

# A Complete Origami Nativity 

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For my grandma, Barbara, for encouraging me to follow my dreams.

With special thanks to Paul Hanson, Mark Bolitho, and Robert Lang for nurturing my love of folding.

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## Foreword

For a debut offering, you'll have to look hard to find a better book than this. It presents a fascinating selection of original designs ranging from the straight-forward to the highly complex. The theme is "Nativity" and whilst the subjects fit in perfectly, I believe that unlike most origami books, the models are not the only reason to invest in this book. I'd go so far as to say not even the main reason. How can this be? Because of the extraordinary depth of knowledge that is presented throughout. The author shows us an alternative way of looking at the design and the sequencing of origami, introducing the reader to a variety of concepts and techniques that they can use not just to complete the designs, but to adapt and extend them in their own work.

Take the example of "smooth folding" (a phrase coined by the author's sister-in-law). The idea is to minimise the appearance of exposed creases in the final model. It's something I actively try to achieve in my own work and I suspect many creators are aware of it, but I've never heard it discussed within a book, let alone given a name. It's not something you can simply "do", it needs careful thought and planning to decide which creases to omit and to balance the finish with the (counter-intuitive) increase in folding complexity to achieve a cleaner look. Many other important and useful techniques are similarly explored.

I cannot imagine there will be anyone who cannot improve their folding ability using this book. It will also give much food for thought to anyone interested in the "why" as well as the "how". In short, a near essential book for folders who have already learned the basics. The author has made a detailed and thoughtful study of the subject and this is apparent in the remarkable designs he has created. Clearly, a major new British creator has emerged. A word too for the publishers, who continue to go where most would fear to tread in order to promote a true understanding of paper-folding as an art as well as a recreational activity.


Contents






## Introduction

Modern origami design methods have developed to the point where subjects can be specifically designed, rather than being stumbled upon, as has been the more traditional approach. In recent years, there has been an internet-enabled explosion in both the abundance and complexity of origami designs. Nearly any subject you might want to fold has an origami representation available somewhere.

Yet I find there is something especially appealing about not just having individual forms, but an entire thematic collection with a consistent aesthetic. By folding all the models in this book, you can create a complete origami Nativity scene to scale.


However, this book's true purpose is not just to teach you how to fold a selection of specific subjects. The traditional Nativity scene offers a rich variety of subjects ideal for showcasing origami techniques with human figures, animals, geometric subjects and objects large and small. While folding each piece, you will also gain insight that can be used when you fold other models. In places, you'll even be encouraged to make your own entirely original origami designs.

Each model has diagrams with full accompanying text, followed by one or more additional extension activities. These extensions provide a further challenge building on the standard design, and a much greater opportunity to learn. In some cases, the extension task is a small variant included to practise a particular technique, while in others it is different enough to be reasonably called a whole new model.

Along the way, there are examples of a number of methods in the standard origami toolbox, such as dividing the paper into $3^{\text {rds }}$ and $5^{\text {ths }}$, or creating $30^{\circ}$ angles. There are also many techniques introduced, such as box pleating, wet folding, modular locks, and some of the mathematics which underlies origami design.

Despite the emphasis on learning techniques, there is plenty here to satisfy the experienced folder completing all of the diagrams and extension tasks will still provide dozens of hours of challenge. Whether you are a complete novice or an adept origami artist, I hope you enjoy these models and are inspired to modify and improve wherever you see fit.

## Difficulty

Each model has a difficulty rating with the following meaning：

## Simple

## 资

This model is essentially as simple as origami gets，so is accessible to anyone．You should still read the terminology section to familiarise yourself with the most standard terms，symbols，and manoeuvres．Start with these models if you have no origami experience．

## Low Intermediate 藘

This model is a little longer and more involved than a simple model，but fundamentally there isn＇t anything too challenging．

## High Intermediate 號

It is assumed that the folder has mastered the standard folds and is looking to move on to trickier manoeuvres．You should review the advanced symbols and manoeuvres on pages 71 and 72 for models of this difficulty level．

## Complex

## 资资梁

You should be comfortable with all named origami manoeuvres．Models will be challenging，and little explanation will be given for common folds．Do not expect any limit on diagram length when folding a complex model．

## Super Complex

Nothing is off－limits，and there is normally something particularly fiendish about a super complex model． The folder should be familiar with not just all individual folds shown in the diagrams，but also with generally reading and folding from a crease pattern，as this is the most sensible way of sharing a super complex model．


## Model Order

The models in this book are arranged in difficulty order of the standard subject. The extension tasks vary in difficulty and are simply put after the corresponding standard model, but they also roughly increase in difficulty throughout the book. This means if you are an inexperienced folder you may wish to start at the beginning of the book and work through the standard models in order.

## Tip

Don't be afraid if some of the models look tricky. Start at the beginning of the book, and work your way up to the advanced models gradually.


## Terminology

Raw corner - The actual corner of the paper.

Vertex - A point where creases and/or edges meet.

Corner - A point where the paper forms an angle less than $180^{\circ}$.

Edge/folded edge - An edge created by folding.

Flap - A portion of paper that can swing freely about an axis without affecting any other parts of the paper.


Raw edge - The actual edge of the paper.

Crease - A line that has previously been folded but is now flat. In a diagram, creases terminate a short distance away from a raw edge unless that crease is specifically being used in that step.

For the sake of clarity, there is often a deliberate gap between features drawn in a diagram - for example, between the raw edge and the vertical crease here.

White side - The side of the paper which is usually intended to be hidden in the final model.

Coloured side - The side of the paper which is usually intended to be seen in the final model.

Angle bisector - The term used to describe a line that halves an angle.
Base - The partially-folded stage of a model when all required flaps have been created, but there has been no artistic shaping.

Crease pattern - The folds used to create the base of a model (see page 74).

Diagrams - The set of images (and accompanying text) used to convey a folding sequence.

## Note

While understanding the terminology is useful, the text is merely a supplement to the image and not enough to fold from alone.
Use the images as the default method of reading the diagrams.

## Standard Symbols

Below are the symbols not connected to any specific type of folds, which are sufficient to fold the standard models in the first half of the book. See Advanced Symbols on page 71 for symbols which only appear in extension models and standard models in the second half of the book.

Turn over - By default, the symbol means flip the paper over from left to right. Sometimes, the symbol will be at an angle to show, e.g. flipping from top to bottom.


Reference point - A small blue circle highlights an important position.

In general, think of a blue circle as meaning "look here!".

Zoom in/out - A large blue circle with zoom lines shows more detail than can be normally seen for one or more steps. This might also show an x-ray view when a hidden layer cannot be seen from the current viewing angle.


Repeat - Repeat the numbered steps on the region(s) shown.

3.

4.

5.

6.

Repeating one action on the reverse within the current step is shown in the same way. Unless stated otherwise, or if there are clear contextual directions to the contrary, assume that repeating on the other side means turning the paper over about a vertical axis, performing a vertical reflection of the manoeuvre, then turning the paper over again about a vertical axis.


These two sequences have the same result.

3.


5.

.
6.

## Standard Manoeuvres

These are the named types of fold you will see in the standard models in the first half of this book. Advanced Manoeuvres can be found on page 72 .

Note that in this book, where applicable in the text the hyphenated form of a manoeuvre is the verb and the non-hyphenated form is the noun.

Fold/valley fold - The standard manoeuvre is the valley fold, and often an instruction will just say "fold". A dashed line shows where to make the fold, and this line is called the fold-line.

An arrow with a symmetric solid black arrowhead shows the motion of the paper.

A "valley" refers to the line made by a valley fold.
Mountain fold - A mountain fold is the opposite of a valley fold - if the paper were viewed from the other side it would be shown as a valley fold. This involves folding some of the paper behind, and the fold-line is shown with a dot-dot-dash pattern.

A single-sided white arrowhead is used for a mountain fold to show the motion of the paper.

A "mountain" refers to the edge made by a mountain-fold.

Unfold - A symmetric white arrowhead shows an unfold.
This will often be combined with a valley fold in one step, and the text will say "fold and unfold". The symbol for this is a double-ended arrow with a solid black head for the valley fold and a white arrow for the unfold. Pay particular attention to which end of the arrow is which, as the solid black arrowhead will always show the easiest motion for the valley fold (and the hollow arrowhead shows the subsequent unfold).

Reverse fold/inside reverse fold - An inside reverse fold is used where there are at least two layers of paper meeting at a line - this line is the spine of a reverse fold. A reverse fold uses a mountain fold on the front layer and a valley fold on the back layer to push the spine in on itself and change its orientation from a mountain to a valley.

An arrow shows the direction of motion, and a solid white push arrow will often be used.

Outside reverse fold - An outside reverse fold is the opposite of an inside reverse fold - the valley fold is on the front layer and the mountain fold is on the back. An outside reverse fold wraps the paper around itself and results in the spine being outward-facing on the region folded.


Squash fold - A squash fold is used to open out and flatten a folded edge symmetrically. This is much like performing an inside reverse fold, but leaving the spine open.


Swivel fold - An asymmetric squash fold is called a swivel fold.


Rabbit ear - A rabbit ear combines three valley folds and a mountain fold at once. Usually this brings all three sides of a known triangle to lie along one line.

A rabbit ear can be made by performing a valley fold followed by a swivel fold.


Pleat - A valley fold and a mountain fold (that don't intersect) performed together is called a pleat. Often this is done with a pair of parallel lines.

Pleats can include multiple layers - a diagram will usually have a
 symbol to show what is happening to layers of paper that can't be seen directly.

Preliminary fold - One of the standard origami bases is called the preliminary base. The folds used in making this base are replicated frequently enough that it has become a manoeuvre in its own right to refer to a similar type of fold. A folding sequence for the preliminary fold can be seen in steps 3 to 7 of the Frankincense on page 26 .


Open/inflate - When opening up a part of the paper, the arrow shown on the right may be used.

## Paper

## What type of paper should I use?

Standard origami paper (kami) is coloured on one side and white on the other and is reasonably thin, which means it is suitable for most models in this book. This is the cheapest and most readily available origami paper to use and is almost always in precut squares (though watch out for poor quality kami where the paper isn't exactly square-shaped!). In some circumstances you will may prefer to use other paper, sometimes for purely aesthetic reasons, or because you need paper that holds its shape more readily or is more durable.

The text before each standard model will say when you should consider using paper that isn't kami. For the extension models, you can use your judgement on what paper will be the most suitable.

## What shape of paper do I need?

Origami usually begins with a square, but the generally accepted principle is that if a starting shape can be easily folded from a square, then that shape is also a valid starting point.

Using a sheet of paper with a large perimeter-to-area ratio significantly decreases the complexity of a model in general, so the origami purist might view this as something of a cheat. However, with any subject that is long and thin - snakes being the typical example - there is little to be gained by folding the paper a number of times until a sufficiently long and thin shape is achieved. In fact, there is a significant loss - the model becomes undesirably thick and paper is needlessly wasted. For environmental reasons alone it seems perfectly reasonable therefore to begin with a rectangle in certain cases, as is shown with the Shepherd's Crook. All other models in this book do use a square sheet.

## How big should my paper be?

Throughout this book, I'll refer to the width or dimensions of the paper to avoid confusion with papermaking size (see page 24). The ideal paper width for simple origami is a square with a side length around the $15 \mathrm{~cm} / 6^{\prime \prime}$ mark, and this is the standard paper width you will encounter.

As you become more adept, models will become more complex, which leads to a higher crease density. This should be compensated for by your ability to fold more intricately. Nonetheless, there will be instances where it is easier to use larger paper even for simpler models. On step 1 of each model is an image of the final model relative to the starting width of paper.

Each model in this book has a recommended width for the first time you fold a model, which is intended to be the most comfortable width when the model is folded on its own. However, to create a complete to-scale Nativity scene puts a significant constraint on the paper width on some models (e.g. the Gold, Frankincense, and Myrrh). These models have been intentionally designed to accommodate this width issue, so that they are all foldable at the scaled width as well. However, the difficulty rating of each model is based on the firsttime width only, so be prepared for more of a challenge if you wish to fold the scaled version of a model.

The paper dimensions mentioned in this book are all multiples of $5 \mathrm{~cm} / 2^{\prime \prime}$, for the sake of convenience and adhering to convention.

## Shepherd's Crook

First time:
伩 $15 \mathrm{~cm} / 6^{\prime \prime}$
To-scale: $\prod_{\downarrow} 20 \mathrm{~cm} / 8^{\prime \prime}$

## 棌



This model is the simplest one in this book, and is the only model that doesn't begin with a square sheet.

You can use the method below to get one or more sheets with the right proportions, instead of measuring the paper.


[^0]2. Fold the top raw corners down to lie on the crease you just made.

3. Fold the left and right raw edges onto the vertical crease.

6. Outside-reverse-fold so that the rightmost section becomes vertical.

4. Fold the paper in half.

Tip
The folds in steps 5 and 6 don't have any given reference points. Just make the folds lightly in roughly the right place, and adjust them afterwards if you aren't happy with the final model.

. Outside-reverse-fold the tip up.


First time: $20 \mathrm{~cm} / 8^{\prime \prime}$

To-scale: $\llbracket \downarrow 20 \mathrm{~cm} / 8^{\prime}$

## Note

This is an extension model if you are looking for the next simplest model after the Shepherd's Crook, turn to the Manger on page 14.

The Shepherd's Crook is rather wide, but a more elegant crook can be made by starting from a thinner sheet. I deliberately haven't stated the exact proportions, in order to encourage some experimentation and creativity.

Begin with steps 1 to 4 of the Shepherd's Crook. Then make the curved hook using a series of crimps (see page 72) to make a gentle curve.

Use fairly thick paper, because thin paper won't be sturdy enough to hold the shape of the model.


## Smooth Folding

## What is smooth folding?

Smooth folding means folding an origami model in such a way that there are no visible creases on the exterior of the final form. Smooth folding is on a spectrum; it is possible to partially smooth fold a model by omitting some creases but not others.

## Why smooth fold?

Smooth folding is done primarily to improve the appearance of a model. After all, creases that run across the surface of a model can be somewhat unsightly. Therefore, when making a model for a display it is best practice to smooth fold it. Many models photographed in this book were smooth folded (i.e. not necessarily made by following the folding sequence as diagrammed). The diagrams show the best way to make the structure of the model for a first-time folder. The avid folder will remake models and experiment to find ways to avoid these creases, even though this makes folding much more difficult.

There is also a practical benefit to smooth folding; excessive creases can hinder the folding process by cluttering up a model. Sometimes certain steps, such as collapses, are easier when only the minimum number of folds are present.


Left: Manger - notice how the vertical paper on the left face bends due to creases across the surface. Right: Smooth Manger - the equivalent region remains flat.

## How do I smooth fold?

In many cases, designers deliberately present a folding sequence in diagrams which make a model fully or partially smooth. If this isn't the case (or if you want to refine a partially smoothed sequence), a little planning work is needed.

## Planning

One of the most intuitive ways to plan a smooth model is to fold the model once and then colour all visible surfaces of the paper on the final model. Then unfold the paper flat. The coloured regions will show where you must be careful to avoid leaving creases when folding subsequent versions of the same model.

## Folding

The most straightforward way of following a "fold and unfold" instruction is to align the fold, completely flatten the whole sheet of paper with the new fold in place, and then unfold the paper again. This leaves creases running all the way from one side of the paper to the other.

