise. A more precise statement would have been that each state space pertains to, or is attached to, the like named region of Minkowski space so as to have the possible domains of its wavefunctions associated to itself uniquely. Namely, it might have been stated more clearly that if $\psi$ is a function of $x^i, x^j, x^k$ then $\psi \in A'$. However, the paper’s main result was that a third order operator can output the value $(\Phi \pi)^3$ required for $\alpha_{MCM}$, and that observed delay correlations would lend further support to the way we have hypothesized $\hat{M}^3$. The distinction of the domain of each function space was not very important for the main result and this detail was glossed over by putting the domains into the spaces of functions on those domains when the intention was only to group them together.

The state spaces are said to contain the coordinate spaces because the coordinate spaces are the domains of the functions in the state space.

The hypersurface of the present is given by $\delta(t - t_0)$, as per usual. Some readers
of FSC insist that they cannot, or could not, understand the obvious relationship between the Dirac $\delta$ function and a surface selected from a bulk. It is claimed that the absence of further words such as “given by” overwhelmed and destroyed their knowledge of the only possible relationship between a $\delta$ function and a surface. The hypersurface of the present is also called “all of space.” The manner in which a $\delta$ function selects all of space from all of spacetime is well known to the paper’s intended readership. What we usually do with the hypersurface of the present in QM is to integrate over all of it. For instance, the position space wavefunction is normalized by integrating $\psi^* \psi$ over all of space. Quantum mechanics usually ignores $d^4x$ but the “dance” prescribed in FSC is such that we need to differentiate among the $d^3x$’s at various $x^0$’s. (This distinction underlies the prediction for delay correlations.) The intended readership was assumed to have some familiarity with the physicist’s basic mathematical toolbox but many detractors have admitted no such familiarity.

The hypersurface of the present is given by the $\delta$ function in the way that one might select the volume $V$ of all of space from the volume $VT$ of all of spacetime by inserting $\delta(t - t_0)$ into $\int d^4x$. The selection of such surfaces by $\delta$ functions is standard. While it is true that the surface is not the $\delta$ function itself identically, the reader is given a choice by the brevity. They may understand the relationship between surfaces and $\delta$’s, or they may choose not to. Usually papers are read with the intention to understand them but MCM publications are most often read with the intention to say that they are wrong. In this case, the reader can say, “I understand the relationship between surfaces and $\delta$ functions,” or he can say, “Even though I understand the relationship, I will assume the author does not and that he has left out words such as ‘given by’ not as a slight abuse of grammar but rather as a reflection of his absurd stupidity and total ignorance, which I have assumed a priori.” Furthermore, the hypersurface of the present being given by a $\delta$ has absolutely nothing to do with the paper’s main result. It is only mentioned to compare the present’s quality of singular thinness to the extended bulk of the past and future, and to complement thereby the stated division of Past:= $[t_{\text{min}}, t_0)$, Present:= $[t_0]$, and Future:= $(t_0, t_{\text{max}}]$. The observer is said to be isomorphic to the $\delta$ because the $\delta$ that selects the hypersurface of the present is comoving in spacetime with the observer. If the observer’s proper time is $t_{\text{now}}$, then that shows up in the stated mechanism as $\delta(t - t_{\text{now}})$. Isomorphic means “corresponding or similar in form and relations” and the association of the observer at proper time $t_{\text{now}}$ with $\delta(t - t_{\text{now}})$ is exactly that. Now we have worked through the introductory remarks in FSC. The remainder of the excerpt proposes the $\hat{M}^3$ operator whose non-unitarity and functioning are discussed in Section 1.
After briefly explaining the relationships among \{\mathcal{A}, \mathcal{H}, \Omega\}, the mathematical property of \(\dot{\mathcal{M}}^3\) was stated as in Equation (A.2). It was suggested that the mechanism proposed for \(\dot{\mathcal{M}}^3\) would result in observable delay correlations. Delay correlations were observed in the Babar data forthwith [26]. The fixation of detractors of the terseness of FSC belies a low comprehension if not a malicious intent to wrongfully naysay. A positive reader should have come away with the understanding that \(\alpha_{\text{MCM}}^{-1}\) can be extracted from some rather ordinary quantum mechanical formalism, that \(\alpha_{\text{MCM}}^{-1}\) differs from \(\alpha_{\text{QED}}^{-1}\) by about 0.4%, and that observed delay correlations were expected to serve as experimental support for the stated mechanism. In the remainder of this appendix, we continue a critical review of FSC. The intention is to address all possible criticisms that a non-positive or overly pedantic reader might seize upon in lieu of the main results.

The MCM is such that the universe is like a quantum particle. Since the universe contains smaller quantum particles of its own, an apparent scale invariance and self-similarity in the model directed this writer’s attention toward fractal models of cosmology. Coming quickly to the prolific body of work due to El Naschie, a formula was encountered for the fractal dimension of a Cantorian spacetime [90].

\[
D = 4 + |\varphi|^3, \tag{1}
\]

where \(\phi\) is the golden ratio. The formula attracted this writer’s attention profoundly, as described in Reference [91], and the formula

\[
\alpha_{\text{MCM}} = (\Phi \pi)^3 + 2\pi \approx 137, \tag{2}
\]

was quickly obtained. The example of the 2D box given in FSC was devised to support an explanation for where such a number might come from. (Appendix C gives a famous precedent for such a program in reverse engineering.) The inclusion of \(t\) for one of the sides of the box was a hard concept in FSC because \(t\) is used for time evolution in quantum mechanics, but the subsequent introduction of the second time \(\chi^4\) sidesteps the issue. The sides \(D\) and \(L\) of the box—a duration and a length—were chosen to satisfy \(\Phi \, D = 2L\). Written as \(D = 2|\varphi|L\), this is in the same general form of \(C = 2\pi \, R\) giving the circumference of a circle in terms of the radius. This is somewhat interesting as a geometrical confluence, and it was stated as such in FSC. Unstated was that Equation (2) was conceived as a circularization of the rectangular Equation (2), meaning \(4 \rightarrow 2\pi\) and \(|\varphi|^3 \rightarrow (\Phi \pi)^3\). In that way, in the context of the author’s abstract thinking, the confluence of the dimensions of the 2D box was slightly more significant than what was recorded in FSC.
The well known wavefunction of a particle in 2D box with sides of lengths $D$ and $L$ is

$$
\psi_{nm}(x, \chi) = \frac{2}{\sqrt{DL}} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi \chi}{D}\right) .
$$

(3)

A 2D universe is a box spanned by one dimension of space and one of time, so putting the particle in this box was like putting it in finite model of the universe. Having fixed the box’ aspect ratio, the duration was chosen as $|\varphi|$. This convention has survived into the current version of the unit cell where we set $A \in \Sigma^{-}$ at $\chi^4 = \varphi$. One of the main purposes of the unit cell is to put a universe, possibly even infinite universe, inside an abstract box finite dimensions. The chosen dimensions are such that

$$
\psi_{nm}(x, \chi) = 2\sqrt{2}\Phi \sin\left(2n\pi x\right) \sin\left(\Phi m\pi \chi\right) .
$$

(4)

This is not a simultaneous eigenvector of $\partial_x$ and $\partial_\chi^3$ as would be required for FSC’s Equation (19):

$$
\hat{Y}\psi_{11} = (\partial_x + \partial_\chi^3)\psi_{11} = \alpha_{\text{MC}M} \psi_{11} .
$$

(5)

Having defined $\hat{U} := \partial_x$ and $\hat{M} := \partial_\chi$, we would operate on Equation (4) as

$$
\partial_x\psi_{11}(x, \chi) = 2\pi \phi_1(x, \chi) , \quad \text{and} \quad \partial_\chi^3\psi_{11}(x, \chi) = (\Phi \pi)^3 \phi_2(x, \chi) ,
$$

(6)

where $\phi_1$ and $\phi_2$ are two wavefunctions demonstrating an incompatibility with Equation (5). The statement $\hat{U} := \partial_x$ refers to

$$
\hat{U}(t, t_0) = \exp\left\{ \frac{-i\hat{H}(t-t_0)}{\hbar} \right\} = \exp\left\{ \frac{-i(t-t_0)}{\hbar} \left[ \frac{\hat{p}^2}{2m} + \hat{V}(\hat{x}) \right] \right\} ,
$$

(7)

wherein the $\hat{H}$ depends on $\hat{p} \propto \partial_x$.\footnote{This form of $\hat{U}$ is developed in Appendix B.} To get $2\pi$ out of $\psi$, we have operated with $\partial_x$. Since $\hat{U}$ uses $\partial_\chi^2$, there would be an implicit square root somewhere or the aspect ratio of the box would need to get $\sqrt{\pi}$ added into it, or perhaps one would pull $2\pi$ out of the denominator of $\hbar = \hbar/2\pi$ to cancel the second $2\pi$ generated by $\partial_x$. While we have written in equation $\hat{Y} = \hat{U} + \hat{M}^3$, we have used it as $\hat{Y} = \partial_x + \partial_\chi^3$. This could have been better clarified. As it is, the ambiguity is relegated to the $:= \; \text{symbol}$ and the hard functioning of the result is given by the the strict equality in Equation (5).

Equation (5) requires a simultaneous eigenfunction of $\partial_x$ and $\partial_\chi^3$. One such function is

$$
\tilde{\psi}_{nm}(x, \chi) = Ae^{i2n\pi x} e^{i\Phi m\pi \chi} .
$$

(8)
\[ \tilde{\psi} \] would be obtained by rescaling \( x \) and \( \chi \) to be small relative to the dimensions of the box. Far from the edges of the universe-as-a-box, the solutions are plane waves. In this case, one might have supposed plane waves with the given wavenumber and frequency. It is true that free particle plane wave solutions are essentially the opposite of the particle in a box, but it also true plane waves in the universe are ultimately constrained to be particle in a box states due to the \( L^2 \) condition of square integrability. In any case, observable operators have real-valued eigenvalues and Equation (9) does the trick with operators \( i\partial_x \) and \( i\partial^3_\chi \). The required rescaling is implicit in FSC’s Equation (10) which sets the integration bounds across the time side of the box at \( \pm \infty \). Overall, the heavy reliance on the := symbol was purposed to avoid making many statements the analytical form of \( \psi \). The main point was that it was reasonable to expect that such a \( \psi \) should exist. Noting that even the inclusion of this present paragraph would have increased the word count of FSC by 10% or so, certain details were deliberately omitted. The main purpose was to suggest a toy model wherein one obtains \( \alpha_{\text{MCM}} \) as the eigenvalue of some observable operator.