However, smooth folding can require the folder to only make a fold in a specific section of the paper, rather than all the way across. An instruction will say "only mark firmly in the region shown" when this happens. This partial folding often means keeping part of the paper three-dimensional where it would be easier to fold it flat, or terminating the fold-line of a valley fold before it meets a reference point, such as another crease or an edge. There may not even be any references for where to terminate such lines. Under these circumstances, it can take a delicate touch to make folds precisely.

All of this means that smooth-folded versions of a model are more difficult than their creased counterparts. Therefore, make sure you are familiar with a model before attempting a smooth version.


Three valley folds in ascending order of difficulty from left to right. Left: The fold-line spans the whole width of the paper; the paper can be fully folded flat between the fold and the unfold. Centre: The fold-line terminates at a known reference. Right: The fold-line terminates at a point which has no references.

While this may be obvious, I should note that making mistakes can be costly when smooth folding. Paper has a natural tendency to bend along existing creases, so models with sections that should be flat are most effective when kept flat from the start. So although creases can be hidden, they will still remain in the paper to some extent once they've been made, ending up as unwanted weak points where the paper may bend.

## Tip

To hide an unwanted crease you can run a thumbnail along it until the paper is flat. The effectiveness of this varies with the paper type.

## Examples in this book

There are four extensions in the first half of this book specifically dedicated to smooth folding. These are in increasing order of difficulty:

- Smooth Manger (page 16)
- Smooth Gold (page 23)
- Smooth Frankincense (page 29)
- Smooth Myrrh (page 33)

All other diagrammed models are already designed to compromise between a simpler, shorter folding sequence and a perfectly smooth model. However, there is nothing to stop you attempting smoother versions for more practice, provided you are prepared for an increase in difficulty.


First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$


The lock in step 13 of this model relies on the tension of the paper to keep the inserted flap in the pocket. Thick paper is better for this, and there are no places where lots of layers overlap, so there is no downside to using thick paper with this model.


3. Fold the raw edges to the central vertical crease and unfold.

6. Turn the paper over.

4. Fold the raw corners in to the vertical creases.

7. Fold the bottom corners up to the top centre.

10. Swing the marked corners out to the sides. The paper will not lie flat from now on.

12. Pull out the coloured layer of paper so the layer is vertical.

13. Tuck the white raw corner into the pocket shown. Repeat behind.

5. Fold the raw edges to the central horizontal crease.

8. Fold the corners down to the centre of the diagonal edges.

11. Push the bottom corner shown until it is part of a flat edge.



First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$


The Manger holds its shape well, but some of the creases made along the way are a hindrance to the lock in step 13, because the paper has a tendency to bend along those creases where it would ideally lie flat.

For just one extra step compared to the previous diagrams, the below sequence can be used to prevent forming the central vertical crease. After this, make steps 7, 8, and 9 very lightly, and fold only the necessary portions of these steps more firmly at the end of the model. Alternatively, for a more difficult challenge, you could try to achieve the same end result with your own new sequence to avoid the folds in steps 7, 8, and 9 entirely.

4.

3.

6. Continue from step 5 of the Manger.


## Note

As this is an extension model, you may need to refer to Advanced Symbols on page 71.

This model is a little more advanced than the Manger, and as an additional challenge, there is no text to accompany the diagrams. It is important not to become reliant on text, because text should only be a supplement to the diagrams (and many diagrams you will encounter elsewhere do not have any text anyway).

The Manger with Legs uses a border graft to add legs to the Manger. See page 86 for a full explanation of this.




15.

18.

21.
22.

24.

23.



First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$ $5 \mathrm{~cm} / 2^{\prime \prime}$

## Note

See page 34 before folding the to-scale Gold.


In a world with coins, a gold bar is perhaps not the most practical form of giving gold, so this model is intended to be symbolic more than realistic.

The lock in step 16 is used to hold the model together, but isn't particularly strong, so choosing paper that holds folds well is useful here. Fittingly, this means that foil paper is a good option.


1. Fold and unfold the centre vertical and horizontal lines.

2. Fold the top and bottom raw edges to the centre and unfold.

3. Fold the top and bottom raw edges to the creases you just made and unfold.

4. Fold the left and right raw edges to the centre and unfold.

5. Fold the bottom right raw corner diagonally so the the right raw edge lies on the topmost crease.

6. Fold the left and right raw edges to the creases you just made and unfold.

7. Fold part of the diagonal edge to lie on the horizontal crease as shown. Ensure the fold-line goes through the highlighted point.

8. Unfold so that the paper is flat. The paper should be oriented in the same way as in step 7.

9. Turn the paper over.

10. Turn the paper over in the direction of the diagonal fold.

11. Repeat steps 7 to 11 three times so that the paper has both a vertical and a horizontal line of symmetry.

## Tip

Develop a habit of looking at the picture of the next step - in many cases, seeing the result of a manoeuvre is just as instructional as the text or an image of the step itself.

15. Fold the first layer of paper underneath. Repeat behind.

13. Fold the lines shown, to make the paper three-dimensional. The paper won't lie flat from now on.

16. Tuck the flap in between the layers. Repeat behind.

14. Bring the raw edges underneath on all four sides. The valley folds are the only new folds here.


First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$ $5 \mathrm{~cm} / 2^{\prime \prime}$
 $15 \mathrm{~cm} / 6^{\prime \prime}$

It is only marginally more difficult to fold a smooth version of the Gold than the standard version, and the payoff is well worth the effort.

The crease pattern of the final model is shown again on the right, but with the visible portion of the model highlighted in yellow. This is the region where you need to avoid extra creases, and this means reducing the length of the fold-lines in steps 1, 2, 5 and 7; use the adjusted steps below.


1.

5.

2.

7.

Tip
You won't have references for how far to fold the lines in step 2 until you have done step 5 . Fold sparingly, then adjust to make the full fold once the location is more apparent.

Gold: 8-16

8. Perform steps 8 to 16 of the Gold. For the repeats in step 12, use step 7 on this page.

## Wet Folding

## What is wet folding?

Wet folding is a technique that involves deliberately dampening the paper, in order to provide more shaping and rigidity to an origami model than is achievable with dry-folding.

One of the steps in paper-making is sizing, which is when compounds are added to the pulp to reduce ink bleeding and water absorption. Paper with a lot of size is often thicker and becomes resistant to retaining folds or curved shapes.

Wet folding uses the size of the paper to the folder's advantage. When the size is partially dissolved, it becomes more malleable and can be sculpted to have rounded curves. When the paper then dries, it sets and retains its shape more readily than if water hadn't been used.

Thick paper is preferred for wet folding, because it sets more solidly than thin paper and is less inclined to rip while the paper is wet.

## How do I wet fold?

In order to wet fold, many sources recommend fully dampening the paper before beginning any folding. We might call this "full wet folding", and this is a rather different experience from folding when the paper is dry. The paper is very soft when wet folding, so make folds carefully to avoid ripping, especially where many creases intersect. Also add water in small amounts at a time: more water can easily be added, but the only effective option to remove water from over-saturated paper is to wait for it to dry.

If you are wet folding primarily to set the paper rigidly, it is sufficient (and much easier) to dry fold the model first, then spray it with a fine coating of water and use tools such as wire ties and clamps to hold the paper in the desired position until it sets. This "wet setting" technique was used for many of the models photographed in this book, particularly the later, more complex ones.

These pages show a few behind-the-scenes photographs of models drying using this method.



## Tip

Wet folding can be used to give an exact curve to the base of the Frankincense by letting it dry while on a round object like the bottle above.

## Tip

To get a model like the Twohumped Camel to stand on all four legs once it has dried, make sure it sets while standing up.

$\square$
To-scale: $\square$ $5 \mathrm{~cm} / 2^{\prime \prime}$

## Note

See page 34 before folding the to-scale Frankincense.

This model starts by forming a preliminary base. If you are familiar with this base, feel free to use your favourite method to achieve the same result (provided that you first make the extra creases in step 2).



1. Fold and unfold the opposite raw edges together.

2. Fold the right raw corner to the left raw corner. Ensure the fold-line goes through the intersections shown.

3. Fold the left and right raw corners to the centre and unfold. $\stackrel{4}{4}$

4. Open the raw edges and squashfold the left raw corner to lie on the raw corners at the bottom.

5. Squash-fold the raw corner as in step 4.
6. Fold the raw edges to the centre then unfold. Repeat behind.

7. Fold between the intersections shown then unfold.

8. Bring the raw corner up to the intersection shown and unfold. Repeat behind.

9. Fold the raw corner underneath. Repeat behind.

10. Fold the left flap to the right on the front and the right flap to the left behind.

11. Bring the raw corner upwards by folding using the creases made in step 8. The paper won't lie flat until step 15.

12. Fold the edges to the centre and unfold.

13. Thin the flap using folds made in the previous step, while bringing the raw corner down. See the next step for a reference.


14. Note the references shown here.

Turn the paper over.

20. Rabbit-ear the raw corner down to the right.

21. Fold one flap to the right in front and one flap to the left behind.

22. Flatten the top point and bring the narrow raw corner up to the left.


23. Fold the paper under using an existing crease.

24. Roll the narrow flap to become the handle.

25. Thin and slightly curl the spout with mountain folds.

26. Make soft folds to round the central section.


First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$ $5 \mathrm{~cm} / 2^{\prime \prime}$


To make the Frankincense without any extra creases at all is debatably more trouble than it's worth, because it means not creasing the diagonal between the end of the handle and the spout tip (though you are most welcome to try this - this is shown in the photo above and crease pattern on the right).

However, the other prominent creases can be removed without too much trouble - these are the creases that go directly through the centre of the paper.

In order to do this, you will not be able to keep the centre of the paper flat like it is in the diagrammed sequence. Even with the long diagonal creased, this is a much harder challenge than the previous smooth models (the Smooth Manger on page 16 and Smooth Gold on page 23). Have a go at those two first (if you haven't done so already), to get some practice with the kind of delicate folds you will
 need to make.

$\square$
To-scale: $\square$ $5 \mathrm{~cm} / 2^{\prime \prime}$


Like the Gold and Frankincense, the Myrrh in this book is really a generic representative shape - it isn't meant to be a historically accurate container that would have been used for resin storage.

The order 7 rotational symmetry is due to a common way of forming a pyramid which is showcased here: the central angle is divided into 8ths and then one sector is folded away to push up the central point up. You can adapt this model to have a different order of symmetry if you wish, but 7 turns out to be one of the most convenient to use.



1. Fold and unfold the diagonals.

2. Turn the paper over.

3. Fold the opposite raw edges together and unfold.

4. Make a preliminary fold.

5. Squash-fold the left flap. Repeat behind.

6. Fold firmly through all layers and unfold.

7. Reverse-fold the corner.

8. Fold two flaps to the left on the front and two flaps to the right behind.

9. Fold the flap underneath the front layer.

10. Fold the corner up underneath the front layer. This is easier to do if you unfold the model slightly.

11. Gently open the top of the model, keeping only the folds from steps 11 and 17 in place.

12. Mountain-fold the edge underneath on an existing crease.


The main diagonal and orthogonal creases made in steps 1 and 3 of the Myrrh are the only unneeded creases in the model. To make a smooth Myrrh, these have to be folded so that the only visible marks are at the raw corners or raw edges.

There is an alternative, however. It feels more natural to try to keep the initial folds if possible, because they are fundamental to locating the rest of the creases. This motivates a rotation of the structure of the design by a 16 th of a turn, so that these diagonal and orthogonal folds take the role of the mountain folds made by the squash folds in steps 5 and 7. This makes the Myrrh perhaps the most interesting of the models with a smooth extension, since there hasn't previously been any reason to create a structural change.

By performing this rotation, we find some pleasant geometry (and end up with a slightly larger model). However, some of the references are rather inconvenient when it comes to actually folding the model which, when combined with not having step-by-steps diagrams, makes this the most difficult of the smooth models.

To fold this altered version, use the crease pattern on the right (see Crease Patterns on page 74).



## Miniature Origami

There are many self-imposed origami challenges which have become so numerous that they have formed a whole category of origami known as novelty challenges.

In my opinion, the most notable novelty challenges are those that directly hone a particular skill: some examples include folding with one hand only, folding without seeing your paper, and following or writing instructions that include text but no diagrams.

One of the most popular novelty challenges (and the most useful for developing technical origami skill) is folding miniature models. This means folds are extremely close together and the folder has to be very precise.

This is your challenge with the three gifts of Gold (page 20), Frankincense (page 26) and Myrrh (page 30) - to fold them to scale. You may wish to fold the smooth versions at the same time.

When folding a model with smaller paper than usual, the consequences of one inaccuracy can be severe, so take your time to get the folds in the right place.


When folding miniature models, a "folding tool" such as a cocktail stick or a pair of tweezers, may help to manipulate the paper.



## Baby Jesus

First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$ $10 \mathrm{~cm} / 4^{\prime \prime}$

This model uses the bird base, which is perhaps the unspoken global favourite of all the traditional origami bases. Even though the bird base is short and simple enough that an experienced folder can fold it within a minute, people have still developed a plethora of different sequences to arrive at it.

For the sake of authenticity, I present the method I personally use to make a bird base, but feel free to use whichever sequence you prefer.



1. Fold and unfold the diagonals.

2. Turn the paper over.

3. Fold opposite raw edges together and unfold.

4. Fold the raw edges to the diagonals, only marking firmly in the sections shown.

5. Make a preliminary fold.

6. Reverse-fold using existing creases.

7. Fold and unfold the flap on the top layer.

8. Fold and unfold the top layer.

9. Fold the corner to the crease intersection.

10. Fold the corner to the crease intersection.

11. Fold the flap on the right across to the left.

12. Fold the flap on the right across to the left.

13. Turn the paper over.

14. Fold the raw corner up so that the fold-line goes through the intersections shown.

15. Fold the corners to the intersections shown and unfold.

16. Fold and unfold, marking firmly only as far as the crease you just made.

17. Fold the tip down to the imaginary dotted line.

18. Fold the flap down so that the fold-line goes through the crease intersections shown.

19. Swivel-fold using creases made in steps 15 and 16.

20. Fold the corners into the centre.

21. Swivel-fold the points as far as they will go.

22. Turn the paper over.

23. Fold the right flap across using a vertical foldline, then do the same for the left flap.

24. Mountain-fold the edge underneath so that the foldline is directly above the flap below.

25. Valley-fold using a foldline that goes between the two points shown.

26. Lift up the flap without making a crease.

27. Tuck the corner under the raw edge.

28. Tuck the points under the horizontal edge.

29. Turn the paper over.


First time: $\qquad$
To-scale: $\square$ $10 \mathrm{~cm} / 4^{\prime \prime}$


The design for the Baby Jesus can be modified to become more three-dimensional and remove extraneous edges, at the cost of a bit more complexity. Below are diagrams for a version with these alterations.

This model includes some sinks - you may wish to see Advanced Manoeuvres on page 72 if you are unfamiliar with sinks.

There are also quite a few folds without any references. The choice of location has a significant impact on the shape of the final model. This makes the Christ Child an excellent model for practicing three-dimensional shaping and learning to experiment for yourself. You will need to consider the paper you use to hold the three-dimensional shape effectively: standard kami is usable but can have a tendency to tear. Foil-backed paper or wet folding both work well here.

For an additional challenge, there is no text to accompany the diagrams.


1. Start by folding the first 13 steps of the Baby Jesus.

2. 


3.

4.


6.

$\square$



16.

17.

21.

22.

23.

19.

24.

Step 25 is designed to hold the top raw corner firmly. To tuck it most effectively, work out where it needs to be, then unfold some of the previous steps, which you can refold after the tuck.


26.

0



First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$


The Stable is the only modular origami design in this book, which means that multiple sheets of paper are folded into units, and then the units are combined to make the subject (ideally by folding, not by gluing). Modular designs are good for folders who want to spend a lot of time folding without too much complexity. Many people find the repetition of making the same unit(s) multiple times to be quite therapeutic.

The sheets of paper used to make the Stable must all be squares of the same width, so the width is not listed next to each unit individually.

You will need to fold the following number of units using a total of 16 sheets of paper:

- Post $\times 4$ (page 44 )
- Beam $\times 7$ (page 46 )
- Ridge Board $\times 1$ (page 48 )
- King Post $\times 2$ (page 50)
- Roof $\times 2$ (page 52)

Tip
Before committing to folding the Stable, fold the hardest units first, to check you are comfortable with those. The King Post is the hardest of the units here.

After making these units, follow the assembly instructions on page 54 to put the units together. Note that this model is rated as high intermediate difficulty due to the hardest part of the model (the assembly), but it will take longer to make the Stable than other models with the same difficulty rating.

Use medium thickness paper - it needs to be sturdy enough that the Posts can hold the weight of the upper sheets, but thicker paper will make the locks much more difficult.

If you wish, you can add the Rooftop (page 66) to this Stable for a quick enhancement without having to fold the hardest units of the Stable Barn.

Note
All sheets of paper must be the same width, so this is not listed next to each unit individually.


1. Fold in half and unfold.
2. Fold in half, but only mark the right hand side. Then unfold.

3. Fold all the way across, then unfold.

4. Fold and unfold.

5. Fold and unfold but only mark the right hand side.

6. Fold and unfold.
7. Turn the paper over.

8. Fold and unfold but only mark the right hand side.

9. Fold the raw edge down to the first crease.

10. Fold the raw edge back up to meet the top edge.

11. Refold the pleat from steps 9 and 10.

12. Fold the left edge over while interlocking the pleated layers - tuck the near top left corner into the pocket on the right, but make sure that the raw corner ends up on the top.

13. Unfold steps 9 and 10.

14. Pleat the paper on existing creases.

15. Fold the right edge over.

16. Open the hole down the centre of the tube to form a hollow square prism.

17. Fold the lower section of the paper inside to lock the tube in place. Make sure the flap on the inside of the tube is pushed to the back edge to match with the top.

## Beam

## 湤湂



1. Fold the bottom raw edge to the top and unfold.

2. Fold and unfold into 8ths.

3. Fold the top and bottom raw edges to the crease and unfold.

4. Fold and unfold, but only mark firmly at the raw edges of the paper.

5. Fold the raw edges into the centre.

6. Fold the raw edges out.

7. Unfold step 5 (but not step 6).

8. Fold diagonal creases and unfold. You only need to fold firmly on the near layer of paper.

9. Fold diagonal creases and unfold.

10. Pleat the paper.

11. Pull out four corners.

12. Wrap the bottom layer up to lie on the top.

13. Fold the raw edge up.

14. Turn the paper over.

15. Reverse-fold four corners.

16. Reverse-fold the two white trapezia inside.

17. Turn the paper over.

18. Fold on existing creases. The model will be three-dimensional from now on.

19. Tuck the raw corners into the pockets.


## Ridge Board



1. Fold in half and unfold.

2. Fold the left side over using the existing vertical crease.

3. Fold and unfold the left and right raw edges to the leftmost crease.


Tip
Make sure your folds are sharp. The best method to achieve this is to reinforce each fold by running a thumbnail or a folding tool along the fold-line, once the fold has been made.
2. Fold the left raw edge to the crease.

4. Fold the right raw edge to the middle and unfold.

5. Unfold steps 2 and 3.

7. Turn the paper over.

9. Fold and unfold, marking firmly only at the raw edges.

10. Fold the raw edges to the creases you just made.
13. Fold the edge over, tucking the corners into the pockets under the raw edges.

12. Fold the edge over.


11. Turn the paper over.

14. Turn the paper over.

15. Open up the tube, forming a hollow rectangular prism.

## King Post <br> 旗彩



1. Fold in half and unfold.

2. Fold the right raw edge to the middle and unfold.

3. Fold and unfold, only marking firmly at the raw edges.

4. Fold the left raw edge to the crease.

5. Unfold steps 2 and 3.

6. Fold the left side over using the existing vertical crease.

7. Fold and unfold the left and right raw edges to the leftmost crease.

8. Fold and unfold through both layers in the regions shown.

9. Fold and unfold diagonal lines in the regions shown through both layers.


10. Fold the raw edge down to the bottom.

11. Turn the paper over.

12. Fold the right edge over, tucking it into the pocket on the left.

13. Pull the edge out and tuck it under the raw edge as shown.

14. Open the tube, forming a rectangular prism.

15. Reverse-fold the sides using existing creases.

16. Fold the tip down.

## Roof



1. Fold in half and unfold, only marking firmly at the bottom raw edge.

2. Fold and unfold to the creases you just made, folding all the way up the paper.

3. Fold the raw corners down to lie on the first crease.

4. Fold and unfold the left and right raw edges to the crease you just made, only marking firmly at the raw edge.

5. Fold and unfold the raw edges to the creases made in step 2.

6. Fold the diagonal edges together.

7. Fold the raw corners up, so that the raw edge lies on the creases you just made.

8. Fold the raw edge up as shown.

9. Pull out some paper from behind the near layer. This brings a point outwards to make a small white triangle on each side.

10. Turn the paper over.

11. Fold the edge up so that it is vertical and at a right angle to the rest of the paper.

12. Mountain-fold the sides using existing creases so that they become vertical and are at right angles to the rest of the paper.

## Stable Assembly湤潞



The Posts, King Posts and Beams will be assembled in the configuration above. Step-by-step instructions for this are below.


1. Tuck the upper tab of the Beam into the pocket at the top of the Post.

2. Tuck the lower tab into the pocket.

3. Attach another Post on the right using the
same method as with steps 1 and 2.

4. Repeat steps 1 to 3 with the other two Posts and another Beam.

5. Set aside these six units (to be attached in step 11).

6. Tuck the lower tab of another Beam
into the pocket of a King Post.

7. Tuck the upper tab of the Beam into the pocket.

8. Repeat steps 6 to 8 with the other King Post and another two Beams.

9. Repeat steps 6 and 7 with another Beam on the right.


10. Attach the units set aside in step 5 in the same manner as with steps 1 and 2.

11. Tuck the corners of the King Posts under the raw edges on the bottom face of the Ridge Board.
12. Tuck the flaps of the Roof units under the raw edges on the sides of the Ridge Board.



The Stable Barn offers a much larger step up in difficulty than any previous extension models. It includes diagonal rafters, a stronger join at the top of the model, a more sophisticated roof, and more space in the building itself than the Stable.

You will need to fold the following units using a total of 18 sheets of paper. As with the Stable, all sheets must be squares of the same width:

- Post $\times 4$ (page 44 )
- Beam $\times 2$ (page 46 )
- Ridge Board $\times 1$ (page 48)
- Rafter $\times 4$ (page 58)
- Roof Joint $\times 2$ - (page 62 )
- Upper Roof Slat $\times 2$ (page 64)
- Lower Roof Slat $\times 2$ (page 65)
- Rooftop $\times 1$ (page 66)

Once you have made all of the units, follow the assembly instructions on page 68.


Fold two units as shown in these diagrams (type A). Then fold two units which are mirror images of these (type B).


1. Mark the centres of each raw edge and unfold.

2. Fold the raw edges to the horizontal creases as shown and unfold.

3. Fold to the marks you just made and unfold.

4. Fold a line that goes between the intersections shown, marking firmly only in the bottom half of the paper, then unfold.

5. Fold the top raw edge to the crease you just made and unfold. Mark only on the right side of the paper.

6. Fold to the marks you just made and unfold.

7. Fold a line that goes between the intersections shown, marking firmly only over the previous crease, then unfold.

8. Fold the top raw edge to the crease you just made and unfold.

9. Fold the raw edges to the creases as shown and unfold.

10. Fold parts of the diagonal as shown and unfold.

11. Fold vertical lines that go through the intersections shown and unfold.

12. Turn the paper over.

13. Fold lines that go through the crease intersections shown and unfold.

14. Fold the top and bottom raw edges to the creases you just made and unfold.

15. Fold lines that go through the crease intersections shown and unfold.

16. Fold lines that go between the intersections shown.

17. Turn the paper over.

18. Fold halfway between two horizontal lines and unfold.

19. Fold lines that go between the crease intersections shown and unfold.

20. Fold lines that go between the crease intersections shown and unfold.

21. Fold the raw edges to the horizontal creases as shown and unfold.

22. Fold lines that go between the crease intersections shown and unfold.

23. Pleat on existing creases. Leave the folded parts perpendicular to the paper.

24. Reverse-fold the raw edge.

25. Fold up using existing creases.

26. Mountain-fold the top raw edge back using an existing crease.

27. Fold the raw edge to the left.

28. Wrap the raw edge around to the right using existing creases.
29. Squash-fold the bottom section to
lie horizontally.
30. Squash-fold the bottom section to
lie horizontally.


31. Reverse-fold the raw edge using existing creases.

32. Mountain-fold the top section to lie horizontally.

33. Tuck the raw edge into the pocket.

34. Repeat steps 26 to 34 on the right. You may have to partially undo the mountain-folds you made in steps 31 and 32 .


Type A (type B is a mirror image of this).

## Roof Joint



1. Fold in half and unfold.

2. Fold the raw edge over to the crease.

3. Unfold steps 2 and 3.

4. Fold and unfold the left and right raw edges to the leftmost crease.

5. Fold and unfold.

6. Fold and unfold in the regions shown.

7. Fold the bottom raw edge down.

8. Fold the right raw edge over.

9. Turn the paper over.

10. Fold the bottom raw edge up to the centre.

11. Fold the top raw edge down.

12. Turn the paper over.

13. Open the tube.


14. Turn the model over.
15. Fold a right angle in the tube using existing creases. This is a particularly tricky step, so be patient here.


16. Fold and unfold. Mark the raw edge only.

17. Fold the raw corners over so that the raw edges meet the crease you just made.

18. Fold the raw corners over so that the valley folds align with the ends of the previous folds.

19. Fold and unfold. Mark the raw edge only.

20. Fold the raw corners over.

21. Fold the raw corner back to the right so that the fold-line aligns with the edge behind it.


22. Perform steps 1 to 5 of the Upper Roof Slat.

23. Bring the raw corner which is underneath the white region up to the front.

24. Fold and unfold through all layers.

25. Fold the raw corner over slightly.

26. Fold the raw corner over to the right so that the intersection shown lies on the edge.

27. Fold the flap across to the left as
shown.

28. Turn the paper over from top to bottom.

29. Turn the paper over from top to bottom.

## 4 Toofiop



1. Fold in half and unfold.

2. Fold the top right raw corner to the intersection shown, marking firmly only in the bottom half of the paper, then unfold.

3. Turn the paper over.

4. Fold and unfold, only marking at the raw edges.

5. Repeat step 2 with the other three raw corners.

6. Fold raw edges to the creases you just made, only marking at the raw edges, then unfold.

7. Fold and unfold between the intersections shown.

8. Fold and unfold in the regions shown.

9. Mountain-fold in half.

10. Fold and unfold through both layers.
11. Swivel-fold using existing creases. Repeat behind.


12. Fold and unfold through both layers.

13. Reverse-fold each corner in and out.

14. Fold and unfold the raw corners. Repeat behind.

15. Fold the raw corners behind using the creases you just made. Repeat behind.

16. Form the mountain fold shown and tuck the corner underneath. Repeat behind.

17. Form the mountain fold shown and tuck the corner underneath. Repeat behind.

## Stable Barn Assembly <br> 鿟棌棌



1．Perform steps 1 to 5 of the Stable Assembly．


Stable Assembly：6－7

3．Attach the Rafters to the Roof Joints in the same manner as steps 6 to 7 of the Stable Assembly．

2．Slide the Roof Joints into the Ridge Board．Push them as far as they will go－the white parts of the Roof Joints will touch each other．


4．Attach the Rafters to the Posts in the same manner as steps 1 to 2 of the Stable Assembly．

5．Set this frame aside until step 11.


6．Slide a Lower Roof Slat into an Upper Roof Slat．


7．Mountain－fold the raw corner underneath．

8. Note flaps A, B and C which will be tucked into pockets in step 10. Turn the interlocked units over.

11. Tuck the flaps of the Rooftop under the raw edges of the Ridge Board.


## Interlude

It's time to let you in on a secret. If you've folded the models up to this point, it may have seemed like the extension models were designed as harder variants of the standard models, but in actuality, the extension models were all designed first, and the standard models were later made as simplified versions.

Indeed, the standard and extension models paired together so far are just different representations of the same subject - it's really just a choice of difficulty and/or stylistic preference which version you choose to make. This isn't the case for most of the models from this point onwards - the extension models are different subjects and intended to be made in addition to the standard versions to complete the Nativity scene.

In the following sections, the standard models were designed first and provide plenty of difficulty in their own right. This has two main consequences.

Firstly, this interlude is where the difficulty ramps up considerably. If you've folded some of the extension models before this point, you may have already referred to Advanced Symbols on the next page and Advanced Manoeuvres overleaf. If not, you will now need to look at these and be prepare to encounter them abundantly in all models from now on.

The second consequence is that the extension challenges will have a different feel, with your creativity taking the place of explicit diagrams in most cases. The difficulty of some of these extension models will really test even the most experienced folders.


In a sense, the first half of this book can be thought of as mostly focusing on folding, with tips and notes reflecting this. From now on, certain folding techniques will be assumed. For example, skills developed from the smooth extensions (such as folding carefully between crease intersections) will be integrated into the diagrams as standard, wherever feasible. Additionally, the tips and notes from now on are more geared towards design, with a scattering of mathematics to explain some of the design elements. This isn't compulsory reading by any means, but to gain the most from this book you would do well to review and get the gist of these sections.

I hope you enjoy the models from this point on - I can assure you it will take much longer to reach the end of this book than it has taken to get to this interlude!

## Advanced Symbols

Hidden/imaginary line - A relevant line that cannot be seen directly will be dotted. Hidden lines are obscured by nearer layers of paper and include edges and creases. Imaginary lines include extensions of edges and outlines of where paper will end up after making a fold.


Rotate - Turn the paper in the direction shown by the number of turns given.

A circle means rotate on a plane parallel to the page.


An ellipse means rotate about an axis in the direction of the squash - the larger arrow shows the front. This will only be used for threedimensional steps.




Right angle - This symbol shows two perpendicular lines (you may need to imagine one of the lines being extended).


Divisions - Split the length into the proportions shown. By default this means dividing into equal sections to perform the step, but occasionally this means divide into a certain ratio using a given folding sequence. Even if there is no sequence, you should always try to fold accurately when you see this symbol.