The usual definition for an expectation value is

\[ \langle \hat{Q} \rangle = \iiint dx \, dy \, dz \, \psi^*(x, y, z) \hat{Q} \psi(x, y, z) , \tag{9} \]

with the integral over all of space. FSC’s Equation (9) was

\[ \langle \hat{M}^3 \rangle := \int d\chi \, \psi^*(\chi) \delta(\chi) \partial^3_\chi \psi(\chi) . \tag{10} \]

If \( \delta(\chi) \) is the Dirac \( \delta \) function, we obtain

\[ \langle M^3 \rangle := (\Phi \pi)^3 \int d\chi \, \psi^*(\chi) \delta(\chi) \psi(x, \chi) = (\Phi \pi)^3 |\psi(0)|^2 , \tag{11} \]

which does not satisfy the stated \( \langle \psi_{11} | \hat{M}^3 | \psi_{11} \rangle = (\Phi \pi)^3 \) equality or even have the correct units. At the time of the publication of FSC in 2011, this writer was under the wrong impression that there exists a class of spike functions called \( \delta \) functions, one which is named after Dirac. A primitive definition of the Dirac \( \delta \) function is

\[ \delta(x - x_0) = \begin{cases} \infty & \text{for } x = x_0 \\ 0 & \text{otherwise} \end{cases} , \tag{12} \]
but the $\delta(\chi)$ appearing in Equation (.10) was purposed as a generalized spike function

$$\ddelta(x-x_0) = \begin{cases} 
\infty & \text{for } x = x_0 \\
1 & \text{otherwise} 
\end{cases},$$

(.13)

from which the given use case for a vanishing path of integration at infinity follows. However, since it is entirely reasonable that a reader would assume that $\delta(\chi)$ is the Dirac $\delta$ function, the use case for a vanishing path of integration at infinity as in FSC’s Equation (10) does not follow from FSC’s Equation (9) in an intuitive way.

Using $\ddelta$ and referring to Figure 13, the meaning of FSC’s Equation (10)

$$\int_{-\infty}^{\infty} dt \psi^*(t) \ddelta(t) \psi(t) = \int_{C_+} dz |\psi(z)|^2 + \int_{C_\varnothing} dz |\psi(z)|^2 + \int_{C_-} dz |\psi(z)|^2,$$

(.14)

is better understood. The method from complex analysis referred to in FSC is the Cauchy residue theorem. If one wants to integrate along some axis, often it is simpler to add a path at infinity from one end of the axis to the other such that a closed path of integration encircles one or more poles. One assumes that the integrand vanishes along the path at infinity and it follows that the integral over the axis is equal to $2\pi i$ times the sum of the encircled poles’ residues. Presently, the box condition guarantees that the integrand vanishes everywhere except on the interior of the box so the $C_\varnothing$ need not be any more complicated than a path around the edge of the box. Since $\psi$ in eigenvector of $\hat{M}^3$, the underbraced quantity

$$\langle M^3 \rangle := \left(\Phi \pi \right)^3 \int d\chi \psi^*(\chi) \ddelta(\chi) \psi(x, \chi) = \left(\Phi \pi \right)^3,$$

(.15)
must be equal to unity. In the absence of $\delta$, a properly normalized $\psi$ will always satisfy
\[ \int d\chi |\psi(\chi)|^2 = 1 \] .
(16)

With $\delta$ defined as in Equation (13), however, the integrand hits a pole at $\chi=0$. By adding the $C_\varnothing$ path, identifying polar complex variables $z=\chi^4 e^{i\chi^4 \varnothing}$, and writing
\[ \int_{C_+} dz |\psi(z)|^2 = \lim_{\varepsilon \to 0^+} \int_\varepsilon^\varphi dz |\psi(\chi^4_+, 0)|^2 \approx \int_0^\varphi dz |\psi(\chi^4_+, 0)|^2 \]
\[ \int_{C_\varnothing} dz |\psi(z)|^2 = \int_0^\beta dz |\psi(\Phi, \chi^4_\varnothing)|^2 \]  
\[ \int_{C_-} dz |\psi(z)|^2 = \lim_{\varepsilon \to 0^-} \int_{-1}^\varepsilon dz |\psi(\chi^4_-, 0)|^2 \approx \int_{-1}^0 dz |\psi(\chi^4_-, \beta)|^2 , \]
we remove the pole. The acquisition of $\beta$ in the process is associated with the progression from neighborhood of fractional distance to the next: the higher level of aleph. Associating negative length with the negative time $\chi^4_-$, the total length along the $\chi$ direction is $\Phi + (-1) = |\varphi|$ so we have preserved the duration $D$ originally chosen. We also preserve (up to a sign) the post-FSC notion that the distance from $H$ to $\Omega$ should be $\Phi$ times the distance from $H$ to $A$. Overall, the purpose of putting $\delta$ into the structure of the integral over all of $\chi$ is to force the Present→Future→Past→Present structure. With $\delta$ added as a topological obstruction, $\chi$ is not simply connected. Adding the $C_\varnothing$ restores the simply connected property.

A few remarks about normalization follow FSC’s Equations (9) and (10.) As in Equation (15), the integral over the whole box has to be normalized to unity even with $\delta$ added as a new topological boundary condition enforcing the $\hat{M}^3$ structural constraints. In a typical normalization, one multiplies by a constant in an ad hoc way. $\delta$ contains an infinity in it, however, exhibiting the property of multiplicative absorption. Then one takes $A\delta = \bar{\delta}$ so that normalization is automatically imposed without the supplemental introduction of ad hoc constants. $\delta$ is called renormalizable not in the sense of the unrelated QFT renormalization, but rather because the multiplicative absorptive property facilitates normalization as needed.

The remainder of FSC develops the algebra from which Einstein’s equation would be derived in Reference [3]. It is emphasized heavily in subsequent work that although the MCM derivation of Einstein’s equation may appear to have been goal seeked, or reverse engineered, no such thing was the case. The algebra in which Einstein’s equation was found was assembled in FSC [24] almost a year before the relativistic
result was found and reported [3]. The comment on Palev statistics and the citation
to a paper regarding the “chronon” reflects a comment made to this writer by David
Finkelstein who went into the Professor Emeritus status shortly before this writer
was admitted to Georgia Tech. The suggestion of the relevance of Palev statistics
was never investigated and may have been a monkey wrench thrown into the works
by a notorious and miserly detractor of the MCM (see Section 33.) As one further
remark on the algebra developed, the reader invited to notice that FSC’s Equation
(18) is a rich algebraic structure indeed. It is not not cited as a thesis in the main
body of this paper, but this algebra should be reconstructed in the language of Galois
theory. The structure is quite rich and this writer has never seen another like it (which
may reflect this writer’s limited exposure to abstract algebra.)

FSC concludes with a suggestion for the experiment very soon after which time
reversal symmetry violation was discovered [26]. In previous work [25], dark energy
had been described as delay correlation of sorts and the algebraic structure around
FSC’s Equation (15) \( \varphi^{**} \neq \varphi \) was meant to break “time reversal symmetry.” Partic-u-larly, if the state space of states in the past is different than the space of states
in the present, it was suspected that the duration between an event and its observa-
tion would have observable correlations. Since the experimental quantity in context
was the fine structure constant, it was suggested that the delay correlations would
manifest in its observed value.
Appendix B: Focused Review of Quantum Mechanics

The Translation Operator

To build up the usual operator formalism which shall be extended with $\hat{M}^3$, we begin in the basis of position eigenstates. This is the usual path of development because it connects so well with the picture of classical physics, and the position basis is preferred for talking about $\hat{M}^3$ since it is suppose to connect the quantum and gravitational theories.

By definition, position eigenstates are eigenvectors of the position operator $\hat{x}$:

$$\hat{x}|x\rangle = x|x\rangle . \tag{.18}$$

The position operator $\hat{x}$ has a continuous spectrum and the position basis is complete:

$$1 = \int dx |x\rangle \langle x| . \tag{.19}$$

If we want to move a particle from $x_1$ to $x_2$, the machinery of quantum mechanics is such that we will operate on $|x_1\rangle$ with some operator such that $|x_2\rangle$ is the result, or output of the operation. We will call that operator the translation operator and label it $\hat{J}$. Evidently, it satisfies

$$\hat{J}(\Delta \vec{x})|\vec{x}\rangle = c |\vec{x} + \Delta \vec{x}\rangle . \tag{.20}$$

This equation comes directly from the physics. $\hat{J}$ moves $|\vec{x}\rangle$ to $|\vec{x} + \Delta \vec{x}\rangle$. Now it remains to reverse engineer what the actual analytical form of the operator is. Exactly as we have proposed that $\hat{M}^3$ should move $|\psi\rangle$ like so, like so, and like so, and then left determining the actual machinery of $\hat{M}^3$ to a later endeavor, this is what is done with $\hat{J}$ and other operators. One conceives of an operation, labels the operator that does it, and then works out what it has to be later. It is no way a hoax that we have written Equation (.20) without knowing what mathematical form $\hat{J}$ might take, and neither is the MCM reliance on $\hat{M}^3$ without first defining anything other than business as usual in quantum theory. It is for a good reason that

$$\hat{M}^3|\psi; H_1\rangle = c |\psi; H_2\rangle , \tag{.21}$$

looks exactly like Equation (.20). $\hat{J}$ is the spatial translation operator for arbitrary translations $\Delta \vec{x}$ and $\hat{M}^3$ is another kind of translation operator between unit cells in the MCM cosmological lattice. $\hat{M}^3$ is necessarily more complicated than $\hat{J}$ because it
must also implement a time evolution to the later chronological time on the forward evolved $H$-brane. This is part of the reason why $\hat{M}^3$ is posed as three separate operations.