## Advanced Manoeuvres

Mixed reverse fold - Sometimes a reverse fold will involve multiple spines. Such a reverse fold will combine an inside reverse fold at one spine and an outside reverse fold at another. By default, it should be assumed that the mixed reverse fold doesn't lock any layers. If that isn't the case, a symbol will show the arrangement of layers.


Crimp - A crimp is a combination of two reverse folds performed at the same time (this is analogous to a pleat being a combination of a valley fold and a mountain fold). Sometimes the reverse folds cannot actually be performed individually, but where they can, the text will often say "reverse-fold in and out" rather than directly referring to a crimp.


As with a pleat, a symbol will show the layers of paper where not all layers can be seen.

Petal fold - A petal fold combines two swivel folds. Usually the two swivel folds are mirror images of each other with one on the left and one on the right.


Double rabbit ear - A double rabbit ear combines two rabbit ears. Usually the rabbit ears are mirror images of each other with one on the front and one on the back.


Spread-sink - A spread sink is like a squash fold (and is sometimes called a spread squash or squash sink), but doesn't have any folds terminating at the edge of the paper. These are most easily performed on a point that has a large angle (i.e. close to $180^{\circ}$ ). Like all sinks, a spread sink is resistant to any adjustment after it has been folded, so fold these slowly and carefully on your first attempt.


Open sink - An open sink is like an inside reverse fold except the folds form a complete loop in the centre of the paper rather than terminating at a raw edge on either side. The standard form of sink is the open sink which is usually most easily performed by opening and flattening the region to be sunk, pinching a mountain ridge along the entire folded loop, then inverting the area within the loop.


Closed sink - A closed sink is like an open sink, except it locks the layers of paper together. Hold all flaps together and invert the point (or sometimes an edge) at the top. The paper cannot be fully opened and flattened, unlike with an open sink, since some of the flaps are
 held together. Instead, the paper will resemble a cone during the sink.

Mixed sink - It is possible for a sink to be neither fully open nor fully closed. If a model requires a mixed sink, then either a diagram or the text will specify which flaps need to remain free or become locked - in the latter case, the other flaps can be sunk in whatever way is most convenient.


Double sink - Successive sinks can be combined in one diagram (and by default assume that these will be open sinks). Similarly, sometimes three or more sinks can be combined in this way. The text will usually say "sink in and out" rather than naming a double or triple sink etc. for these general multi-sinks.


Unsink - Very occasionally a point needs to be folded in a way that looks like the undoing of a sink. This is not because a point has been needlessly sunk in, but rather the sunken point has appeared as a result of a other folds. These are some of the trickiest individual manoeuvres,
 so be patient and be prepared to open up the paper significantly to achieve the result.

Collapse - A collapse is the catch-all term for a combination of folds not covered by any other named fold. Naturally, these are often the most difficult steps. These usually occur when folding from crease patterns, or when the diagrammer is unwilling or unable to use a simpler sequence that breaks the manoeuvre into multiple smaller steps.


## Crease Patterns

## What are crease patterns?

After folding an origami model, you will usually keep it folded (this is, after all, probably the intended form). But if you were to unfold a model and look at the lines spread across the paper, you would be looking at its crease pattern.

While every model has a crease pattern, in the sense that all models could be unfolded to show a pattern of creases in the paper, I should clarify that this section about crease patterns really only refers to purist representational origami - where one sheet of paper is used to make one model. Modular origami, for example, doesn't usually have the same design process, so understanding the crease pattern for a modular unit often involves a different sort of analysis.


The full crease pattern for the Shepherd's Crook.

## Why use crease patterns?

Over the centuries, paper has become cheaper and origami has spread across the world. The result was a worldwide explosion of new origami designs of increasing complexity in the tail end of the 20th century. But interestingly, the greatest advancements in origami design have come not from more paper-folding, but from paper-unfolding.

## Understanding structure

The final form of an origami model obscures many of the lessons we can learn from the structure of the model. By "structure", I'm referring to the layout of the different parts of the paper within the square.

By unfolding a model and analysing the location of the creases, we are much better equipped to explore questions like: "How much paper does this flap take up?", "How can I make just this part of the model longer?", and "Based on the length and location of the creases alone, what model will this crease pattern make when folded?".

But learning about a crease pattern isn't just useful from a design point of view.


The simple crease pattern for the Manger shows that crease patterns are not always intimidating.

## Practicality

Complex origami models can now be hundreds of steps long, and making even poor-quality diagrams for this sort of model can take a tremendous amount of time. Crease patterns provide an alternative, and while they generally cannot fully describe a model in the way traditional diagrams would, they can achieve most of the goal with a small fraction of the effort.

Not only this, but it's much easier to share a model and learn from the design at a glance with one image than it is to have dozens of pages of diagrams. It's understandable therefore that many designers opt to use crease patterns to present their work.

## Learning to use crease patterns

There is, however, a major disadvantage to this new norm of using crease patterns rather than traditional step-by-step diagrams. Because crease patterns are typically only used for complex models, it can be very hard for less experienced or less advanced folders to practice using them. It might be argued that this provides a natural barrier to prevent people from trying to go beyond their folding ability. After all, if someone hasn't put in the time to learn about crease patterns, a model presented only in the form of a crease pattern will likely be too difficult for them anyway.

The problem with this line of reasoning is that although such a barrier might be well-intentioned, protecting folders from going "beyond their reach", does nothing to help improve their reach. Folders should be encouraged to experiment and the focus should be on a folder's gradual learning, rather than if they manage to complete a given model.

Crease patterns don't have to be treated as an alternative to diagrams, or only act as a standalone method for presenting a model. Accompanying a crease pattern with even just a few comments or with an image of the model's base will go a long way to improving its accessibility.

My preference would be that all origami diagrammers show the crease pattern at the start of a (representational one-sheet) model, no matter how simple. That way people will be exposed to crease patterns early and learn to read them gradually, rather than suddenly having to jump that hurdle just at the point when they think they are starting to master the art.

This is why even the simplest models in this book have a crease pattern at the start, and why it's worth noting that when the model is simple, the crease pattern doesn't look nearly so daunting as when the model is a complex behemoth. I would encourage anyone at all interested in folding to pay attention to the crease patterns shown next to a model - at the very least, always try to figure out where the paper in the crease pattern ends up in the final model.

## Why don't some crease patterns show all the folds of a model?

It's worth noting that when folding a model from diagrams there are almost always creases made along the way that aren't actually folded in the final model (for example, marks made when finding references). For the sake of clarity, these lines aren't included in the crease pattern.

Additionally, for complex models where there are sometimes lots of intricate folds, the full crease pattern for the final model is frequently not as helpful as a less cluttered selection of creases that show how to partially fold a model. Clarity and ease of folding should trump technical accuracy here.


The complete crease pattern for the Feathered Angel's base. This is rather overwhelming and not as useful for seeing the overall structure or where to begin as the more detailed steps on page 146.

Crease patterns like this ought to have additional details or be broken down to help less experienced folders.

## Structural and artistic creases

It's useful to be aware of the distinction between a structural crease and an artistic crease.
Conventionally, the text in most origami diagrams will shift from referring to "points","flaps" and "edges" to using the names of the individual features of the subject being represented. This is because the first major stage of folding a model is about getting the various parts of the paper into the right places - the folds required for this stage are structural creases in the crease pattern and make up the model's base.

Features of the final model are not clearly defined until the base has been folded, but once the overall structure is in place, we can treat sections independently as parts of the final model. The folds for shaping the model often don't have a specified location, but are made at the discretion of the folder. These folds end up as artistic creases in the crease pattern.

Defining the base is partly a subjective decision. Even with a base as popular as the bird base, there is not a unanimous agreement on whether all of the flaps should be pointing down (the convention used in this book) or if two of the large flaps should be pointing upwards. From now on, act as though each model has one clearly-defined base, even if in the back of your mind you know this is a slight oversimplification.

## Folding with Individuality

The origami designer must have proficiency in both the structural aspect of design, and with the artistry involved in shaping the model into something spectacular. However, the structural stage of origami design can be outsourced in a way that the artistry stage cannot.

With step-by-step diagrams, the distinction between structure and artistry is disguised, leaving room for the folder to experiment with their own individual style. Naturally, it's most convenient for the designer to finish the whole model when making diagrams, but it's an interesting thought experiment to consider the consequences of diagrams always stopping once the base has been reached. The enforced creativity would likely lead to a greater level of diversity amongst origami models. In practice of course, this would risk alienating less experienced folders - a good compromise would be to indicate to the folder where they no longer need to rigidly follow the steps provided.

Conversely, it's quite clear that the purpose of a crease pattern is to convey the structural creases needed for the folder to make the base, and no more. Everything after the base is completed can be left to the individual folder's creative judgement. An experienced folder may actually enjoy having a crease pattern rather than diagrams, since they know they have the freedom to make the model their own.

## How do I fold from a crease pattern?

The first step in learning to fold from a crease pattern is to change your understanding of how an origami model is put together.

Traditional step-by-step diagrams give the impression that models all have a definite sequence to be followed and are made up of distinct, discrete steps. The insistence on naming manoeuvres does nothing to contradict this view. Of course, diagrams and naming standards are very useful for communication and instruction, but they shouldn't dominate the way we think about models.

As seen in the Standard Manoeuvres section on page 6, there are multiple equivalent sequences that can be used to fold even some of the most basic manoeuvres and traditional bases. Folding from a crease pattern should be viewed as the most general of all such methods.

The method for folding from a crease pattern, in its purest form, is this: fold and unfold every line in advance, then, when all of this pre-creasing is done, fold every single line simultaneously. Sometimes there are "stepping stones" when folding the pattern, where the paper can be folded flat midway through, but these types of pattern are not guaranteed.

When you first come across this methodology, having only ever folded from diagrams, it can sound ridiculous - but sometimes this really is the only way. That said, as you become more advanced you will begin to be unfazed by the process of one giant collapse - as ever, experience and temerity are key.

Here are some general tips to fold from a crease pattern:

- Always pre-crease every fold (rather than being in a rush to start the "actual folding").
- Identify (from the pattern) where each part of the paper ends up in the base while the paper is still flat.
- Long lines are generally most important to the design, so when collapsing the model these should often be folded first.
- Many folders recommend starting in the middle of the paper, where the folding is usually most difficult, then working outwards towards the edges.
- Some crease patterns need to be folded uniformly so that all sections are at the same stage of the collapse at any one time. Don't expect to be able to completely finish folding one section before moving to the next. Instead, cycle through each section multiple times, gradually making each section more flat every time you come to it.


## Crease Pattern Conventions

While I can't prescribe the conventions used by other folders when making crease patterns in general, I can provide a few principles that I abide by in this book (unless clearly stated otherwise):

- For the sake of clarity, the white side of the paper is always shown in a crease pattern, regardless of the intended colour of the model.


Although these crease patterns for the Star are the same, the image on the left is much clearer due to the lighter background colour. The side of the paper shown is irrelevant to the side of the paper intended for outside of the model.

- Crease patterns will always show a difference between the mountain and valley folds. Valley folds are dashed blue lines and mountain folds are solid red lines. Of course, if you view the paper from below, the valleys/mountains assignment will switch, so you can view these as the other way around if desired. What matters here is the distinction between the two types of folds. Work out which side of your paper you wish to be visible and assign the valleys and mountains accordingly.


These two crease patterns for the Ox have precisely the opposite valley/mountain assignments and are therefore equivalent - both options will yield the same model.

- Paper that isn't used is shown in a paler colour and won't have any creases. This is most frequently seen when a raw corner is folded under and just needs to be treated the exact same way as the paper above it. Although there may end up being creases on these areas, those creases are indicated elsewhere, so showing them on the crease pattern would be redundant and could also mask the structure of the design. It's doable to fold unused paper out of the way early on before pre-creasing through multiple layers, but usually you can make more accurate folds by pre-creasing while the paper is entirely flat, then folding the unused paper and pre-creasing that part separately.
- If the crease pattern is the only instruction provided for a model, key reference points will be shown using interval divisions, or even a small square showing a folding sequence. These important folds should be made early on, since many other folds will depend on them. Sometimes there are lines which are not actually folded, but are still important for finding a reference point or understanding the structure of the model. These are shown with black dotted lines.


Crease pattern, base, and final model of the Myrrh. Note that the dotted black lines for the octagons in the crease pattern for the Myrrh above are not folded in the base. However, they are shown since it's useful to see these - they are creased in the base and they are used in the final model.

- The vast majority of folds (i.e. every fold that doesn't have a symbol or extra instruction) can be located using other existing reference points. If in doubt, try seeing if a line connects two known points, is halfway between two parallel lines, or is an angle bisector. Sometimes, however, a fold's location is best determined by an application of Kawasaki's theorem (see page 81). The tell-tale sign for this is that one endpoint is known, but the angle of the fold isn't obvious. The solution here is to fold the other lines around the known vertex and just see where the mysterious final fold ends up. As long as you make the paper around the vertex flat (and there is only one unknown angle), Kawasaki's theorem guarantees that there will be a unique angle for this missing fold.
- Creases do not terminate at raw edges (recall that creases terminate before the raw edge in diagrams, which helps identify where the raw edge is, but this aid isn't needed in a crease pattern).


Crease pattern for the Smooth Myrrh. Interval divisions and dotted lines are used to show the locations of creases which do not have obvious references. The Smooth Myrrh has no flat-foldable base, so the central octagon has to be folded in the crease pattern (unlike in the Myrrh)

## Useful Mathematical Theorems

There are two well-known mathematical theorems which are necessary (but not sufficient) conditions for being able to fold a crease pattern so that the paper lies completely flat. This means that the conditions in both theorems need to be true for the crease pattern to be flat-foldable (necessary), but there will still be other things to consider (not sufficient). This information can be particularly useful for those who are drawing a crease pattern, whether they are the original designer of the pattern, or someone trying to fill in details so they can fold from an incorrect or incomplete crease pattern.

Both of these theorems apply in the middle of the paper, but not at a raw edge or a raw corner.

## Maekawa's Theorem

At any flat-foldable vertex, the difference between the number of mountain folds and the number of valley folds is always two.



Valley folds around the vertex Mountain folds around the vertex Is Maekawa's condition satisfied?

A corollary of Maekawa's theorem is that there will always be an even number of creases around a vertex, or equivalently, an even number of angles around a vertex.

## Kawasaki's Theorem

The sums of each set of alternating angles around a flat-foldable vertex are both equal to $180^{\circ}$.


## Two-colourability

An appealing property of flat-foldable crease patterns is that the polygons of the paper bordered by creases and raw edges can be two-coloured, so that no adjacent regions have the same colour. This is actually quite easy to understand: all the regions that remain facing upwards in the folded form will have one colour, and regions which are upside-down after folding will have the other colour. None of these regions can be adjacent, because a fold-line appears precisely at the border between regions facing opposite directions.

Again, note that this property is always present when a crease pattern is flat-foldable, but like the conditions above, this property is not sufficient to ensure a crease pattern is flat-foldable.


Left: Each side of a flat-folded sheet of paper can be assigned a different colour. After unfolding, no two adjacent regions in the crease pattern will have the same colour.

Right: An example of the two-colourability of flat-folded crease patterns using the crease pattern of Kneeling Mary. Note that no two regions of different colours are touching, and how such a colouring naturally shows the alternating angles suitable for checking if the conditions of Kawasaki's

Theorem are met.


First time: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$

To-scale: $\square$ $10 \mathrm{~cm} / 4^{\prime \prime}$

Wearable: $\square$ $80 \mathrm{~cm} / 32^{\prime \prime}$


The traditional image of a crown - a hollow metal ring with spikes - is more accurately called a coronet. A (Western) crown has arches and usually has an elaborate covering.

Origami coronets are normally modular designs, so for a change, here is a design made from just one square. A repeating pattern which is more commonly seen in tessellations (see page 93) mimics the final form of a more traditional modular design. This repeating pattern is based on translational symmetry, and in fact the Coronet and the Antique Crown are unique in this book by being the only models without either reflectional or rotational symmetry.

The base of the Coronet ends up rather thick, so use thin paper. Foilbacked paper is especially effective here because it is thin and metallic.


1. Fold and unfold the central horizontal and vertical lines.

2. Fold and unfold the raw edges to the creases you just made.


3. Fold and unfold to make $8^{\text {ths }}$ horizontally.

4. Fold the lines shown and unfold.

5. Fold the lines shown and unfold.

6. Turn the paper over.

7. Fold the raw corner over.

8. Fold the lines shown and unfold.

9. Fold the lines shown and unfold.

10. Fold the raw corner and unfold.

11. Turn the paper over.

12. Collapse the paper on the creases shown...

13. ...and fold the upper raw edge behind to flatten the paper again.

14. Mountain-fold the front layer underneath so that it lies between two layers of the leftmost upper point.

15. Turn the paper over.

16. Swivel-fold the corner down while bringing the leftmost raw edge across to the right.

17. Squash-fold the raw corner down, bringing the raw edge to the left.

18. Curl the model and tuck the rightmost point into the pocket of the leftmost point.

19. Fold the corner behind.

20. Mountain-fold the entire ridge underneath using the crease made in step 21.

21. Fold through all layers and unfold.


Tip
If you have a tapered object such as a funnel or a makeshift cone, press the Coronet over it to get an exact circular shape.


The number of spikes on the Coronet was determined by a convenient way of dividing the paper, namely into 8 ths.

However, the same pattern for the Coronet can be used to make an arbitrary number of points. The crease pattern on the right shows how you can do this to create eight points - the number of an Antique Crown (also known as an "Eastern Crown"). It may be helpful for you to turn to Crease Patterns on page 74.

Using this pattern to create points works best when the the outermost points overlap and are used as a lock to hold the ring together. Refer to step 22 of the Coronet as an example of this.

You can extend the pattern to make a crown with an arbitrary number of spikes by using a grid with a larger grid number. See page 158 for quick grid divisions if you decide to do this. Be aware that the more points you make, the thicker the final model will be, so large, thin paper will be necessary.


## Grafting

## What is Grafting?

Grafting is an origami design technique used to augment and enhance existing models, usually to add fine details. Learning to modify designs is a stepping stone to creating your own entirely new models, so grasping grafting is one of the best starting points if you are learning to design models for the first time.

The most useful sorts of modifications are those that allow us to add points, flaps, and other details without having to change the main design significantly. Fundamentally, this means adding more paper to use for these new features, which has traditionally been done by using multiple sheets of paper. By contrast, grafting allows the designer to 'add' paper to a design while still folding purely from a square sheet.

Grafting is one of the most versatile origami design techniques; ten of the extension models in this book utilise grafting in some way. In this section we'll look at two specific examples: the Manger with Legs and the Camel with Toes. As with all topics in this book, these pages offer only an introduction to the idea, and you would do well to play around yourself and/or find further reading for more detail.

## Border Grafts

The simplest type of graft is a border graft. This is most easily imagined as folding over the raw edges of the square so that the resulting shape is a new, smaller square, an example of which is shown below. A known design can then be folded from this new square, with some extra paper in the final model where the raw corners and raw edges would otherwise have been. This new paper at the edge can be hard to use well, but the new paper at the raw corners can be easily pulled out and used to make more details that weren't present in the original design.

The compromise made with a graft is that the square obtained after folding over the edges will be smaller than the original square. Therefore, the more paper that is folded over, the larger the amount of paper in the graft, but the smaller the grafted model will be compared with the original.

So that's the idea in a nutshell, though there are details that need to be worked out when doing this in practice.


Here is an example of folding two raw edges of a square over to make a new square. This border graft provides paper that can be easily used at the top left corner.


A symmetric border graft - folding all raw edges allows paper to be easily pulled out and used at all four corners.

## Border Graft Example: Manger with Legs

The design for the Manger with Legs adds a border graft to the standard Manger; the four raw corners of the grafted paper are used for four individual legs.

Studying the crease patterns of the original and grafted models (shown below) illustrates how this graft works. It's also helpful to be familiar with both the Manger and the Manger with Legs in their final folded forms - if you haven't done so already, I'd recommend folding both of these now, then coming back to this section.

In reality, grafting does not involve any paper being added to or extended from an existing sheet. However, for the purposes of origami design, the border graft can be thought of as adding new paper around the existing crease pattern. The 'new' paper used in the graft for the Manger is shown in the central image below. To make all of the legs the same length, the width of paper folded over in the border graft must be the same on all four raw edges. The width of the border on each side is equal to one quarter of the original square's width, or equivalently, one sixth of the new square's width.


Left: Crease pattern for the standard Manger. Centre: A border has been added around the Manger's crease pattern.

Right: The final crease pattern for the Manger with Legs.

Although you don't necessarily have to create a whole new crease pattern for a new design, in this case it's illustrative to see the completed crease pattern for the Manger with Legs. In general, as well as including the newly added feature(s), the crease pattern of the final grafted model will also make some changes to the existing portion of the crease pattern. This is the case with the Manger with Legs above.

In practice, the most expedient way to complete the crease pattern after doing a border graft is to actually fold the model, though an experienced designer may be able to fill in many of the gaps without having to fold the new model at all.

Conveniently, due to the border graft, having the coloured side of the paper end up on the external part of the final model means starting with the paper the other side up. This actually means there is a colour change on the inner part of the Manger with Legs. This change was unintended, but as is often the way in origami design, this sort of thing should be capitalised upon. In this case, the folder can use straw-coloured paper on one side, and a wood colour on the other side.

## Strip Grafts

It can be tricky to fold the extra paper provided by a border grafts for flaps which used the raw edge of the paper on the original model. Border grafts are of no use at all in situations where the designer wishes to add features to a flap which is internal to the model, rather than on the raw edge. Fortunately, grafting can be done in the centre of the paper too using similar principles as border grafting. This is called strip grafting. The simplest way to do this is to pleat the paper to make at least one 'new' strip vertically and one horizontally. The pleated extra paper at the raw edge can be used to add details to the end of an otherwise plain flap - fingers and toes are the usual examples.

With a border graft, the desired amount of paper is always folded over at the edge, and the only consideration is how much paper needs to be added. A little more thought is required with a strip graft, because the designer has to make sure the pleat is not only the right width, but also at the correct location along the edge of the paper. Below is an example of how this might go wrong.


The correct approach for a strip graft where the pleats must end up $2 / 3$ of the way along the square's width: only the paper not to be used in the graft should be accounted for. Only this paper is divided in the desired ratio, which in this case is 2:1.


An incorrect approach: centering the pleats of the strip graft at $2 / 3$ of the square's width in the original square actually results in a graft which is not $2 / 3$ of the way along the square's width in the pleated paper.

A good way of looking at a strip graft is think of it as inserting pleated paper into the gap made by cutting the original paper in the right place, since this newly added paper wouldn't be counted when determining the location for the graft.


Think of a strip graft as cutting the paper at the right location for the graft and separating the sections, and then inserting the paper for a strip graft. This new sheet of paper then shows where the graft actually needs to be in order to be folded as per the previous figure.

As with border grafting, if both horizontal and vertical grafts are made, and both use the same width of new paper, then the resultant shape is a square, and so a known model can be made using the new smaller square. Note too that strip grafts and border grafts can be used together.

## Strip Graft Example: Camel with Toes

Below is a slightly simplified version of the crease pattern for the Two-humped Camel. The Camel with Toes split grafts the paper at the end of each leg so that there are two toes on each foot. Traditionally, this could have been achieved by cutting the paper, but this can be avoided with the more sophisticated strip graft.

We'll see how this can be done by imagining cutting the paper, adding a rectangle into the gap to account for the extra paper we need for the toes, then sticking the paper back together to make a square again. In fact, we'll actually see two ways this might be done, with one option much more preferable than the other.


A simplified crease pattern for the Two-humped Camel.

## Option 1

Imagine we cut perpendicularly into the paper at the part of the crease pattern where the ends of the legs are. However, we soon hit a snag: the cuts from the forelegs and hind legs meet in the centre of the paper The simplest step forward is to keep cutting all the way across the paper. This is shown below:


Left: Start cutting into the paper at the tip of each leg. Right: Keep cutting all the way across.

We then separate the paper until the gaps are the desired width. In order to make the toes the same length on each leg, the gap width must be the same at the forelegs and the hind legs. Then we fill in these gaps with paper. One option for this is the crease pattern on the right below:


Left: The regions of paper are separated. Right: The paper for the strip grafts is inserted in the gaps.

There's nothing incorrect about this graft, but a lot of paper has to be added for just one extra point at the end of each leg. Each strip has one end at the tip of one leg, but the other end of the strip terminates in the middle of a raw edge and is unused. We might realise that if the original design had happened to have the ends of the legs at exactly opposite points on the square, both ends of one strip graft would have been sufficient to cover two legs. Or equivalently, each leg would only need one end of a strip.

Thankfully, we can get this result by tweaking the process from this first attempt.

## Option 2

Let's begin again, by imagining cutting into the paper at the tip of each leg, but instead of cutting all the way across the square, we'll stop when the cuts meet. This leaves us with two rectangles at the left and right raw corners, and one large uncut region in the middle.

So where else can we cut? Firstly, let's observe that we want fewer strips this time, so that we don't need to insert as much new paper as with the first attempt. This means that, ideally, we would cut the paper into as few regions as possible.

Intuitively, since we will be folding along some of these cut lines later to bring the two sides of the graft back together we want to ensure that none of the shapes we are cutting are concave (because concave shapes contain an angle larger than $180^{\circ}$, which cannot lay flat without further folds - see Kawasaki's Theorem on page 81). This means we need to divide this large uncut section of paper into at least two regions.

These observations actually only give us one natural option; making one further horizontal cut is the only way to divide the remaining paper into just two regions with the resultant shapes being convex.


Left: Cut into the paper until the cuts meet. Right: One further cut is needed to insert paper for a strip graft.

Now that the paper is divided into convex regions, we pull the pieces apart. We have complete control over the distance here, but in order to save paper we want this gap to be as small as absolutely necessary to create the toes we want. Again, we should note that to make all toes the same length, the gap on the forelegs and hind legs should be the same. This is good, because unlike with the first attempt, if the gap widths were different we wouldn't actually be able to draw our new square around the pieces in such a way that it precisely touches the raw edges of the pieces from the original square.


The second stage: separate the (imaginary) sections of paper to the desired distance.

If we are being sticklers for anatomical correctness, we could measure the length of a Camel's toes and let this dictate the width of the gap exactly (using the fact that the gap is precisely twice the length of each toe). But for the sake of foldability, it is more sensible to choose a distance such that locations on the resultant paper can be found using simple folding sequences.

One option is to make the longer side of the rectangular pieces exactly half the width of the new square shape. Of course, if this doesn't create a final model that looks right, we could adjust this choice later, but it turns out that this will be suitable.

Having made a choice of the gap length, all that remains is making sure the model can be folded. The grafted region can be collapsed with just a few folds - our restriction to common angles helped to ensure this. One method for doing so is shown below. This makes a new, smaller square, from which the Two-humped Camel can be folded using the diagrams on page 194. But now there will now be extra paper at the end of the legs that can be used for the toes - a successful graft!


The final crease pattern for the Camel with Toes. Less than half as much paper has been added compared with option 1.

## Tessellations and Pattern Grafts

Traditional origami design is focused around making flaps of the right lengths and in the correct locations to represent a subject. This is in contrast to origami tessellations, which are about forming a pattern of folds across an area of paper. Tessellations are becoming an increasingly popular aspect of origami.

One of the simplest ways of designing a tessellation is to use a design where the raw edges form an exact square boundary, whose sides are symmetric, and where all sides are the same. This is achievable with regular pleats made perpendicular to the paper edge. Multiple copies of such a design can be placed side-byside and folded from one sheet. See the Star Tessellation on page 102 for an example of this.


Left: One unit of the star tessellation. Note that the raw edges of the paper form a square boundary which allows the tile to tessellate. Right: Two units folded together from one rectangular sheet.

While a perfectly valid area of origami in its own right, tessellations can also be used in representational origami to create some especially complex models. If a tessellation is folded on a sheet of paper and still leaves a square shape, then that square can be folded with a known design to make a model that will include the tessellation. This is called a pattern graft. This can also be done with any other shape which tiles the plane. Example uses include adding scales to a dragon's skin, bricks to an otherwise plain wall, or feathers to a wing.


Left: A rhombus tiles the plane. Right: A tessellation made with rhombuses. This tessellation can be used as a pattern graft for adding scales to a subject.

Pattern grafts differ from border and strip grafts in a few important ways that make them among the most advanced grafts. Firstly, multiple pleats have to be used, and typically these pleats are much smaller and more numerous than when pleats are used to add points. This means folds have to be precise and intricate.

Pattern grafts are also more difficult to implement effectively from a design perspective: pleats must be spaced across an area, and since these pleats propagate across the entire sheet of paper there may be other areas of the final model that end up with multiple unwanted folded lines running across them. It's also often harder to integrate the unused parts of a graft when there are multiple small pleats than with a larger border or strip graft. This means that unless the designer is careful, a design involving patterning can waste paper and therefore end up rather small compared to the initial width of the paper. The designer should also ensure that the patterned region doesn't have any other folds across it, because the patterning stacks multiple layers of paper on top of each other, which is hard to fold.

As with other grafts, there isn't anything stopping the designer considering pleats at any angle. This can quickly become the stuff of folding nightmares. In order to mitigate this added complexity, pleats are often restricted to being entirely vertical or horizontal, and all the same width. In fact, as long as pleats are parallel to the edges of the paper and are equinumerous in both directions, then the paper will end up being an exact square again after pleating, as desired. This means that after adding a pattern graft to a model the original sequence can be followed from the new square with minimal modification. This method is used with the Feathered Angel on page 146.

Although there are a couple of examples of patterns directly shown in this book, you may wish to try adding your own patterns to other models. In this book, possible subjects include adding a thatched or tiles texture to the Stable Roof, adding patterns to clothing on any of the figures, and patterning wool on the Sheep. If you try your own, remember that in addition to the challenge of designing the pattern and working out where it needs to be in the underlying model, patterns add extra thickness and will make folding the model more difficult.

## Other Grafts

The examples shown so far in this book all use grafts which make a new square shape, but this doesn't have to be the case. For example, each Shepherd (pages 111 and 115) is made by grafting an internal flap onto Joseph to create the beard, and the resultant shape after grafting is a concave polygon.

This broader view of grafting opens up the possibility of modifying designs originally made from rectangles by adding grafts to make a square shape (I'm greatly tempted to use the phrase purification for this process in line with purist origami).

As a final thought, two of the most well-disguised grafts are the common blintz base, obtained by folding all four raw corners into the centre of the paper to create a new square, and the windmill base. Traditionally, the blintz base has been used as a graft to add complexity to other traditional bases (see page 117).