To work out the mathematical representation of $\hat{J}$ based on the physics we have assigned to it, first we will consider infinitesimal translations in 1D. The generator of infinitesimal translations is

$$\hat{J}(dx) |x\rangle = c |x + dx\rangle .$$

(22)

For ease in notation, and because any two states $c_1 |x\rangle$ and $c_2 |x'\rangle$ are the same state, we will set $c = 1$. As per usual in quantum mechanics, we will explore the mathematical structure by inserting Equation (19): the completeness relation. Assigning the dummy integration variable $x'$, we have

$$\hat{J}(dx) |x\rangle = \hat{J}(dx) \int dx' |x'\rangle \langle x'|x\rangle$$

$$= \int dx' \hat{J}(dx) |x'\rangle \langle x'|x\rangle$$

(23)

$$= \int dx' |x' + dx\rangle \langle x'|x\rangle .$$

The quantity $\langle x'|x\rangle$ is the interpreted as the expansion coefficient of $\hat{J}(dx)|x\rangle$ written in the basis of $|x' + dx\rangle$ states. That basis is still just the position basis with position measured from an origin shifted by $dx$ so we will introduce a coordinate transformation to shift it back. Using

$$x'' = x' + dx \implies dx'' = dx ,$$

(24)

we have

$$\hat{J}(dx) |x\rangle = \int dx'' |x''\rangle \langle x'' - dx|x\rangle .$$

(25)

Since $x'$ and $x''$ are only dummy variables, we can forget about the old $x'$ and rename $x''$ as the new $x'$ writing

$$\hat{J}(dx) |x\rangle = \int dx' |x'\rangle \langle x' - dx |x\rangle ,$$

(26)

The expansion coefficient $\langle x' - dx'|x\rangle$ is called “the position space wavefunction” and it is written as $x(x' - dx')$. If we would have labeled the operand in Equation (22)
Next Steps and the Way Forward in the Modified Cosmological Model

|ψ⟩ then the wavefunction would be the more familiar looking ψ( ⟨x' − dx'|). In that case, we would write

$$\hat{J}(dx)|\psi_1⟩ = c|\psi_2⟩ ,$$

so it is obvious why it is better to label the position states with their positions, in the present context, than to label them with Greek letters. Equation (27) does not reflect the physics we have assigned to \(\hat{J}\) and we would have to add some notes to say, “ψ1 is the state of being located at x and ψ2 is the state of being located at x + dx.” That would be stupid but it is demonstrative to emphasize to beginners that the \(\langle x' − dx'|x⟩\) appearing in Equation (26) is just like an ordinary ψ(x) despite it being written here as x(x’). So, we have explained that inserting the completeness relation into the definition of infinitesimal translation makes the wavefunction appear but we have not yet said what the wavefunction is. Since the wavefunction (in position space) is the expansion coefficient in the continuous basis (of position states), we should build up expansion in the discrete basis and then generalize it so that the wavefunction is not mysterious in any way. Then we will return to the analytical form of \(\hat{J}\) in the following subsection.

.0.1 The Interpretation of Basic Formalism in Quantum Mechanics

If one measures position, there are an infinite different number of positions one might observe so we say the spectrum of the position operator is continuous. Let there be some observable operator \(\hat{A}\) such that there are only a finite number of quantized values that might be observed. In the eigenbasis of \(\hat{A}\) we have

$$\hat{A}|a_k⟩ = a_k|a_k⟩ ,$$

which is exactly like the eigenvalue equation for the position operator: Equation (18.) The difference is that there an infinite number of positions x that one might find in a measurement of position by operation with the position operator but there only a finite number of values one might find when measuring observable A. To avoid a complete restatement of all of quantum mechanics here, it suffices to say that the fundamental idea in quantum mechanics is that everything which can be observed may be represented as an operator, and that ever possible value of it which can be observed is an eigenvalue of the operator. The possible values of x touch each other and the spectrum of \(\hat{x}\) is called continuous while there are numerical gaps between the \(a_k\) and the spectrum of \(\hat{A}\) is said to be discrete. The completeness relation for
discrete eigenbases is

\[ 1 = \sum_k |a_k\rangle \langle a_k| . \] (29)

If we operate on \( |a_k\rangle \) with \( \hat{A} \), we are guaranteed to get \( a_k \) since \( |a_k\rangle \) is the eigenvector of \( \hat{A} \) with eigenvalue \( a_k \). However, sometimes one does not know ahead of time what outcome a measurement will give. To determine what will happen when we measure \( A \) on an unknown state \( \psi \), we insert the completeness relation to “expand \( \psi \) in the eigenbasis of \( \hat{A} \)” as

\[ 1 |\psi\rangle = \sum_k |a_k\rangle \langle a_k| |\psi\rangle = \sum_k |a_k\rangle \langle a_k| |\psi\rangle . \] (30)

As in Equation (26), we have obtained an “expansion coefficient” \( \langle a_k| \psi\rangle \). This is the discrete version of Equation (26)’s \( \langle x' - dx'|x\rangle \), which we have called a “wavefunction.” \( \langle a_k| \psi\rangle \) is not a wavefunction, however. It is just a number. \( \langle x' - dx'|x \rangle \) is a wavefunction because it contains the integration variable \( x' \). This can be tricky for beginners and we will belabor the details here very much. It is the intention to make MCM publications so plainly accessible to beginners that even complete novices might easily see through detractors’ stupid remarks and baseless criticisms. Early work in the MCM written for subject matter experts was flawed in this regard because myriad ignoramuses, codgers, and blowhards could levy any criticisms desired without the accountability of third parties being able to judge for themselves. (See Appendix D.)

Now that we have expanded \( \psi \) in the eigenbasis of the observable represented by \( \hat{A} \). Since the expansion coefficients are not functions of any variables, they must be numbers and we can simplify Equation (30) as

\[ |\psi\rangle = 1 |\psi\rangle = \sum_k |a_k\rangle \langle a_k| |\psi\rangle = \sum_k c_k |a_k\rangle . \] (31)

For expansion in the continuous basis, we would have

\[ |\psi\rangle = 1 |\psi\rangle = \int dx' |x'\rangle \langle x'| |\psi\rangle = \int dx' \psi(x') |x'\rangle . \] (32)

Having made clear the role of the the wavefunction \( \psi(x) \) as an expansion coefficient, we will continue in the example of the discrete basis. Then we will say more about
the continuous basis as the context develops. Operating on \( \psi \) with \( \hat{A} \) to affect in the theoretical framework the measurement of observable \( A \) on the state \( \psi \) yields

\[
\hat{A}|\psi\rangle = \sum_k c_k \hat{A}|a_k\rangle = \sum_k c_k a_k.
\] (33)

The interpretation is that a measurement of \( A \) on state \( \psi \) will yield eigenvalue \( a_k \) with probability \( P = |c_k|^2 \). If \( \psi \) was an eigenstate of \( \hat{A} \), then all the \( c_k \) would be equal to zero for every value of \( k \) except one, and then we could write \( |\psi\rangle = c_j |a_j\rangle = |a_j\rangle \) without including the sum because there is no need to sum the terms whose coefficients are zero.

\[=\]

\[\textit{N.b.}, \text{ operation by } \hat{A} \text{ does not return any one value of } a_k \text{ even though a lab measurement of } A \text{ will certainly return a single value. This issue shows what it means that the collapse of the wavefunction is implemented in an } \textit{ad hoc} \text{ way in quantum mechanics. For states expanded in discrete bases this looks like}

\[
|\psi\rangle_{\text{discrete}} = \sum_k |a_k\rangle \langle a_k|\psi\rangle
\]

\[=\]

\[\sum_k c_k |a_k\rangle \xrightarrow{\text{measurement}} |a_j\rangle.\] (34)

Equation (33) shows the operation of \( \hat{A} \) on \( |\psi\rangle \) which does not reduce the state to a single eigenstate. Reflecting the lack of a mathematical operation for wavefunction collapse upon measurement, the long labeled arrow shows that collapse happens somehow. In the continuum, the same behavior is written

\[
|\psi\rangle_{\text{continuous}} = \int dx' |x'\rangle \langle x'|\psi\rangle
\]

\[=\]

\[\int dx' \psi(x') |x'\rangle \xrightarrow{\text{measurement}} \int_{x_0-\varepsilon}^{x_0+\varepsilon} dx' \psi(x') |x'\rangle.\] CHECK???

(35)

This makes the quantum mechanics of continuous observables somewhat (massively) more complicated than the quantum mechanics of discrete observables. In the discrete case, the expansion coefficients for a particular basis were just the numbers \( c_k \in \mathbb{C} \) whose squares are postulated to return probabilities.

\[\text{CONT FORMULA HAS A PROBLEM!!!!!!!!!!}\]

\[1\text{This outstanding measurement notation is due to Sakurai (and Napolitano) [92].}\]
In the case of the eigenbasis of an observable operator with a continuous spectrum of eigenvalues, the expansion coefficient whose square gives the probability of observing a value (very) near to a particular eigenvalue $x_0$ is a function of the eigenvalue in question. The discrete-continuous correspondence $c_k \leftrightarrow \psi(x)$ yields the following probability structure.

$$P_k = |\langle a_k | \psi \rangle|^2 = |c_k|^2 \quad \iff \quad P(x_0) dx' = |\langle x_0 | \psi \rangle|^2 dx' = |\psi(x_0)|^2 dx' . \quad (36)$$

The $dx'$ tells us that the probability of observing state $|\psi\rangle$ with exact continuous parameter $x_0$ is infinitesimal. In practice, it is not possible to measure $\psi$ at mathematical point $x_0$ due to resolution limits of physical devices, general principles of Heisenberg uncertainty, and ultimately due to Planck scale effects. While this writer is usually first to step forward with criticisms for quantum theory, this is a beautiful example of its robust power: we would like to know the exact probability for finding particles at exact points but we can’t and quantum mechanics says we can’t. This is a great success among a few shortcomings that this writer likes to highlight.

Since probability is dimensionless, the expansion coefficients $\psi(x)$ in the continuous basis have to have units of $[\sqrt{x}]^{-1}$. That makes the wavefunction radically different than the $c_k$ discrete expansion coefficients. These units are reflected in the normalization conditions

$$\langle \psi | \psi \rangle = \langle \psi | 1 | \psi \rangle = \sum_k \langle \psi | a_k \rangle \langle a_k | \psi \rangle = \sum_k c_k^* c_k = \sum_k |c_k|^2 = 1$$

$$\langle \psi | \psi \rangle = \int_{-\infty}^{\infty} dx' \langle \psi | x' \rangle \langle x' | \psi \rangle = \int_{-\infty}^{\infty} dx' |\psi(x')|^2 = 1 . \quad (37)$$

Calling attention to this radical alteration of the structure for the eigenbases of operators with continuous spectra, the expansion coefficient $\langle x | \psi \rangle = \psi(x)$ is called the position space wavefunction. We often call the position space wavefunction simply “the wavefunction.” The important thing to know about wavefunctions is that they are the infinite number of expansion coefficients needed to expand and abstract state ket $|\psi\rangle$ in the eigenbasis of some observable with a continuous spectrum. For each of an uncountably infinite number of unique $x$ in the spectrum of $\hat{x}$, there is a corresponding expansion coefficient $\psi(x)$. Mirroring the discrete expansion

$$|\psi\rangle_{\text{discrete}} = \sum_k c_k |a_k\rangle = c_1 |a_1\rangle + c_2 |a_2\rangle + c_3 |a_3\rangle + ... \quad , \quad (38)$$
we would like to write the continuous expansion as

$$\ket{\psi}_{\text{continuous}} = \sum_x \psi(x) \ket{x} = \ldots \psi(x') \ket{x'} + \psi(x'') \ket{x''} + \psi(x''') \ket{x'''} + \ldots .$$  \hspace{1cm} (39)

However, the eigenvalue spectrum \{x\} is an “uncountable” set. That means we could never enumerate the various x eigenstates with natural numbers as we have for the \(a_k\) eigenstates. Luckily, Newton has developed an excellent workaround for us. Written in the notation of Leibniz, the workaround is

$$\ket{\psi}_{\text{continuous}} = \int_{-\infty}^{\infty} dx' \psi(x') \ket{x'} .$$  \hspace{1cm} (40)

This workaround is great and useful but it comes at the expense of some complication. The expansion coefficient \(\psi(x')\) in this case is called the wavefunction. Since the continuous basis we have chosen is the basis of position eigenstates, \(\psi(x')\) is the position space wavefunction. The main other case is the momentum space wavefunction which can be useful in certain applications.

0.2 Back to the Translation Operator

Now that we understand the wavefunction, we will continue from Equation (26), restated here:

$$\hat{J}(dx)\ket{\psi} = \int dx' \ket{x'} \bra{x' - dx} x \rangle = \int dx' \psi(x' - dx) \ket{x'} .$$  \hspace{1cm} (41)

Here, we are very careful to understand that the \(dx\) infinitesimal translation is a different sort of object than the \(dx'\) differential of the integration variable, and we also understand that \(\psi(x') = \langle x' | x \rangle\) is used because \(x(x') = \langle x' | x \rangle\) would look weird. Also, the minus sign in the argument of \(\psi\) seems to reflect translation by \(-dx\) rather than by the \(dx\) that had been intended. This was a feature inherited by our change of variables. Apparently, \(\hat{J}(-dx)\) is the operator that generates translation by \(dx\).

$$\hat{J}(-dx)\ket{\psi} = \int dx' \psi(x' + dx) \ket{x'} .$$  \hspace{1cm} (42)

We have seen that the \(c_k\) expansion coefficients give the probability for finding \(a_k\) in a measurement of observable \(A\) as \(P_k = |c_k|^2\). In general, the \(c_k\) are called probability amplitudes and the product with the complex conjugate \(c_k^*\) gives a real-valued probability. In the continuous case, the probability amplitude is the wavefunction so we get \(P(x_0) = |\psi(x_0)|^2 dx\) which results in a real-valued probability after it is integrated.
across some range. Since it has to be integrated, we call the modulus squared of 
\( \psi(x) \) a probability density. Before we operated with \( \hat{J} \), the wavefunction was \( \psi(x) \). After, it was \( \psi(x + dx) \) and the probability density was \( |\psi(x + dx)|^2 \). Evidently, the translation operator \( \hat{J} \) has shifted the probability density for finding \( \psi \) in some region of space by the expected amount \( dx \). We have succeeded in implementing the desired physics but we have not yet obtained the analytical form of \( \hat{J} \). To get there, we will impose some more physics.

- If \( |x\rangle \) is properly normalized to \( \langle x|x \rangle = 1 \), then the translated state \( |x'\rangle \) must maintain the normalization.

\[
\langle x'|x' \rangle = \left[ \langle x|\hat{J}^\dagger \right] \left[ \hat{J}|x \rangle \right] = \langle x|\hat{J}^\dagger \hat{J}|x \rangle = 1 \implies \hat{J}^\dagger \hat{J} = 1 . \tag{.43}
\]

In other words, we require that \( \hat{J} \) is a unitary operator. In general, unitary transformations preserve the norm of a ket. Whatever the norm was before the transformation, the norm of the transformed ket is “unity” times the old norm.

- Two consecutive translations by \( \Delta x_1 \) and \( \Delta x_2 \) must be equal to a single translation by \( \Delta x_1 + \Delta x_2 \).

\[
\hat{J}(\Delta x_1)\hat{J}(\Delta x_2) = \hat{J}(\Delta x_1 + \Delta x_2) . \tag{.44}
\]

- Translation by \( \Delta x_1 \) and then \(-\Delta x_1 \) must be the identity operation.

\[
\hat{J}(-\Delta x_1)\hat{J}(\Delta x_1) = \mathbb{1} \implies \hat{J}(-\Delta x_1) = \hat{J}^{-1}(\Delta x_1) . \tag{.45}
\]

(This follows from Equation (44) in the case of \( \Delta x_2 = -\Delta x_1 \).)

- In the limit of vanishing displacement, the translation operator must reduce to the identity.

\[
\lim_{dx \to 0} \hat{J}(dx) = \mathbb{1} . \tag{.46}
\]

We still don’t exactly have a picture of the analytical form of \( \hat{J} \) though have obtained have a detailed view of its physics. To move forward, we supplement these physical requirements with a mathematical requirement that \( \hat{J}(dx) \) should be linear in \( dx \) to leading order.

\[
|\mathbb{1} - \hat{J}(dx)| = \mathcal{O}(dx) . \tag{.47}
\]

\(^1\text{Dagger denotes the conjugate transpose which is the ordinary complex conjugate for non-matrix quantities.}\)
Now the magic is made to happen with an ansatz! We guess that the form is

$$\hat{J}(dx) = 1 - i\hat{K} dx ,$$  \hspace{1cm} (48)

for some Hermitian operator $\hat{K}$. It is not difficult to verify that this ansatz satisfies the above conditions. Particularly, the requirement that $\hat{K}$ is Hermitian satisfies $\hat{J}^\dagger \hat{J} = 1$. Often times, it is taken as a postulate of quantum mechanics that the generator of translations $\hat{K}$ is the momentum operator times a constant. Sakurai and Napolitano [92] proceed with a method by which one is able to deduce that the momentum operator satisfies the ansatz. Their method of Taylor series analysis necessarily introduces some gaps in the mathematical rigor at order $O(dx^2)$. This method is perfectly standard in physics and take a postulate also introduces a gap in the first principles approach to understanding where everything comes from. However, all the expressions which follow from the postulate gap are exact while $O(dx^2)$ gaps propagate through all the expressions which follow from the method of Taylor series analysis. Because the MCM has some concept of changing levels of aleph, we prefer to stay away from ignoring the $O(dx^2)$ terms. There is a vague concern that the process by which sequential MCM unit cells are housed in increasing great neighborhoods of fractional distance with respect to infinity might cause $O(dx^2)$ terms to be integrated over, or something, such that they cause problems. By substituting the hand-waving of $\hat{K} \propto \hat{p}$ for the issue of ignoring $O(dx^2)$ terms, we preclude the possibility of any issues coming from the ignored terms on a “higher level of aleph.” However, we do not preclude the possibility that $\hat{K}$ is somewhat more complicated than the $\hat{p}$ times a constant which he have assumed. Such are the issues in the structure of quantum theory.

We postulate that $\hat{K}$ is $\hat{p}$ times a constant. From Equation (48), we can see that $\hat{K}$ does not have the correct units to the be the momentum operator which should have units of mass times velocity. As it is, $\hat{K}$ has units of inverse meters. Dimensional analysis shows that $\hat{p}$ must be divided by something with units of action if it is to play the role of the generator of translations. Sakurai and Napolitano [92] mention that if quantum physics had been developed in history before classical physics, the fundamental units of physics would have been chosen so that this constant of proportionality between $\hat{K}$ and $\hat{p}$ was equal to one. With units already having been set, it works out to be $\hbar$.

$$\hat{J}(dx) = 1 - \frac{i}{\hbar} \hat{p} dx .$$  \hspace{1cm} (49)

Finite translations are obtained by compounding infinitesimal ones. To obtain the
analytical form of
\[ \hat{J}(\Delta x)|x\rangle = |x + \Delta x\rangle, \] (50)
we divide the translation into \( N \) equal parts
\[ \delta x = \frac{\Delta x}{N}. \] (51)

Applying Equation (44), we have
\[ \hat{J}(\Delta x) = \hat{J} \left( \sum_{k=1}^{N} \delta x \right) = \prod_{k=1}^{N} \hat{J}(\delta x). \] (52)

We make the connection to the generator of infinitesimal translations by taking the limit \( N \to \infty \) such that \( \delta x \to dx \). This yields
\[ \hat{J}(\Delta x) = \lim_{N \to \infty} \prod_{k=1}^{N} \hat{J}(\delta x) = \lim_{N \to \infty} \left( 1 - \frac{i\hat{p}_x \Delta x}{hN} \right)^N. \] (53)

This limit is a famous definition of the exponential function so we have
\[ \hat{J}(\Delta x) = \exp \left\{ -\frac{i\hat{p}_x \Delta x}{h} \right\}. \] (54)

If we are able to determine the analytical form of \( \hat{p} \) operating on position states, then we will have found the analytical form of the translation operator for such states.

**The Momentum Operator**

Heisenberg won the 1932 Nobel prize in physics “for the creation of quantum mechanics.” In 1926, Dirac wrote the following [93].

“The new mechanics of the atom introduced by Heisenberg may be based on the assumption that the variables that describe a dynamical system do not obey the commutative law of multiplication, but satisfy instead certain quantum conditions. One can build up a theory without knowing anything about the dynamical variables except the algebraic laws that they are subject to, and can show that they may be represented by matrices whenever a set of uniformising variables for the dynamical system exists [94]. It may be shown, however, that there is no set of uniformising variables for a system containing more than one electron, so that the theory cannot
Almost 100 years later, the problem cited by Dirac remains. Multiparticle systems are still not exactly solvable. Still, the triumph of Heisenberg was triumphant indeed. This shortcoming of the theory is not treated as a failure and many good approximations based on the single electron system are available today. Earlier in 1925, Dirac described the kernel of what Heisenberg had done [95].

"It is well known that the experimental facts of atomic physics necessitate a departure from the classical theory of electrodynamics in the description of atomic phenomena. This departure takes the form, in Bohr’s theory, of the special assumptions of the existence of stationary states of an atom, in which it does not radiate,¹ and of certain rules, called quantum conditions, which fix the stationary states and the frequencies of the radiation emitted during transitions between them.² These assumptions are quite foreign to the classical theory, but have been very successful in the interpretation of a restricted region of atomic phenomena. The only way in which the classical theory is used is through the assumption that the classical laws hold for the description of the motion in the stationary states, although they fail completely during transitions, and the assumption, called the Correspondence Principle, that the classical theory gives the right results in the limiting case when the action per cycle of the system is large compared to Planck’s constant h, and in certain other special cases.