Left: The traditional windmill base. Right: The border graft used for the Manger with Legs. Notice how similar these two bases are.


Left: Traditional blintz base. Right: A rotated variant of the blintz base. You may enjoy experimenting with a rotated graft like this to breathe new life into a traditional base.


First time: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$

To-scale: $\square$ $1.8 \times 10^{6} \mathrm{~km} / 6 \times 10^{9}$ Tip


There are many eight-pointed origami stars, some of which use the proportions of the one shown here, but the majority are modular designs formed from eight separate sheets of paper. Here is a similar effect but from just one sheet of paper, which results in a much more efficient use of paper.

The yellow regions in the crease pattern on the right show the areas visible on the final model, which you may wish to use to make a smooth version of the model.



1. Fold and unfold.

2. Turn the paper over.

3. Fold and unfold both diagonals.

4. Fold the raw edges to the creases made in step 1 and unfold.

## Tip

Make sure your folds in steps $7,8,9$, and 11 go through the corresponding crease intersections accurately, to get sharp points on the finished star.

5. Fold the raw corners to the centre and unfold.

7. Fold the raw edges to the diagonals, only marking firmly up to the first crease. Then unfold.

10. Fold and unfold between the references shown.

6. Turn the paper over.

8. Fold the raw edges to the creases made in step 5 , only marking firmly in the regions shown. Then unfold.

11. Fold and unfold the angle bisectors shown.

12. Fold and unfold the angle bisectors shown.

13. Turn the paper over.

15. Turn the paper over.

18. Fold and unfold the top point to the crease you just made.

19. Open-sink in and out.

17. Fold and unfold through the crease intersection shown.

20. Fold the sides over so that the fold-line is perpendicular to the raw edges. Repeat behind.


24. Fold and unfold through all layers.

25. Unsink four corners. You may find turning the paper over helpful here.

26. Fold the flaps upwards on each side.

27. Bring the raw edges to lie horizontally by collapsing using existing creases.

28. Squash-fold the raw corner.

29. Turn the paper over.

30. Swivel-fold the raw edges in. This will form a small gusset in the centre of each swivel fold.

31. Turn the paper back over. $\infty$

32. Fold the near flaps back down and tuck them into the pockets.

33. Repeat steps 26 to 32 on the other three raw corners.


By adding a border graft to the Star, one of the raw corners can be extended to form a tail/long point (see Grafting on page 86).

One way to achieve this is shown in the crease pattern on the next page - the crease pattern for the original star has been overlaid on a larger square.

You should begin by pre-creasing the folds shown, taking carefully note of the black creases. The creases in the top section are the same as in the standard Star.

Then fold the crease pattern to make the model in the first diagram. The easiest way to do this is to fold the Star on just the top square shown but with an additional collapse at the bottom corner. Then comes the trickiest part - thinning the tail with a series of fiddly sinks on each side, using the black lines in the crease pattern. Each of the two series of sinks should be performed in one manoeuvre.

As with the Star, the yellow regions on the crease pattern show the areas to avoid creasing, should you wish to make a smooth version of this model.


\|


First time: $\square$ $20 \mathrm{~cm} / 8$ " per unit


The crease pattern shown on the right is for one unit that can be repeated to tile the plane. Including such a high number of middle flaps introduces so many issues with the folding sequence that even tiling a few of these together is more difficult than anything else mentioned in this book. If you attempt this tessellation, I highly recommend that you first fold just one unit of this repeating pattern on a separate sheet of paper.

It is particularly important to pre-crease all required folds while the paper is flat before making any lasting folds. Mimic the sequence up to step 26 of the Star, which allows the paper to fold flat. After this point, the paper will not lie flat until the very end. Folding the units around the edges is the best place to begin the collapse, because this locks the final outer shape of the model, which helps force the inner sections into place. If you embark on folding even a few units together, be prepared to put aside many hours, and if you do manage to fold them successfully, congratulations for tackling the hardest model in this book - this is an impressive achievement!

The astute reader may have noticed that this model isn't actually a Nativity model, per se. But as mentioned on page 94 , theoretically speaking, patterns can always be added to an existing model, for example the Star Tessellation one could be used to make a resplendent cloak for one of the Wise Men. In practice, this is something far beyond anything I would recommend attempting, but you may enjoy the thought of it!



## 倁潞



First time: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$

To-scale: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$

Joseph is the simplest of the adult human figures and develops motifs used later in this book, so make sure you are confident folding this model before moving on to the harder figures.

Use thick paper here to hold the structure of the model. There are no downsides to this, because there are no places with lots of overlapping layers.


1. Fold the paper in half vertically and horizontally, then unfold.

2. Fold and unfold the top and bottom raw edges to the centre but only fold firmly where shown.


3. Fold the top raw edge down to crease you just made.

4. Fold the top edge to the central horizontal crease and unfold.

5. Fold the edge to the crease shown and unfold.

6. Fold the left and right raw edges to meet the creases intersections shown and unfold.

7. Fold the raw edge to the top.

8. Unfold step 5.

9. Fold the diagonal lines shown and unfold.


10. Unfold step 3 but not step 5.

11. Fold the diagonals and unfold.

12. Fold the left and right raw edges to meet the creases intersections shown, then unfold. Only mark firmly in the regions shown.

13. Fold and unfold the bottom raw edge to the crease intersection shown, only marking firmly in the region shown.

14. Fold diagonal lines through the intersection shown.

15. Turn the paper over.

16. Fold the raw edge to the crease intersection shown, then unfold. Only mark firmly in the central region.

17. Fold diagonal lines through the intersection shown.

18. Fold and unfold the left and right raw edges to meet the creases shown.

19. Turn the paper over.

20. Fold four pleats simultaneously to form a small corner in the middle of the paper.

21. Fold the small corner down.

22. Pinch mountain folds between the
intersections shown.

23. Perform a number of pleats
simultaneously.
24. Carefully open the top parts of the two pleats made in step 21.


25. Fold and unfold lines between the intersections shown.

26. Pleat on existing creases.

27. Curve the model by making the folds shown, but only make them softly. The paper won't lie flat from now on.

28. Rotate the model so that you are viewing the right-hand side.

29. Reverse-fold the raw edge. Repeat behind.

30. Fold the raw edge to meet the crease. Repeat behind.

31. Swivel-fold the raw edge to the left. You will need to form the mountain fold between the intersections shown. Repeat behind.

32. Fold the raw corner over slightly. There are no exact references.

33. Reverse-fold the edge so that it lies vertically. Repeat behind.

34. Mountain-fold the region behind. Repeat behind.

35. Unfold the pleats so that you can see the entire raw edge. Repeat behind.

36. Squash-fold the right-hand vertex, so that you can fold the raw edge up slightly.

37. Fold the edge over.

38. Refold the paper that was unfolded in step 37, while keeping steps 38 and 39 in place.

39. Make a small spread-sink to sharpen the corner. Repeat behind.

40. Rotate the paper so that you can see the back of the model.

41. Curl the two sides so that the raw corners overlap.

42. Mountain-fold the raw corners underneath to lock the layers together.

43. Fold the belt down

44. Mountain-fold the edge under so that the model balances freely.

45. Shape the arms.

46. Turn the paper over.

47. Closed-sink the point to make a chin (or just mountain-fold it underneath).



Joseph can be modified in many ways to add more features, and one of the more obvious adaptations is to add a beard with an internal graft.

A little work is required to do this in such a way that we don't have pleats running across the model in places we don't want them. This consideration makes it one of the most complicated of the grafts in this book. The instructions on the following pages show one way to form the base whose crease pattern is on the right.

One of the payoffs of this graft is that some of the extra paper can be used to give more freedom to the arms: Joseph's arms have little range of motion, but the shepherd's arms are more detached from the torso, meaning they can be shaped more freely.

After following the step-by-step diagrams, you will need to shape the beard yourself. Alternatively, the crease pattern
 on page 115 shows one way of forming a beard.

First time: $\square$ $25 \mathrm{~cm} / 10^{\prime \prime}$ $\Delta$ To-scale: $\square$ $25 \mathrm{~cm} / 10^{\prime \prime}$

1.

4.

7.

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47.

44.

45.

Joseph: 23-49

48. Continue with step 23 of Joseph, adjusting any steps which aren't applicable, as appropriate.

The crease pattern on the left below is for the base of Shepherd A. Following the diagrams for this base requires you to shape the beard as you see fit.

Alternatively, instead of starting with the diagrams, you may wish to use the crease pattern on the right below to form a base, which shows slightly more detail around the beard. Once you have collapsed the paper into the base, proceed with steps 23 to 49 of Joseph. Some of the creases needed for these steps are also on the crease pattern below (these were made within the diagrams for Shepherd A so are not included on the crease pattern on the left).


For simplicity, the beard graft for Shepherd A has been kept to the same grid as was used for Joseph. It requires a $5 \mathrm{~cm} / 2^{\prime \prime}$ increase in paper width when made in proportion to the to-scale Joseph ( $20 \mathrm{~cm} / 8^{\prime \prime}$ ). You can make a longer beard by adding $10 \mathrm{~cm} / 4^{\prime \prime}, 15 \mathrm{~cm} / 6^{\prime \prime}$ or even $20 \mathrm{~cm} / 8^{\prime \prime}$ for a full floor-length beard.

The crease patterns below are for an extra $10 \mathrm{~cm} / 4^{\prime \prime}$. As above, the right-hand crease pattern shows one possibility for more detail with the beard; see if you can extend the same pattern to fold a group of Shepherds with a variety of beard lengths. The grid divisions on page 158 are useful for this in general.

First time: $\square$ $30 \mathrm{~cm} / 12 "$


To-scale: $\square$ $30 \mathrm{~cm} / 12{ }^{\prime \prime}$


## Circle Packing

## What is Circle Packing?

Circle packing is an origami design method where a subject is approximated by a stick figure, and flaps of a model's base are represented by circles. This section outlines how a designer can create a new origami model by arranging (or "packing") these circles suitably within a square of paper.

The circle packing method is the most general of all methods for arranging flaps. Some other methods are presented later in this book, and each of these methods is called a design geometry.

## Trees

A tree is a particular type of stick figure diagram - specifically one that contains no loops. Often with origami design, it's enough to approximate an intended subject with a tree and then aim to fold that tree. This is most applicable when the subject has many long, thin appendages, because the important properties of the figure are captured nicely by the tree diagram.


Two stick figure diagrams. The example on the left is a tree, but the example on the right is not, because it contains a loop.

## Tree Anatomy

A tree is composed of two types of object: points which are called nodes, and lines called edges. Each edge connects two distinct nodes.

Nodes which only connect to one other node are called leaf nodes, and edges which connect a leaf node to another node are called leaf edges. Any other nodes are branch nodes, and any other edges are branch edges.

For the sake of simplicity, it's customary to remove any branch nodes that only connect to two nodes, and combine the adjacent edges. This makes for a tidier but equivalent tree, for the purposes of origami design.

| Tree | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| Leaf nodes | 1 | 3 | 4 |
| Leaf edges | 0 | 1 | 2 |
| Branch nodes | 0 | 0 | 1 |



Left: A tree with redundant branch nodes. Right: A tidied version of the same tree

## Trees of Traditional Bases

At the bottom of this page is a selection of some traditional bases with their crease patterns. Some familiarity with these bases is helpful; feel free to fold these bases if desired, but this isn't required.

The bases on the left have flaps which hinge on vertical creases, and the bases on the right have flaps which hinge on horizontal creases. The flap length is the shortest distance from the flap's tip to its hinge crease.


The length of a flap (the distance from the hinge crease to the flap tip) is the same length as that on the stick figure.

When a base is represented by a tree, the intention is to preserve only the tree's structure and the flap length of each flap. This sort of tree can be thought of as an idealised shadow of the base cast from light shining in the direction parallel to the hinge creases - see the base and tree for the waterbomb base below.

Leaf nodes in the tree correspond to flap tips on the base, and branch nodes in the tree correspond to the hinge creases on the base.


Some traditional bases with their crease patterns and trees: the waterbomb base (top left), bird base (top right), fish base (middle left), frog base (middle right), blintzed fish base (bottom left), and blintzed frog base (bottom right).

## Circles

A great deal can be learned by comparing the location of points in the tree to the location of the points on the crease pattern.

First, consider any flap of a base which is represented by a leaf edge in its tree. Then imagine a circle centred at the point on the square which becomes the tip of the flap in the folded base. The radius of the circle is the flap length. The key observation is this: the paper which falls within this circle is always on the flap. Any paper outside this circle does not necessarily lie on the flap. We say that a circle is the minimal flap shape for circle packing.

More generally, the minimal flap shape for a design geometry is the shape of paper which must always be used for a (leaf edge) flap, when abiding by the limitations of that particular design geometry. The minimal flap shape for a design geometry depends on the possible angles of flap hinge creases: when all angles are allowed, the minimal flap shape is a circle. Note that flaps on a model don't necessarily attain this shape, the minimal flap shape just shows the smallest amount of paper required.


The rightmost point in the fish base comes from the right raw corner on the crease pattern. This is a leaf edge so can be represented by a circle. All paper within the circle shown lies within the flap in the base.

The corresponding points for branch nodes are a little more complicated than for leaf nodes. Consider one branch node and then every leaf node which is connected to that branch node using a leaf edge. The branch node corresponds to the points on the circumferences of all of these circles.

In the tree, leaf edges connect leaf node to branch nodes. Accordingly, leaf edges correspond to the area between the centre of a circle and its circumference, i.e. the paper within the circle.


Left: The tree for the fish base with the branch node highlighted. The four leaf nodes are all connected to this branch node.
Right: Circle packing for the fish base. Each leaf node has a circle, and the branch node in the centre of the tree is represented by the circumferences of all four circles.


The fish base, but only showing the parts of the circles overlapping the square. This is the standard way of presenting a circle-packed crease pattern. Note that only one quarter of each circle is actually used in the final model.

## Rivers

Once circles for all leaf edges are added to the crease pattern, it's much easier to see the correspondence between the location of the paper in the crease pattern and in the base/tree. However, this still leaves the branch edges unaccounted for.

Within a crease pattern, the paper corresponding to a branch edge is a curved section of paper, a more complicated shape than a simple circle. This curved section is called a river, and has a constant width, equal to the length of the region on the tree.

Unlike with circles, the boundary should not necessarily be taken as a border showing exactly where paper is going to fall within the base. Rather, the important property of a river is that it divides the paper into two distinct regions. These two regions correspond to the two disconnected sections of the tree that would remain if the branch edge were removed.

A branch edge always connects two branch nodes. Each "bank" of the river corresponds to one of the two branch nodes.

Of the traditional bases on page 117, the only one that includes rivers is the blintzed fish base.


Left: The left-most branch edge (green) on the tree of the blintzed fish base corresponds to a river.
Centre: The most common representation of the river.
Right: The actual, angular path in the crease pattern.

## Circle Packing in the Traditional Bases

Below are the same bases from page 117, but with the circle packing shown for each of them.


Traditional bases with their circle-packed crease patterns and trees: the waterbomb base (top left), bird base (top right), fish base (middle left), frog base (middle right), blintzed fish base (bottom left), and blintzed frog base (bottom right).

## Design Methodology

Now with an understanding of trees, circles, and rivers, we have enough groundwork to consider a general methodology for designing an origami model:

1) Start with a subject.
2) Approximate the subject as a tree.
3) Find a base which matches the tree. This can be done by:
a) Making circles for all leaf edges;
b) Making rivers for all branch edges;
c) Arranging these circles and rivers so that the perimeters touch, as dictated by the branch nodes.
4) Fold the base and shape it into the intended subject.

The bulk of the process happens in the third step, but it is not difficult to create some configuration that technically satisfies the requirements. In fact, there is free software available on the internet that does precisely this step.

However, the real skill of the designer is to do so in a way that accommodates the fourth step. There are many things that need to be considered here, such as:

- The amount of paper needed to make the flap width
- Ease of folding the base
- Convenient reference points
- A consistent aesthetic across the model
- Being able to utilise the raw edge of the paper for a colour change
- Relative number of layers of paper to create thicknesses appropriate for different parts of the subject

To my knowledge, there is currently no software or algorithm which comes close to accounting for all of the above points.

However, the most objective consideration lends itself very naturally to computation because it is purely numerical: efficiency.

## Efficiency

The extent to which a particular model uses all the available paper is called efficiency. Efficient models end up large relative to the starting sheet of paper, whereas inefficient models have more wasted paper, so they tend to end up bulkier, smaller relative to the starting paper, and harder to fold.

Efficiency is measured relative to the best configuration of circles and rivers. The optimal configuration is the one which minimises the width of the square that can contain these features. Equivalently, the optimal configuration is the one which maximises the flap lengths relative to the square's width. An example showing this equivalence is shown below.

For simple trees, the optimal configuration can found exactly (and proven to be optimal). For more complex trees, the best that can be done is to calculate an upper bound for efficiency by comparing to the best configuration found so far.


Left: The fish base, which is the optimal arrangement of circles for its tree.
Centre: A valid, but rather silly arrangement of the same circles. The width of the fish base square is $56 \%$ of the width of the large square, so the central arrangement is only $56 \%$ efficient.
Right: The same silly arrangement, but the square is scaled down to be the same width as the square for the fish base. The flaps length are only $56 \%$ of the flap lengths for the fish base, meaning the arrangement is $56 \%$ efficient.

Below are some key factors to consider when trying to make efficient models.

## Minimising waste

Circles and rivers don't actually need to touch, even when they come from adjacent edges in the tree: the only requirement is that they don't overlap. However, any paper that separates circles and/or rivers will have to be folded away and wasted. Wherever feasible, this waste should be avoided, hence why it is often assumed that adjacent edges in the tree correspond to touching features in the crease pattern.

After drawing all circles and rivers, there will still be paper left between these features. This paper can sometimes be utilised for making wide flaps, but from the perspective of making the tree, it is wasted. Circles which are tightly packed together have less wasted paper than when there are large gaps. Interspersing small circles between large ones should be the default way of preventing waste in this area.

Another possible source of waste comes from having windier or longer rivers than necessary. You should be aware of this generally, because windy rivers are often very hard to fold. This means that, from a folding perspective, some of the best rivers will simply be rectangles.

## Truncated circles

Parts of a circle or river that don't overlap the square aren't actually part of the model, and are therefore usually ignored in diagrams. Only $1 / 2$ of the area of a circle on a raw edge is needed, and $1 / 4$ of a circle on a raw corner. Therefore, large circles that would take up a large area of the square when placed in the middle of the paper are used more efficiently when they are centred on a raw edge, or better yet, a raw corner. As a rule of thumb, place the largest flaps at raw corners when designing a model using circle packing.

## Other design geometries

Circle packing is the most general of all design geometries which means, among other things, that the crease patterns for all other design geometries are a subset of those that can be made with circle packing.

We have seen that the minimal flap shape for circle packing is a circle. Other design geometries have different minimal flap shapes, which arise as a consequence of the particular angles allowed in the geometry. Given two design geometries, $A$ and $B$, if $A$ is a subset of $B$, then the minimal flap shape of $A$ is at least as large as the minimal flap shape of $B$. This will mean that flaps pack together at least as well in $B$, so $B$ is at least as efficient as A. Even though the minimal flap shape for a design geometry can be curved, all actual instances of flaps in a design will be polygonal, and so the general term for packing flaps in other design geometries is polygon packing.

Since circle packing is the highest level of all design geometries, all minimal flap shapes that we encounter later will be at least as large as this circle.


A hierarchy of some design geometries. The minimal flap shape is shown next to each one - see the section for each individual design geometry for an explanation of the minimal flap shape.

# Kneeling Mary <br>  

First time: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$

To-scale: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$



1. Fold and unfold.

2. Fold and unfold three raw edges to the centre creases.

3. Fold and unfold a line between the intersections shown, only folding firmly where shown.

4. Fold and unfold a line between the intersections shown, only folding firmly where shown.

5. Fold and unfold the raw edges to meet the intersections shown.

6. Fold and unfold the raw edges to the creases made in steps 5 and 6.

7. Fold and unfold the raw edges to the creases you made in step 8.

8. Fold and unfold diagonal lines.

9. Fold lines between the intersections shown and unfold.

## Tip

Some people prefer to precrease lines by turning the paper over and pinching mountain folds, rather than making valley folds. You may wish to try this for steps 13 and 14.

13. Fold lines between the intersections shown and unfold.

14. Fold lines between the intersections shown and unfold.

17. Fold and unfold diagonal lines in the regions shown.

18. Fold and unfold small diagonal lines in the region shown.

19. Turn the paper over.

20. Fold and unfold between the intersections shown.

21. Fold between the intersections shown, then unfold.

24. Fold and unfold diagonal lines between the intersections shown.

22. Fold between the intersections shown, then unfold.
25. Fold the two diagonal mountain
creases together as shown and unfold.


23. Fold between the intersections shown, then unfold.

27. Collapse on existing creases.

28. Fold the small corner down.

29. Open up the top part of the paper.

30. Collapse using existing creases to form a vertex at the top of the model...

31. ...continue collapsing using the creases shown and flatten the paper.

32. Open the right hand edge and pull it down. The paper won't lie flat.

33. Reverse-fold the right raw edge in while pushing up a small mountain fold near the top of the paper.

35. Repeat steps 32 to 34 on the left.

36. Push the edges in as shown to form a vertex which folds upwards on existing creases...

39. Rotate the model about a vertical axis so that you are looking at the right side of the model.

34. Reverse-fold the flap back to the right.

37. ...collapse the bottom raw edge using existing creases and flatten the paper.

40. Pull the raw edge out and flatten the corner at the top. The paper won't lie flat until step 50 .

41. Crimp the paper using the folds shown. Repeat behind.

42. Reverse-fold the raw edge. Repeat behind.

43. Fold the raw corner up. Repeat behind.

44. Fold the raw edge over. Repeat behind.

47. Fold a corner up using mostly existing creases. Repeat behind.

48. Reverse-fold. Repeat behind.

49. Collapse the arm to the side so that it is horizontal and lies flat. Repeat behind.

50. Turn the paper over.

53. Fold the edge back down.

56. Pleat the paper.

59. Fold the model to bring the top section perpendicular to the lower section.

60. Sink the corner of the chin (or mountain-fold it under).



First time: $\square$ $25 \mathrm{~cm} / 10 "$

To-scale: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$


The Kneeling Mary is (obviously) in a kneeling position, which may be desired, but it isn't possible to pose the model differently. In order to have enough paper for the model to be in a standing position, the design has to be altered. This can be easily addressed with a border graft. The crease pattern on the right shows the position of the original square for the standard Mary.

As usual, we can reap other benefits from a graft with just a little thought. The extra paper at the top is used to make a slightly longer hood and the paper at the sides allows for longer arms, which makes posing easier.

The Standing Mary uses box pleating with a $16 \times 16$ grid, compared to the Kneeling Mary, which uses a $12 \times 12$ grid. As with the Kneeling Mary, note how many of the creases are based on the grid but are not oriented at a multiple of $45^{\circ}$ to the grid.


## Box Pleating

## Pure box pleating

Consider the crease patterns below: the one on the left is for the base for the Coronet, and the one on the right is for the Sheep. You can find one of the most striking differences between the two by looking at the angles of the creases. All of the creases for the Coronet are at some multiple of a $45^{\circ}$ angle to the other creases. Compare this to the Sheep's crease pattern, where some of the creases are vertical and some are at a $45^{\circ}$ angle, but most are not.

In general, we say that a model is purely box pleated if its crease pattern is like the Coronet's, where all creases are at a multiple of $45^{\circ}$ to the others.

Usually, although not always, the edges of the square will align with creases in the purely-box-pleated crease pattern, so the vertical and horizontal creases are parallel or perpendicular to the raw edges of the paper.


Crease patterns for the Coronet and the Sheep.

## Advantages of pure box pleating

What can't be seen from the crease pattern alone is the time it takes to fold each model. When measuring by the number of steps in the folding sequence, it takes around $50 \%$ longer to fold the base of the Sheep than the Coronet. This isn't due a difference in the number of creases - both have approximately the same number of creases - but because many of the steps for the Coronet involve making multiple similar folds at the same time. For example, step 3 of the Coronet has four separate horizontal fold-lines, but their regularity means the diagram is clear and the folds can be made quickly. This is in contrast to the Sheep, where many unique folds have to be made one at time and the folder has to concentrate a little more to make sure the folds are in the correct position.

Between the Sheep and the Coronet, the model with the box pleating restriction is the simpler model to fold. But is this true more generally? Looking at the other models in this book, we find that the only models besides the Coronet and Antique Crown which are purely box pleated are the Shepherd's Crook and the units for the Stable, and indeed these are among the simplest models in the book.

However, we run into a bit of an issue when looking for other purely-box-pleated models, although some models come very close. If we look at the crease pattern for the Kneeling Wise Man, for example, the model is entirely purely box pleated aside from a small region near the centre.


The crease pattern of the Kneeling Wise Man - notice how only a very small number of the creases prevent the model being classified as purely box pleated, but their endpoints all lie on a squarebased grid. (If you're wondering why we should include these creases in the crease pattern, recall from the comment on page 76 that the definition of a base is partly subjective.)

## Box pleating

Regarding the rogue creases in the Kneeling Wise Man's crease pattern, if we overlay an evenly spaced square grid on the crease pattern we see that all creases, including the offending diagonal ones, have endpoints that lie on this grid. This isn't a given - traditional origami models preferred to use convenient angles as a basis for their creases rather than grids (see $22.5^{\circ}$ Geometry on page 180), so when the endpoints of creases do all terminate on a grid, it suggests that this grid was the deliberate underlying focus.

We need to allow for this grid-based approach when talking about designs such as the Kneeling Wise Man, and so we define a design geometry distinct from pure box pleating. We say a model is box pleated if its crease pattern can be overlaid with a square grid such that all of the creases for the base are either some multiple of a $45^{\circ}$ angle to the grid, or have both endpoints on the grid. For clarity, this will sometimes be referred to as generalised box pleating. Typically, all of the creases on box-pleated models do have endpoints on a grid.

If the sides of the square also lie on this grid, then the grid can be made by dividing the width and length of the paper by an integer. We'll call the smallest such integer the grid number.

With this more general definition of box pleating, we find that in addition to the models identified so far, the following models all have this property (with their grid number in brackets): the Manger with Legs (6), the Stable Barn (various, from 8 to 40), Joseph (32), the Shepherds (40 and 48), Kneeling Mary (12), and Standing Mary (16). All of these models are symmetric, which is why all of the grid numbers are even.

But we aren't done yet, because in our search we also notice some models have sections that look like they are on an underlying grid, even if the whole model isn't. For examples, see the three crease patterns below.


Crease patterns for the Gold (left), Angel (centre), and Lamb (right). Each of these patterns has been overlaid with a full or partial grid. Each grid aligns with creases in the crease pattern.

## A region-based definition

Let's go back to the Kneeling Wise Man, which currently falls under the definition of being box pleated. After performing a border graft to make the Standing Wise Man, we no longer have a box-pleated model. But the entire top half of the design for the Standing Wise Man is nearly identical to the Kneeling Wise Man's crease pattern. What can we say about crease patterns which are partly box pleated?

We can think of local regions of the paper as being box pleated, rather than being rigid about the whole crease pattern having to abide by a global rule. So we'll say that a region of a model is box pleated if the crease pattern for that local region is box pleated, i.e. if there is an underlying grid such that all creases in that region are at some multiple of $45^{\circ}$ to the grid or have both endpoints on the grid. We can make an analogous local definition about being purely box pleated too, by simply removing the part about crease endpoints being on the grid.


The top region of the Standing Wise Man is box pleated.

## Advantages of box pleating

Having established what box pleating is, we can now come back to the idea of simplicity in origami. Simplicity can be categorised fairly well into three parts, all of which are generally simpler when a model is box pleated compared to a non-box-pleated counterpart.

## Folding Simplicity

It's rather difficult to measure folding difficulty objectively, but if the ease of making box-pleated folds isn't already apparent to you, I hope the following is compelling.

When folding simple models there are only a limited possible number of exact folds that can be made. We call these referenced folds, and much academic work has gone into defining the axioms that can be used to specify a referenced fold. Normally, all structural creases are defined using referenced folds.

Referenced folds are in opposition to arbitrary folds, where the folder has to choose (or guess) where their fold goes (which is notoriously irksome for many beginner folders).

We're going to begin to list the possible referenced folds, starting by considering just one fold. With just a bit of experimentation you will find that there are only four referenced folds that can be made from an initial square. This can be proven rigorously with an axiomatic approach, but this won't be done here.


There are four referenced folds that can be made when starting to fold from an uncreased square sheet of paper.

In order to make our list as succinct as possible, we only want to count fundamentally different folds. This means that we will group folds which are just rotations or reflections of each other together, and call each grouped object a state. The four folds above can be grouped into two states.


There are two states that can be made with just one referenced fold. The state consisting of the two orthogonal folds is represented here by the vertical fold, and the state consisting of the diagonal folds is represented by the downward diagonal fold (the choice of representative is arbitrary).

The states that can be made with a maximum of two folds can be enumerated fairly simply too, and are shown on the next page. The top row contains the states that are still purely box pleated, and the bottom row contains the rest, none of which are purely box pleated. In fact, half of these 14 total states using only one or two referenced folds are not purely box pleated.


There are 12 states that can be made with exactly two referenced folds. The 5 states in the upper row are all purely box pleated, while the 7 in the bottom row are not purely box pleated.

Since virtually every origami model begins by folding at least two referenced folds, we might be interested to look at the frequency with which these states occur across all origami designs.

A full-blown study of this is well beyond the scope of this book, but what we can say with confidence is that all of the purely-box-pleated states are commonly used in traditional models. However, most of the others are not seen in any traditional models, with only the bottom right state being at all frequent. Considering not just traditional models, but all origami designs I've personally encountered, I don't think I've ever come across the $5^{\text {th }}$ of the non purely-box-pleated states in any instructions, and the $6^{\text {th }}$ is nearly as rare.

The number of states that can be made from more than two referenced folds increases at a rapid rate, so I won't attempt to enumerate those here. Nevertheless, it's worth observing that the proportion of boxpleated states diminishes as the number of folds increases, yet these rare box-pleated states are still the most commonly encountered.

All this is to demonstrate that designs have a definite preference towards box-pleated states. This is no doubt in part due to a perception of elegance, but I would suggest it is as much, if not more, due solely to the simplicity of folding them.