"In a recent paper [96] Heisenberg puts forward a new theory, which suggests that it is not the equations of classical mechanics that are in any way at fault, but that the mathematical operations by which physical results are deduced from them require modification. All the information supplied by the classical theory can thus be made use of in the new theory. [sic]

"We are now in a position to perform the ordinary algebraic operations on quantum variables. The sum of \([matrices] \ x \ and \ y\], with the nm matrix

¹Classical electromagnetic theory predicts that electrons undergoing centripetal acceleration in atomic orbits should radiate energy and fall into the nuclear. All classical charged particles radiate energy and the non-radiation of electrons in atomic orbitals was one of the main non-classical problems in the early days of atomic physics.

²In celestial mechanics, it is understood that a large asteroid impact might subtly alter the orbital radius of a planet around the sun. In atomic physics, the situation is totally different. When a photon comes and hits an atomic electron, it cannot alter the electron’s orbit slightly. If the photon does not have energy to knock the electron all the way to the next fixed stationary state, then the photon will scatter elastically from the electron. This phenomenon describes the nature of quantum mechanics. In celestial mechanics, there are a continuum of orbital radii allowed for a planet to orbit the sun but in the atomic version of the solar system with electrons orbiting nuclei, the electron is only allowed certain discrete, or quantized, orbits.
element of $x$ denoted $x(nm)$] is determined by the equations
\[ \{ x + y \}(nm) = x(nm) + y(nm) \] ,
and the product by
\[ xy(nm) = \sum_k x(nk)y(km) \] ,
\[ [sic] \] An important difference now occurs between the two algebras. In general
\[ xy(nm) \neq yx(nm) \] ,
and quantum multiplication is not commutative, although, as is easily verified, it is associative and distributive. The quantity with components $xy(nm) [sic]$ we shall call the Heisenberg product of $x$ and $y$, and shall write simply as $xy$. Whenever two quantum quantities occur multiplied together, the Heisenberg product will be understood. Ordinary multiplication is, of course, implied in the products of amplitudes and frequencies and other quantities that are related to sets of $n$’s which are explicitly stated."

Quantum quantities are what we now call observable operators. The quantities we observe commute in the usual way but their representations in quantum theory do not. The principle manifestation of the Heisenberg’s quantum algebra is the commutator of position and momentum
\[ [\hat{x}_j, \hat{p}_k] = i\hbar\delta_{jk} \implies \hat{x}\hat{p}_x \neq \hat{p}_x\hat{x} . \quad (.55) \]
In the discrete basis of atomic energy states system for which this language was first devised, the non-commuting matrices only countably infinite dimensional. In position space, the $\hat{x}$ and $\hat{p}$ operators are uncountably infinite dimensional matrices with Dirac $\delta$ function eigenvalues but the non-commutativity is the same.

It is a beautiful feature of quantum mechanics that observables which be known simultaneously have operators that commute. If two observables can’t be known at the same time, their operators don’t commute, meaning the commutator $[\hat{a}, \hat{b}] \neq 0$. Intuitively, we know that 3D position in the lab so we expect that the $\hat{x}$, $\hat{y}$, and $\hat{z}$ observable operators should commute. That position and momentum can’t commute also follows from physical thinking. To measure momentum, one measures speed, mass and direction. To measure speed, time is measured between two positions. Once a speed is determined, which of the two positions might we associate with it
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simultaneously? Since we have only measured speed between two positions, we cannot rightly associate either of them with the speed. If we were to associate the average of the two positions with the measured speed, that would require an assumption of constant velocity between the two positions. This would be unphysical because we measured the average velocity between the two positions and have no way to know if it was constant on the interval. So, we can be sure that \( \hat{p} \) won’t commute with \( \hat{x} \). Other than that, we need to determine the analytical form of the momentum operator if we are going to answer the previous question about the translation operator \( \hat{J}(\Delta \vec{x}) \) which depends on it.

The guiding principle regarding for form of \( \hat{p} \) is that it has to return eigenvalue \( p \) when it operates on a momentum eigenstates. Following along with the goal to determine the form of \( \hat{J}(\Delta \vec{x}) \) acting states in the position representation, we will consider momentum eigenstates in the position representation. Momentum eigenstates in the momentum representation can only be \( \delta \) functions, and since the position representation is the Fourier transform of the momentum representation, generally speaking, the momentum eigenstate in the position representation has to be a plane wave. Omitting factors of \( 2\pi \) and \( \hbar \), we have

\[
\psi(x) = \int dp' e^{-ip'x} \psi(p') = \int dp' e^{-ip'x} \delta(p' - p) = e^{-ipx} .
\]  

(M.56)

Momentum can be to the left or right so we may ignore the minus sign to write the matrix elements of a momentum eigenstate in the position representation as

\[
\langle x | p \rangle = \psi_p(x) = e^{ipx/\hbar} .
\]  

(M.57)

Since the matrix elements form a continuum, we call \( \psi_p(x) \) the wavefunction. The subscript \( p \) tells us that this wavefunction of the momentum eigenstate with eigenvalue \( p \). For a given \( p_0 \) and \( x_0 \), the expression \( \langle x | p \rangle \) gives the probability amplitude that a particle with momentum \( p_0 \) will be found at position \( x_0 \). In other words, \( \langle x | p \rangle \) is the wavefunction of the momentum eigenstate. Formally, we might say that there exist normalized solutions to the Schrödinger equation in the form

\[
\psi_p(x) = A \exp \left\{ \frac{i(px - Et)}{\hbar} \right\} = c(t) \exp \left\{ \frac{ipx}{\hbar} \right\} ,
\]  

(M.58)

but it suffices to ignore the time part and assume plane wave position states. However, the partial derivative is used to acknowledge that \( \psi \) is usually a function of \( x \) and \( t \).
By optical inspection, one determines that the momentum operator has to be \(-i\hbar \partial_x\).

\[
\hat{p}_\psi(x) = -i\hbar \frac{\partial}{\partial x} \exp \left\{ \frac{ipx}{\hbar} \right\} = p \exp \left\{ \frac{ipx}{\hbar} \right\} = p\psi_p(x) .
\] (59)

If we had used the \(e^{-ipx}\) wavefunction, then we would have gotten the \(-p\) eigenvalue which is correct for a plane wave moving in the other direction. Ultimately, we take it as a postulate of quantum mechanics that the position representation of the momentum operator is

\[
\hat{p} = -i\hbar \frac{\partial}{\partial x} .
\] (60)

The Heisenberg algebra follows directly as

\[
[x, \hat{p}]\psi_p = \hat{x}\hat{p}\psi_p - \hat{p}\hat{x}\psi_p
\]
\[
= -i\hbar \frac{\partial}{\partial x} \psi_p + i\hbar \frac{\partial}{\partial x} (x\psi_p)
\]
\[
= xp\psi_p + (ih\psi_p + ihx \frac{\partial}{\partial x} \psi_p)
\]
\[
= xp\psi_p + (i\hbar \psi_p - xp\psi_p)
\]
\[
= i\hbar \psi_p .
\] (61)

Now we may plug \(\hat{p}\) into Equation (54) to write

\[
\hat{J}(\Delta x) = \exp \left\{ -i\frac{\hat{p}\Delta x}{\hbar} \right\} = \exp \left\{ -\Delta x \frac{\partial}{\partial x} \right\} .
\] (62)

Testing it on the \(\psi_p\) wavefunction, we find

\[
\hat{J}(\Delta x)\psi_p(x) = \exp \left\{ -\Delta x \frac{\partial}{\partial x} \right\} \exp \left\{ \frac{ipx}{\hbar} \right\}
\]
\[
= \exp \left\{ -i\frac{p\Delta x}{\hbar} \right\} \exp \left\{ \frac{ipx}{\hbar} \right\}
\]
\[
= \exp \left\{ i\frac{p(x - \Delta x)}{\hbar} \right\}
\]
\[
= \psi_p(x - \Delta x) .
\] (63)

This agrees with Equation (.42.) Another way to understand what is going on is to
Figure 14: The action of the translation operator $\hat{J} \propto \hat{p}$ on an arbitrary wavefunction in the position representation. While the translation application from, say, $H$ at $x$ to $\Omega$ at $x+\Delta x$ is obvious, more complicated operations are required for MCM applications. The MCM operation must alter the shape of $\psi(x)$. Such operations are usually considered time evolutions. This figure is adapted from Reference [97].

write

$$\hat{J}(\Delta x)\psi_p(x) = \psi'_p(x) \quad \text{and} \quad \psi'_p(x + \Delta x) = \psi(x) \quad .$$

This tells us that the translated wavefunction $\psi'_p$ at the shifted position is equal to the original wavefunction $\psi$ at the unshifted position, as in Figure 14.

Now that we know what the momentum operator is, we may proceed with the derivation of the momentum operator as the generator of translations. We previously skipped this around Equation (.48) by assuming

$$(\hat{J}(dx) = 1 - i\hat{K} dx \quad \iff \quad \hat{K} \propto \hat{p} \quad .$$

Translated by some small amount, the momentum eigenfunction is

$$\hat{J}(-\delta x)\psi_p(x) = \psi_p(x + \delta x) = \exp\{ip(x + \delta x)\} = e^{ip\delta x} e^{ipx} = e^{ip\delta x} \psi_p(x) \quad .$$

We expand the displacement term as

$$\psi_p(x + \delta x) = [1 + i\delta x + O(\delta x^2)] \psi_p(x) \quad ,$$
and compare to the Taylor series expansion of $\psi_p(x + \delta x)$ around $x$

$$\psi_p(x + \delta x) = \psi(x)_p + \delta x \frac{d}{dx}\psi_p(x) + ...$$

$$= \left[ 1 + i\delta x \left( -i \frac{d}{dx} \right) + ... \right] \psi_p(x).$$

Equating $O(\delta x)$ terms between Equations (.67) and (.68), we find

$$-i \frac{d}{dx}\psi_p(x) = p\psi_p(x),$$

confirming that we have the correct form for the momentum operator. In the limit of infinitesimal $\delta x$, we ignore the $O(\delta x^2)$ part of Equation (.67) to write

$$\psi_p(x + \delta x) = (1 + ip\delta x) \psi_p(x)$$

$$= \left[ 1 + i\delta x \left( -i \frac{d}{dx} \right) \right] \psi_p(x)$$

$$= (1 + i\hat{p}\delta x) \psi_p(x)$$

$$= \left[ 1 - i\hat{K}(-\delta x) \right] \psi_p(x)$$

$$= \hat{J}(-\delta x)\psi_p(x).$$

By ignoring the $O(\delta x^2)$ terms and assuming that we can set terms of equal order in $\delta x$ equal between Equations (.67) and (.68), and by assuming that $\hat{J} = 1 - i\hat{K} dx$, we have made a derivation showing that the momentum operator is the generator of space translations.

Momentum in quantum mechanics goes on to be very complicated. Mainly, it is only possible to define the momentum operator as the mass times the derivative of the position for non-vanishing vector potential $\vec{A}$. This equality gives what is called the canonical momentum operator and written $\hat{p}$. In general, however, we have

$$\frac{\partial}{\partial t} \vec{x} = \frac{\hat{p} - \frac{e}{c}\vec{A}(\vec{x})}{m}.$$ 

Thus, we introduce the kinematical momentum operator

$$\hat{\Pi} = m\frac{d}{dt}\vec{x} = \hat{p} - \frac{e}{c}\vec{A}(\vec{x}).$$
The main difference between the canonical and kinematical momenta is

\[ [\hat{p}_k, \hat{p}_j] = 0 \quad \text{but} \quad [\hat{\Pi}_k, \hat{\Pi}_j] \neq 0 \]  \hfill (73)

It is known that the vector potential is not unique and the tricks that one can play with \( \vec{A}(\hat{x}) \) are the main inroads to theories of gauge freedom, or gauge theories. Usually the choice of one \( \vec{A}(\hat{x}) \) or another is called \textit{fixing the gauge}. In turn, this defines the kinematical momentum operator which replaces the \( \hat{p} \) we have postulated above.

**The Time Evolution Operator**

It is said that time doesn’t exist in quantum mechanics. What is meant is that states Hilbert space depend on spatial variables but not time. The time dependence is added to states as a phase factor. In the case of time-dependent Hamiltonian operators, there is a Hilbert space of energy eigenstates corresponding to every possible \( \hat{H}(t) \). Even in the case of a time-independent Hamiltonian, there is still a Hilbert space of energy eigenstates at every time \( t \). This can get glossed over since the eigenstates in each Hilbert are all the same. Indeed, the eigenstates of every observable operator belong to a Hilbert space at a specific time which is distinct from the space of states at any other time. The position space representation gives an outstanding thinking device for understanding the time structure of Hilbert space. If the wavefunctions are functions of the \( x^i \) spatial variables, then we may identify \( \hat{x}^i \) as the spanning basis of some hypersurface of constant proper time. For each of an infinite number of such surfaces, there exists a Hilbert space of wavefunctions which are functions of the \( x^i \) spatial variables at time \( t_0 \). So, while time dependence is usually added to quantum states as a phase factor outside of the Hilbert space, adding time dependence into Hilbert space results in states of the form \( \psi(x^i; t_0) \). The wavefunctions are not functions of time because \( t_0 \) is a constant.

To develop time evolution, we will introduce the symbol \( |\alpha; t_0, t\rangle \) to be the state at at time \( t > t_0 \) of a system that was observed to be in state \( \psi \) at time \( t_0 \). The connection to what we have done in the previous chapter is

\[ \lim_{t \to t_0} |\psi, t_0; t\rangle = |\psi\rangle \]  \hfill (74)

and we will also use notation presently such that

\[ |\psi, t_0; t\rangle = |\psi, t\rangle \]  \hfill (75)
We have previously implemented the spatial translation operator $\hat{J}$ such that
\[
\hat{J}(\Delta \vec{x})|\vec{x}\rangle = |\vec{x} + \Delta \vec{x}\rangle ,
\] (76)
and now we will develop the time translation operator
\[
\hat{U}(t, t_0)|\psi, t_0\rangle = |\psi, t_0; t\rangle .
\] (77)

The added argument $t_0$ tells us that $\hat{U}(t, t_0)$ only operates on the Hilbert space of states which exist at time $t_0$. We will see that this is redundant for time independent Hamiltonians but it is not redundant in general.

The requirements imposed on $\hat{U}(t_0, t)$ are mostly the same as those imposed on $\hat{J}$.

- If $|\psi, t_0\rangle$ is properly normalized to $\langle \psi, t_0|\psi, t_0\rangle = 1$, then the time evolved state $|\psi, t_0\rangle$ must maintain the normalization.
\[
\langle \psi, t_0; t|\psi, t_0; t\rangle = \langle \psi, t_0|\hat{U}^\dagger \hat{U}|\psi, t_0\rangle = 1 \implies \hat{U}^\dagger(t_0, t)\hat{U}(t_0, t) = 1 .
\] (78)

- Two consecutive time evolutions, $\hat{U}(t_1, t_0)$ followed by $\hat{U}(t_2, t_1)$, must be equal to a single time evolution by $\hat{U}(t_2, t_0)$.
\[
\hat{U}(t_2, t_1)\hat{U}(t_1, t_0) = \hat{U}(t_2, t_0) .
\] (79)

- Translation by $\Delta x_1$ and then $-\Delta x_1$ must be the identity operation.
\[
\hat{J}(-\Delta x_1)\hat{J}(\Delta x_1) = \mathbb{1} \implies \hat{J}(-\Delta x_1) = \hat{J}^{-1}(\Delta x_1) .
\] (80)

(CHECK?????????????)

- In the limit of vanishing displacement, the translation operator must reduce to the identity.
\[
\lim_{t \to t_0} \hat{U}(t, t_0) = \mathbb{1} .
\] (81)

- $\hat{U}(t_0 + dt, t_0)$ should be linear in $dt$ to leading order.
\[
|\mathbb{1} - \hat{U}(t_0 + dt, t_0)| = \mathcal{O}(dt) .
\] (82)

The unitarity condition of Equation (78) is required for the preservation of the probability interpretation. This is demonstrated when we require the sum of the squares of the expansion coefficients in a particular basis must sum to unity at all
times, as in Equation (4.37). To demonstrate, we expand in the discrete basis of $|a_k\rangle$

$$|\psi, t_0\rangle = \sum_k |a_k\rangle \langle a_k| \psi, t_0\rangle = \sum_k c_k(t_0) |a_k\rangle .$$

(4.83)

The meaning of $c_k(t_0)$ is exactly the same as the previous meaning of $c_k$. We add the time dependence because the probability for finding the $a_k$ eigenvalue in a measurement on $|\psi\rangle$ might not be constant in time. For example, if one prepares a system in an excited state, it will become less and likely that one will observe the system in the excited state as time goes on. On long time scales, systems tend to return to the ground state and/or come to thermodynamic equilibrium.

Assume that $\psi$ is normalized at $t_0$. Multiplying Equation (4.83) from the left with $\langle \psi, t_0 |$ yields

$$1 = \langle \psi, t_0 | \psi, t_0\rangle = \sum_k c_k(t_0) \langle \psi, t_0 | a_k\rangle = \sum_k c_k(t_0) c^*_k(t_0) = \sum_k |c_k(t_0)|^2 .$$

(4.84)

Since $t_0$ is an arbitrary time, this has to hold for any $t \neq t_0$. Therefore

$$1 = \langle \psi, t_0; t | \psi, t_0; t\rangle = \sum_k \langle \psi, t_0; t | a_k\rangle \langle a_k| \psi, t_0; t\rangle = \sum_k c_k(t) c^*_k(t) = \sum_k |c_k(t)|^2 ,$$

(4.85)

where $c_k(t)$ is defined to have the obvious meaning. Time evolution can alter the expansion coefficients in the expansion of an abstract state in a certain basis but the sum the coefficients’ absolute squares always adds up to one. This tells us that the probability of finding the state in one of the possible eigenstates at any time $t$ is always 100%.

As with the generator of translation $\hat{J}$, we will assume

$$\hat{U}(t_0 + dt, t_0) = 1 - i\hat{\Omega} dt ,$$

(4.86)

and then proceed to determine $\hat{\Omega}$. Studying $\hat{J}$, it was not mentioned that these ansatzes are not exactly unitary. Presently, we have

$$(1 - i\hat{\Omega} dt) (1 + i\hat{\Omega} dt) = 1 + \hat{\Omega}^2 dt^2 ,$$

(4.87)

though unitary operators satisfy

$$\hat{O}^\dagger \hat{O} = \mathbb{1} .$$

(4.88)
As is usual by now, we ignore $O(dt^2)$ terms. We will proceed as if it were unitary and call $\hat{U}$ the unitary time evolution operator. There are some principles of classical mechanics which motivate the Hamiltonian as the generator of time evolutions but we will simply postulate

$$\dot{\Omega} = \frac{1}{\hbar}\hat{H} \ . \ \ (.89)$$

$\hat{H}$ the Hamiltonian operator constructing by promoting all instances of positions and momenta in the classical Hamiltonian to their corresponding quantum quantities, or operators.

Now we will derive the fundamental equation for $\hat{U}$. The composition property of $\hat{U}$ is given by Equation (.79.) Combining the compositive law with the ansatz, Equation (.86), we have

$$\hat{U}(t + \delta t, t_0) = \hat{U}(t + \delta t, t)\hat{U}(t, t_0) \ \ \ \ \ \ \ \ (\text{.90})$$

Moving $\hat{U}(t, t_0)$ to the left, we proceed as

$$\hat{U}(t + \delta t, t_0) - \hat{U}(t, t_0) = -\frac{i}{\hbar}\hat{H} \delta t \hat{U}(t, t_0) \ \ \ \ \ \ \ \ (\text{.91})$$

In the limit $\delta t \to dt$, the left side contains the definition of the derivative with respect to $t$.

$$i\hbar \frac{\partial}{\partial t} \hat{U}(t, t_0) = \hat{H} \hat{U}(t, t_0) \ . \ \ (\text{.92})$$

As it turns out, Equation (.92) is the Schrödinger equation for the time translation operator. We obtain the Schrödinger equation for states by multiplying from the
right with $|ψ, t_0\rangle$. This yields,

$$iℏ \frac{∂}{∂t} \hat{U}(t, t_0)|ψ, t_0\rangle = \hat{H} \hat{U}(t, t_0)|ψ, t_0\rangle \tag{.93}$$

This is the Schrödinger equation. If we know how $\hat{U}(t, t_0)$ evolves, then we don’t need the Schrödinger equation for states. We can operate directly on the states with the time evolution operator $\hat{U}$ to generate states an arbitrary time given that we know the state at $t_0$. Therefore, we will solve the Schrödinger equation for $\hat{U}(t, t_0)$.

First we will examine the time independent case of $\hat{H} \neq \hat{H}(t)$. The familiar looking (hopefully) Differential Equation (.93) is solved by optical inspection as

$$\frac{∂}{∂t} \hat{U}(t, t_0) = \frac{1}{iℏ} \hat{H} \hat{U}(t, t_0) \implies \hat{U}(t, t_0) = \exp\left\{-i\frac{\hat{H}(t - t_0)}{ℏ}\right\} \tag{.94}$$

As a reminder that not all differential equations are solved by optical inspection, we write

$$\frac{∂}{∂t} \hat{U}(t, t_0) = \frac{1}{iℏ} \hat{H} \hat{U}(t, t_0) \implies \hat{U}(t, t_0) = \frac{1}{iℏ} \hat{H} \tag{.95}$$

The term on the left contains the total differential of $\hat{U}$ with respect to $t$ which can be replaced immediately but we introduce the formal $u$-substitution

$$u = \hat{U}(t', t_0) \implies du = \frac{∂}{∂t'} \hat{U}(t', t_0) dt' \tag{.96}$$

This yields

$$\int_{u(t_0)}^{u(t)} \frac{du}{u} = \int_{\hat{U}(t_0, t_0)}^{\hat{U}(t, t_0)} \frac{d\hat{U}}{\hat{U}}, \tag{.97}$$

so we continue from Equation (.95) as
\[ \int \hat{U}(t_0, t) \frac{d\hat{U}}{\hat{U}} = \frac{1}{i\hbar} \hat{H} \int_{t_0}^{t} dt' \]

\[ \ln \hat{U} \bigg|_{\hat{U}(t_0, t_0)}^{t} = \frac{1}{i\hbar} \hat{H} t' \bigg|_{t_0}^{t} \]

\[ \ln \left[ \hat{U}(t, t_0) \right] - \ln \left[ \hat{U}(t_0, t_0) \right] = \frac{1}{i\hbar} \hat{H} (t - t_0) . \]

The second term on the left is the identity by Equation (.81) and the log of the identity vanishes. Taking the exponential of both sides yields

\[ \hat{U}(t, t_0) = \exp \left\{ -\frac{i\hat{H}(t-t_0)}{\hbar} \right\} . \]

This is the unitary evolution operator for a state at time \( t_0 \) subject to a time independent Hamiltonian. Although the ansatz stated in Equation (.86) was not exactly unitary, the present form of \( \hat{U} \) given in Equation (.99) is exactly unitary because it is the exponential of a Hermitian matrix. An important statement in the construction of the MCM is \( \hat{U} := \partial_x \) and now we see the dependence

\[ \hat{H} = \frac{\hat{p}^2}{2m} = \frac{1}{2m} \left( -i\hbar \partial_x \right)^2 \implies \hat{U} = \exp \left\{ \frac{i\hbar t \partial^2}{2m \partial x^2} \right\} . \]

in the case of the free particle Hamiltonian.

If the Hamiltonian is a function of time \( \hat{H}(t) \), and if \( [\hat{H}(t_1), \hat{H}(t_2)] = 0 \) for any \( t_1, t_2 \), the solution proceeds identically except that we cannot take \( \hat{H}(t) \) out of the integral as we have in Equation (.95.) The result is

\[ \hat{U}(t, t_0) = \exp \left\{ -\frac{i}{\hbar} \int dt' \hat{H}(t') \right\} . \]

If the Hamiltonian is a function of time, and if \( [\hat{H}(t_1), \hat{H}(t_2)] = 0 \) for any \( t_1, t_2 \), the solution is much more complicated.

Observables that commute with the Hamiltonian are constants of time evolution which is generated by the Schrödinger equation. In general, one defines a correlation amplitude \( C(t) \) as a measure of the difference between \( |\psi, t_0\rangle \) and \( |\psi, t_0; t \rangle \). Depending on the energy of the system, states’ expansion coefficients resemble the expansion
coefficients at an earlier time to a greater or lesser degree.

\[ \hat{U}(t, t_0) = \exp\left\{ -\frac{i\hat{H}(t - t_0)}{\hbar} \right\} , \]  

\[ \hat{U}(t, t_0) = \exp\left\{ -\frac{i}{\hbar} \int_{t_0}^{t} dt' \hat{H}(t') \right\} . \]  

If \([\hat{H}(t_1), \hat{H}(t_2)] \neq 0\), \(\hat{U}\) is very much more complicated being written most generally as a Dyson series. Examples of the three increasing difficult cases of \(\hat{U}\) are a spin magnetic moment in (i) a static field, a (ii) a time varying field with a constant direction, (iii) a field varying in strength and direction. While we have not previously mentioned torsion during \(H_1 \rightarrow H_2\) transport, it is known that GR does not conserve spin without torsion. If the MCM is going to implement of scheme of quantum gravity, then it is reasonable to expect torsion effects along \(\chi^4\). This will invoke the third, most complicated representation

\[ \hat{U}(t, t_0) = 1 + \sum_{k=1}^{\infty} \left( -\frac{i}{\hbar} \right)^k \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \ldots \int_{t_0}^{t_{k-1}} dt_n \hat{H}(t_1)\hat{H}(t_2)\ldots\hat{H}(t_k) . \]

Here, the \(k = 1\) term is a single integral, the \(k = 2\) term is a double integral, etc. The \(k = \infty\) term is an infinite dimensional integral, notably. Although any finite \(\Delta t = t - t_0\) necessarily contains an uncountable infinity of different times \(t_k\) at which the Hamiltonian does not commute, the Dyson series is a decent approximation.
Appendix C: Schrödinger’s Equation

The main problem among the theses in the body of this paper regards finding the equation for $\hat{M}^3$. Schrödinger’s search for his eponymous equation is the main historical precedent for such an endeavor. The following concise statement appears in The Feynman Lectures on Physics [98].

“Where did we get [the Schrödinger equation] from? It’s not possible to derive it from anything you know. It came out of the mind of Schrödinger.”

Dirac wrote the following in 1926 [93].

“A new development of the theory has recently been given by Schrödinger [99]. Starting from the idea that an atomic system cannot be represented by a trajectory, i.e., by a point moving through the co-ordinate space, but must be represented by a wave in this space, Schrödinger obtains from a variation principle a differential equation which the wave function $\psi$ must satisfy. This differential equation turns out to be very closely connected with the Hamiltonian equation which specifies the system, namely, [the time derivative of a state is equal to a constant times the Hamiltonian operator acting the state.]”

Regarding Schrödinger’s variation principle, Galler, Canfield, and Fredericks write the following [100].

“In January of 1926, Erwin Schrödinger changed the face of physics forever. In his first paper, ‘Quantization as an eigenvalue problem (Part I)’ [99], Schrödinger presents a naive argument to ‘derive’ the Schrödinger wave equation and then proceeds to solve for the nonrelativistic energy eigenstates of hydrogen.[1]

Naive means “showing a lack of experience, wisdom, or judgment.” In the opinion of this writer, the authors of Reference [100] have misread Schrödinger. More likely than that the argument was naive, Schrödinger wrote down his equation in the course of playing with the theory, saw that it worked, supposed it must come from the action

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1While the influence of Whittaker’s mechanics textbook [101] on Dirac’s work is well documented, the authors of Reference [100] cite the similar but lesser known reliance of Schrödinger on Schlesinger’s differential equations textbook [102].
principle, and then reverse engineered the action extremized by his equation. The Einstein–Hilbert action was first formulated in this way. The MCM derivation of the fine structure constant presented in Reference [24] was the same. As we might have simply written Reference [24] as a single line reading, “The simplicity of the formula \((\Phi \pi)^3 + 2\pi \approx 137\), as well as the familiarity of the formula’s elements, suggest that it might be of interest to researchers investigating the origins of \(\alpha\),” Schrödinger might have simply left his equation without any motivation at all and left his Stark effect prediction to stand on its own. Obviously Schrödinger’s work far surpassed this writer’s because he not only supposed that there existed a differential equation for the \(\hat{H}\), but he found it and solved it and reported his predictions for the Stark effect.

On the other hand, Schrödinger was fortunate enough to have a trajectory related to dimensionful action, and the equation he was looking for was only the heat equation. In the MCM, it is not yet known exactly what principle might govern \(\alpha\)MCM—it’s not a trajectory so there is no appeal to the action principle—and the implied elliptic curve equation is much more complicated than a heat equation. However, as Schrödinger was motivated to include a justification for his equation which can be misconstrued as naive, and so has the explanation for \(\hat{M}^3\) been called naive, at best.

We conjecture that the same reading of Schrödinger as naive supports detractors criticisms of Reference [24]. In the case of Schrödinger and in the present case, the reverse engineered solution was somewhat clunky in the end because we were not able to backtrack the thinking to something familiar from which to then proceed as if from first principles. Even while Schrödinger’s equation is named after him, the action principle argument he presented in his first reports [99, 103] has not found much love among subsequent generations of physicists. Among a few ways to get to the Schrödinger equation, c.f.: Equation (??), the way that Schrödinger did it is not taught today [100]. By contrast, the mechanism reverse engineered for \(\alpha\)MCM has born fruit many times over. The MCM lattice was constructed to house the action of \(\hat{M}^3\), and in turn that lattice offers a beautiful solution to the fundamental problem of QFT as well as the prediction for no spin-0 fundamental particles. Many other utilities for \(\hat{M}^3\) appear among the 77 theses in the body of this paper. However, where Heisenberg brought the ideas of Bohr to mathematical completion, and where Schrödinger brought forward the ideas of de Broglie before having his own work completed by Dirac, and where everyone who has seen further has done so by standing on the shoulders of giants, the work of finding the differential equation for \(\hat{M}^3\) is impeded by the meddling tentacles of the United States of America.

In this appendix, we will examine a few ways to get to the Schrödinger equation
including the man’s own variational method. The purpose in part is to highlight that reverse engineered solutions often look different than forward engineered ones, especially when the reverse engineering leads to new principles such the action of a wavefunction or some $\hat M^3$ operator. In another part, the connection of the Schrödinger equation to the classical action principle is most relevant to the MCM. In the end $\hat M^3$ operates on wavefunctions (or states) and whatever the equations of motion are that might come from its equation, they will also satisfy an action principle. Having already come to Schrödinger’s equation in supposing that the Hamiltonian is the generator of time translations, we will proceed with Schrödinger’s method of variation as in Reference [103]:

“The theory which is reported in the following pages is based on the very interesting and fundamental researches of L. de Broglie [104] on what he called ‘phase-waves’ (‘ondes de phase’) and thought to be associated with the motion of material points, especially with the motion of an electron or proton. The point of view taken here, which was first published in a series of German papers [99], is rather that material points consist of, or are nothing but wave-systems. This extreme conception may be wrong, indeed it does not offer as yet the slightest explanation of why only such wave-systems seem to be realized in nature as correspond to mass-points of definite mass and charge. On the other hand opposite point of view, which neglects altogether the waves discovered by L. de Broglie and treats only the motion of material points, has led to such grave difficulties in the theory of atomic mechanics—and this after century-long development and refinement—that it seems not only not dangerous but even desirable, for a time at least, to lay an exaggerated stress on its counterpart. In doing this we must of course realize that a thorough correlation of all features of physical phenomena can probably be afforded only by a harmonic union of these two extremes.”

By the time $\hat M^3$ was proposed in Reference [24], physics had once again come to grave difficulties after nearly a century of development and refinement. In 2009 and in the following years, it seemed to this writer not only not dangerous, but even desirable, to lay an exaggerated stress on the counterpart. However, many of this writer’s detractors’ entire careers were encapsulated in a bubble of ideological stagnation. It was completely lost on some what are the ordinary protocols for overcoming such difficulties because they had not personally witnessed them in their lifetimes, i.e.; Kuhn’s *The Structure of Scientific Revolutions* [105]. Some detractors saw the flavor
of extreme conception in $\hat{M}^3$ and brushed it off solely on the quality that it may be wrong, possibly calling it not even wrong despite a full awareness that extreme conception is well motivated and useful in the face of extreme difficulties.

Although the derivation has been called naive, Schrödinger states [103] that the foremost of the chief advantages of his new thinking is that the laws of motion are deduced from Hamilton’s principle. The derivation proceeds as follows.

WHAT IS THE HAMILTON JACOBI EQUATION????

Before Schrödinger found the Schrödinger equation, he found the Klein–Gordon equation.

Schrödinger heard of Louis de Broglie’s work and said to himself (and others) ”if particles are waves, there should be a wave equation”.

The modern way to arrive at Schrödinger’s equation is

ALTERNATIVE METHOD

BALLANTYNE
Appendix D: Note on Previous Work

The early work seemed like it was written by someone who hadn’t put in the work doing years of advisement with a tenured professor. Instead of taking that as evidence that years of advisement with a tenured professor were in order, this writer’s haters took it as evidence that the opportunity for such advisement should be categorically denied. While one would think it proper that the work of someone never having begun a rigorous program of academic advisement would seem like it was written by such a person, this writer’s haters identified that as so grievous a problem that all access to academia should be terminated in full. Indeed, the work of one not having undergone advisement is supposed to seem non-standard and/or inferior. If not for that, the entire program of advisement would be a worthless waste of time adding nothing to a student’s knowledge or abilities. So, while the early work demonstrated a deep thirst for such advisement, the thirst was interpreted as reason to a deny drink. After all, one who already has water shouldn’t be thirsty. Sometimes when a baby is born, the doctor cuts it off from the placenta before all the blood goes into the baby. Maybe it is thought that if the baby was healthy, it would have already had all the blood it needed. Such was the condition of this writer’s time academia. It was wrongfully terminated after the early work demonstrated that the author had not yet matriculated through the academic system, as had all others whose work this writer’s work was held against.

Often one encounters words such as, “The author thanks so and so for help during the preparation of this manuscript.” In the absence of such words, the help of an adviser in coming into compliance with accepted standards is implicit. This writer has had no such help. The usual picture of a lone academic laboring in solitude is ill-fitting because...

It is tempting for this writer to compare the shortcomings of his own work to the shortcomings of others’ work. Ultimately, this would only serve to besmirch the work of other while failing to demonstrate that work units of polished perfection appear often many decades after the theory’s initial ideation. For instance, we might cite Dirac’s definition of his δ function [106] as

\[
\int_{-\infty}^{\infty} x \delta(x) = 1 .
\]

This is plainly wrong. An integral without a differential is ill-formed. However, any reader with even a slight inkling of being able to follow the author can fill in the implicit \( dx \) and easily understand Dirac’s meaning. We would compare this to
Next Steps and the Way Forward in the Modified Cosmological Model

detractors of Reference [24] who were unable to fill in the gap in their minds regarding the conditions under which a $\delta$ function was associated to bulk. In footnote 7 of Reference [50], Feynman cites the uncontrollable disturbance theory as the reason for the weird behavior in the double slit experiment. This was also wrong. Dirac writes the following [107].

"Take the function $f(abc...)$ which is equal to unity when $a = \alpha', b = \beta'$, $c = \gamma'$, ..., ($\alpha', \beta', \gamma'$, ... being eigenvalues of the observables $\alpha$, $\beta$, $\gamma$, ) and which vanishes otherwise, and form the function $f(\alpha\beta\gamma...)$. Let us evaluate the average of this function for the dynamical system in a certain state. If the state corresponds to the normalized vector $|X\rangle$ (a fixed vector in the Heisenberg scheme of quantum mechanics), and $\langle X|$ is the conjugate imaginary vector, then this average may be written as a scalar product

$$\langle X| f(\alpha\beta\gamma...) |X\rangle.$$ 

According to ordinary ideas of probability, [this expression] would be just the probability of $\alpha$ having the value $\alpha'$, $\beta$ having the value $\beta'$, $\gamma$ having the value $\gamma'$, and so on, for the state concerned. However, [this expression] is in general complex. Thus the theory allows one formally to give a value for the probability of non-commuting observables having specified numerical values, but this probability is in general a complex number, so it does not have an immediate physical application. All the same, if the probability is close to zero it can be interpreted as meaning that the observables $\alpha$, $\beta$, $\gamma$, ... are unlikely to have the values $\alpha'$, $\beta'$, $\gamma'$, ..., so there is a limited application for it.

==============
The limited number of citations: Feynman, Zee, Isham, Griffiths.
Lack of familiarity with others’ results does not imply invalidity of my own results.
Huge citation volume reflects colloquia and conferences.
Some good physicists browse arXiv every day.
==============
Dirac Delta. WHICH PAPER??????????
Several Feynman issues. + DIRAC
==============
"It may be wrong..."
"All of this was better known to Hamilton"
Some will say that although this writer ended up solving the most important unsolved problem in mathematics, his early work was worthless trash, not simply lacking the polish and guidance of an adviser.

Ampere didn’t even know about the Ampere–Maxwell law.
Appendix E: Certainty vs Uncertainty

- Which of $\Sigma^\pm$ first
- State space assignments
- Epistemology
- What is prediction?
- Which are the ontological basis assignments? We might use $\hat{\tau}$ instead of $\hat{2}$ also.

WHICH FIRST

Emphasizing the overall condition of assignments not being finalized, one might thing that the open topology of $dS_4$ is better suited to the white hole side of $\emptyset$, and that the black hole side ought to be the side with closed $AdS_4$ topology.

EPISTEMOLOGY

For the algebraic part cycling through an RHS instead of the unit cell’s branes, it is assumed that $\hat{M}^3$ operates on observable states and returns observable states. If $S_1 \subset S_2 \subset S_3$ form an RHS, then we may attach either of $S_1$ or $S_2$ to the observable step in $\mathcal{H}$ without disturbing the overall Copenhagen interpretation. However, there exists a famous issue in the epistemology of quantum mechanics regarding whether uncertainty is physically real or only a reflection of the observer’s ignorance. In the alternative epistemology, which is valid, it is possible that $\hat{M}^3$ will operate on classical states $\delta(x) \in S_3$, such that the uncertainty shifted out of $\mathcal{H}$ and into the psychological interim of the bulk of the unit cell. What is definite is that $\hat{M}^3$ is a threefold process which operates on states in some state space and returns states in that state space at a later time. The three steps should implement a dynamical wavefunction collapse which is absent from the mathematical foundations underlying any issue of epistemology. It is understood that $\hat{M}^3$ must accomplish wavefunction collapse and part of the work which remains is to examine all possible permutations of the assignment of an RHS’ state spaces to the respective manifolds cycled by the the MCM operator. At what step of $\hat{M}^3$ does the collapsed $\delta$ function state appear? Such varied organizations of quantum mechanics as the Dirac, Schrödinger, Heisenberg, and Feynman formulations underscore great freedom in the assembly of the theory, and
all of them fall under the Copenhagen interpretation and a canonical epistemology. Given such wide ranging freedom in the representation quantum theory itself, any one proposal for the RHS machinery of $\hat{M}^3$ is bound to be highly speculative, pending the clarification of further requirements, as needed. For as many representations of QM as there are, one might find a representation of $\hat{M}^3$ in each of them.

**PREDICTION**

$\hat{M}^3$ describes prediction and testing. Prediction in quantum mechanics is not so easy, however. Most or all processes appear stochastic. If one sends a particle through a pair of slits, it will appear to show up at any random place on the screen. Only after many such random events are recorded will a pattern on the screen tend to show wave behavior or particle. Before beginning the double slit experiment, it is possible to predict, “No particles will ever show up at these interference fringes,” but it is not possible to predict where a particle might show up outside of those fringes. This, then, begs a difficult question. If scintillation dots appear on the double slit screen in a stochastic manner, how might we ever implement some system of deterministic wavefunction collapse such that the end product of the evolution from the source, through the slits, and then to the screen ends with a $\delta$ function state at location where scintillation is observed? One ready answer is that the elapsed chronological time in the $A \rightarrow H$ step should be small enough that the $\delta$ function does not spread out beyond the radius of the dot which is appeared on the screen. When one does not make the famous measurement at the slits, particle is observed to leave the emitter. Schrödinger evolution through spacetime suggests that particles move through space as waves, pass through the slits, and then interfere on the screen as waves. $\hat{M}^3$ evolution suggests that the particles leave the universe at the emitter and then come back to the screen, hopefully as points spread out stochastically in the Schrödinger pattern. The $\hat{M}^3$ part forces the particle behavior and the Schrödinger part forces the particle to mostly stick to the classical trajectory, up to some fuzziness. In this way, $\hat{M}^3$ is intended to complement the time evolution operator $\hat{U}$ which is determined by the Schrödinger equation. If one does makes a measurement at the slits, then the $\hat{M}^3$ path of evolution leaves the universe at the emitter, comes back to the slits, leaves again, and then comes back to show particle like behavior distributed within the particle like Schrödinger prediction.
References