I would hasten to add that most traditional models that fall under the definition of box pleating were not necessarily designed to be box pleated, or deliberately optimised to be easy to fold. But through a sort of "survival of the fittest" mechanism, the simple models are the ones that have stood the test of time and been passed down to us. And these models have strong correlation with those models that display at least partial box pleating.

## Design Simplicity

Box pleating allows a designer to design sections of a model independently from other parts. Any section of a crease pattern consisting of parallel, uniformly spaced creases can be shifted across the grid. So entire sections can be moved along or perpendicular to the direction of the creases, provided the shift is in a discrete multiple of the grid number. The larger the grid number, the more granularity and control the designer has to move sections around.

All this means that if a crease pattern comprises features whose regions are all box pleated, then the location and positions of individual features can be fine-tuned. This benefit of box pleating is in contrast to general circle-packed origami design (see Circle Packing on page 116), because with circle packing, changing one area of the design can have a significant knock-on effect on the rest of the paper.

Designing using box pleating is still by no means a trivial task, and it is common in complex models. In fact, the prevalence of complex box-pleated designs, points to the power of method. Designing using box pleating is so effective that it has become something of a default in the mainstream. It's sometimes assumed that a complex model relies on a grid, and I for one find it refreshing to see a complex model that doesn't conform to this mould - see $22.5^{\circ}$ Geometry on page 180 for one such alternative.

## Design Efficiency: Square Packing

Pure box pleating and generalised box pleating are design geometries (see page 116), so both of them have an associated minimal flap shape. For pure box pleating, the minimal flap shape is a square, because the four sides of a square are at the only possible angles for flap hinge creases. For each flap, each square's side length is twice the flap length and the square is oriented in line with the underlying grid. This square fully contains the circle that would have arisen from circle packing, so each square uses over $25 \%$ more paper than the circle.


Left: Stick figure for a flap with unit flap length. Centre: The circle for the flap with circle packing. This circle has area $\pi \approx 3.14$. Right: Square packing for the flap, with area 4 , which is more than $25 \%$ bigger than the area of the circle.

Still, it would be inaccurate to say that pure box pleating is over $25 \%$ less efficient than circle packing, as a general rule. When the flaps are all of the same length, the square regions pack together very tightly and most, if not all, of the inefficiency is mitigated. However, when the squares are of quite different widths, the extra paper consumed by the largest squares becomes more noticeable. In this case, circles are the clear winner in efficiency. Circles can pack together diagonally at any angle, and small circles can fill much of the otherwise wasted area in the gaps between large circles.


Left: Waterbomb base and tree. Centre: The circled-packed crease pattern for the waterbomb base. Right: Because the underlying design geometry is actually pure box pleating, the flaps of the waterbomb base can all be represented by non-overlapping squares.

In short, pure box pleating is most suited to regions when there are a lot of small, similar-length flaps, and circle packing is best used on large flaps when there are smaller flaps elsewhere in the design. Using diagonal lines in generalised box pleating is a good way of helping with the efficiency shortcomings of pure box pleating, because the minimal flap shape is reduced from a square to a circle (see Beyond this Book on page 204). But the difficulty of learning to do this effectively counteracts much of the design benefits that box pleating provides to begin with.

In short, the simplicity of box pleating should be used where the convenience isn't compromising the model, but not as a crutch to be plastered across every design without due consideration.


The traditional fish base has flaps of two different lengths. Even such a simple base has no purely box-pleated equivalent: pure box pleating cannot match the circle-packed structure for efficiency.

## Aesthetic simplicity

The final simplicity of box-pleated designs is the appearance. The origin of the term "box pleating" was due to how well it lends itself to making box shapes, which can be both a strength and a weakness.

If a design only employs certain angles, only those angles will appear in the final form. This can be an asset, for example with the wooden supports in the Stable units, where the repeating parallel lines add to the architectural feel. At other times, this can cause difficulty, for example when trying to create an organic form. Despite their best efforts, the folder cannot always disguise the parallel lines and right angles inherent to box-pleating, leaving the model with an unwanted boxy appearance.

As such, the same message from above is echoed here: when designing a model, make sure to consider whether the ease of designing with box pleating will sacrifice too much of the look of the model. After all, a pleasing final appearance is surely the ultimate end goal of the vast majority of designs.


For inorganic and/or three-dimensional structures, such as the Stable Barn (left), box pleating can be the ideal origami design method. However, box pleating isn't suited to a subject like an eightpointed star (such as the Star, right), because of the $22.5^{\circ}$ angles in the centre.


First time: $\square$ $40 \mathrm{~cm} / 16{ }^{\prime \prime}$

To-scale: $\square$ $30 \mathrm{~cm} / 12 "$


The folding sequence for the head of the Angel is one of the most intricate sequences among the diagrammed models in this book, which is why such large paper is recommended for the first attempt.

Using the sequence shown here, there are a few creases that end up on each wing - after you have folded the model once, you may wish to fold a smoother version. Alternatively, a more elaborate version with feathered wings is given as the extension on page 146.


1. Fold and unfold the vertical and horizontal centre lines.

2. Fold and unfold the diagonals.

3. Fold and unfold, only folding firmly in the region shown.

4. Fold angle bisectors and unfold. You only need to mark firmly in the top half of the paper.

5. Fold the raw edges over so that the fold-lines go through the intersections shown.

6. Fold and unfold through both layers, only marking firmly up to the raw edge as shown.


7. Fold and unfold through the intersections shown in both layers.

8. Turn the paper over.

9. Fold and unfold.

10. Swivel-fold the top raw corner down to meet the raw corner at the bottom in order to form the long diagonal valley fold.

11. Swivel-fold the paper to form a new vertical valley fold.

12. Squash-fold the flap symmetrically.

13. Petal-fold the raw edge upwards to make a corner.

14. Unfold step 16.

15. Repeat step 15 on the left.

16. Fold the flap over to form a new vertical valley fold.

17. Fold and unfold angle bisectors.

18. Fold the flap down.

19. Fold and unfold.

20. Open-sink.
21. Pull the two raw corners out to the sides to open up the front section of paper. The paper won't lie flat until step 30.

22. Push the raw edge upwards and underneath the pleats, then flatten the paper.

23. Open-sink.

24. Turn the paper over.

25. Fold the upper layer down to the first crease. The paper won't lie flat.

26. Spread-sink the corners to flatten the paper.

27. Fold the narrow flap up.

28. Closed-sink the narrow flap.

29. Fold the top section over.

30. Reverse-fold the top corners inside.

31. Pull out the paper as far as it will go.

32. Mountain-fold the corners over - the fold-line aligns with the inner edge.

33. Wrap the edge around to form a hood.

34. Unsink the inner corner. This is tricky - you may wish to unfold the paper significantly to achieve this.

35. Unsink the second corner. Again, you may wish to unfold the paper significantly to achieve this.

36. Fold both flaps upwards.

37. Fold the point down.

38. Open-sink the near flap in.

39. Open-sink the sides of the head. Note that the mountain folds are not vertical.

40. Reverse-fold in and out, but only fold firmly where there are existing creases.

41. Reverse-fold two points up diagonally.

42. Fold the edges upwards.

43. Fold the edges to the centre.

44. Closed-sink the chin (or you can mountain-fold this behind if you prefer).

45. Collapse using existing creases.

46. Reverse-fold the corners.

47. Fold the corner down.

48. Turn the paper over.

49. Fold the wings out using fold-lines that go through the intersections shown.

50. Fold angle bisectors and unfold.

51. Fold the edges over using a valley fold between the intersections shown (this isn't quite a vertical line). Other new creases also have to be made to flatten the paper.



60. Turn the paper over.

63. Separate the layers of paper so that the inner layers go to the back (you may have already done this).

65. Swivel-fold the edges.

64. Turn the paper over.

66. Curl the two raw corners together so they overlap.

67. Tuck the raw corners between the layers.
68. Mountain-fold the corner underneath, so that the model can stand freely.



First time: $\square$ $40 \mathrm{~cm} / 16^{\prime \prime}$

To-scale: $40 \mathrm{~cm} / 16^{\prime \prime}$


The best results with patterning are when the area being enhanced has no folds across it and is relatively large compared to the model. This makes the Angel's wings an ideal example for the technique.

The difficulties with pattern grafting mentioned on page 93 can be partially overcome because of the design of the Angel. A vertical section of paper on each of the lower left and right sides is hidden in the final model, which is the perfect "waste site" for unused parts of the vertical pleats. Meanwhile, each horizontal pleat covers both wings at once, since the Angel is symmetric, meaning these pleats have little waste. As a small bonus, a pleat can be used as the edge of the sleeve, and also to obtain another corner on the hand that can be made into a thumb, which is consistent with the style of the other humanoid figures in the Nativity.

An underlying grid is the simplest way of finding reference locations for the pleats. Dividing the paper into a power of two makes this as easy as possible; a $64 \times 64$ grid is the smallest such grid that provides a reasonable pleat width relative to the Angel. Using 8 of these pleats vertically and horizontally reduces the paper to a $48 \times 48$ grid after pleating (each pleat goes back and forth, so there are 16 intervals in each direction). Therefore, starting with a $40 \mathrm{~cm} / 16^{\prime \prime}$ square reduces to $30 \mathrm{~cm} / 12^{\prime \prime}$ after pleating - the same width needed for the original to-scale Angel. It's also convenient to align some of the other creases to this new grid to make pre-creasing easier.

Begin by pre-creasing the folds below, then make the pleats in the order shown. Note that the lower horizontal pleats are not evenly spaced (as one of the folds on the wing can be used as a pseudo-9th pleat), and four are at an angle, so the paper won't lie flat.


Fold the Angel from the new (bounding) square. Subsequent creases can be aligned to the underlying grid see page 76 for a full crease pattern. You will need to apply some creativity to make some parts of the model, notably the feathers on the wings. Try to make the model your own way rather than trying to exactly replicate the image of the final model shown here.


First time: $\square$ $25 \mathrm{~cm} / 10 "$

To-scale: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$


An origami model is closed back if the paper on the back is connected and the open part of the paper is facing downwards. This is normally preferred, especially with animals on all fours (and is the case with all the animals in this book).

However, it can be rather inefficient to make a strictly closed back vertical figure, because there is only a very small proportion of the model's surface which is facing downwards. Choosing which parts of a design are open or closed is up to the designer and should be considered in conjunction with the intended viewing angle. Unlike the other humanoid figures in this book, this model must be leaning forwards to be closed back, making it well-suited for the kneeling position here.

Use paper that holds its shape well, for example foil-backed paper, or try wet folding here, to make sure the folds from step
 64 onwards remain in place.


1. Fold the diagonals and unfold.

2. Fold the top raw corners together, marking firmly only where shown, then unfold.

3. Fold the raw edges to the creases you just made and unfold.

4. Fold the left raw corner to the crease you just made, marking firmly only at the raw edge and unfold.

5. Fold the raw corners to the intersection you just made and unfold.

6. Fold the points shown together and unfold.

7. Fold the bottom raw corner to meet the intersection shown and unfold.

8. Fold halfway between the creases shown and unfold.

9. Fold between the creases shown and unfold.

10. Fold a horizontal line that goes through the intersection shown.

11. Fold halfway between the creases shown and unfold.

12. Fold between the creases shown and unfold.

13. Fold the raw corners to the intersection shown and unfold.

14. Fold the intersections shown together and unfold.

15. Turn the paper over.

16. Fold the raw corners to the intersections shown and unfold.

17. Fold diagonal lines through the intersections shown and unfold.
18. Fold the raw corners to the intersections shown and unfold.


19. Fold the raw corners to the intersections shown.

20. Fold the intersections shown together and unfold.

21. Fold between the creases shown and unfold.

22. Fold between the creases shown and unfold.

23. Fold the edges over using a crease which only exists on the far layer.

24. Fold the edges back over.

25. Fold and unfold between the intersections shown.

26. Repeat step 25 on the left.

27. Turn the paper over.
28. Fold between the crease intersections shown and unfold.

29. Fold and unfold between the intersections shown.

30. Collapse the paper by folding five pleats simultaneously as shown.

31. Reverse-fold two hidden corners.

32. Fold the flap up.

33. Closed-sink the sides in.

34. Repeat step 29 on the left.

35. Pleat the paper through all layers.

36. Turn the paper over from top to bottom.

37. Fold the flaps up, releasing any locked paper.

38. Reverse the parallel edges in. You will need to spread out the paper using creases made in step 28 to achieve this, as shown for the right hand side.

39. Closed-sink the edges.

40. Open out the layers. The paper won't lie flat until step 44.

41. ... form a preliminary fold at the top and flatten the paper.

42. Reverse-fold the raw edges in and out. You will need to continue the pattern of pleats on the second layer.

43. Reverse-fold the small corners.

44. Mixed-reverse-fold the corners, reversing the pleats so that valleys become mountains and mountains become valleys.

45. Pull out the raw edges to make corners.

46. Fold the four flaps back.

47. Fold four flaps over on each side.

48. Reverse-fold the corners up. Unlike step 46, you don't need to reverse the orientation of the pleats.

49. Pull out paper from under the flap.

50. Fold the raw edges underneath.

51. Open out the top of the paper and pleat the front layer...

52. ...continue the pleat to wrap an entire layer around which forms a three-dimensional hat.

53. Fold a layer behind on each side to narrow the face.

54. Reverse-fold a small flap.

55. Reverse-fold to sharpen the points.

56. Repeat steps 58 to 59 on the right.
57. Bring the left side over to the right by curving the paper rather than folding it.

58. There are no exact references from now on. Mountain-fold the edge over. Repeat behind.

59. Bring the back down and form a sleeve using folds as shown. Repeat behind.

60. Form a knee. Repeat behind.

61. Closed-sink to round the corners Repeat behind.

62. Shape the arm. Repeat behind.

63. Pleat the paper at the raw corner to create a shallow vertex.

64. Make mountain folds and valley folds as shown. Repeat behind.

65. Fold the corner around the shoulder. Repeat behind.

66. Mountain-fold the raw corner underneath to lock step 69 in place.

67. Push the vertex in with a shallow closed sink to suggest the shape of the lower legs.


First time: $\square$ $30 \mathrm{~cm} / 12$ "

To-scale: $\square$ $30 \mathrm{~cm} / 122^{\prime \prime}$


A border graft can be used to add legs to the Kneeling Wise Man. One way of doing this is with the crease pattern on the right - the grafted paper is in a paler colour.

The underlying grid for the box-pleated portion (the head and arms) is based on dividing the diagonal into 60 ths (see overleaf for a quick method for how to divide the diagonal into 15 - the largest odd divisor of 60 ). Note that the crease pattern isn't fully flat-foldable due to the small gussets on the left and right sides.

Having folded the base from the crease pattern on the right, there is a lot of scope for shaping the model, especially the lower half. Have a go at creating your own version rather than trying to replicate the model in the photograph above.


## Quick Grid Divisions

Many models, notably box-pleated ones (see page 131), require dividing the paper into an $n$ by $n$ grid to find references. When $n$ is even, divisions in a less granular grid can be folded in half - powers of two are particularly straightforward since they only require folding in half as many times as needed.

However, when $n$ is odd, it's helpful to be aware of folding sequences that can make these $n$ by $n$ grids so that measuring isn't needed, and it goes without saying that mathematically exact sequences are preferred to approximations (or guesswork). There are a number of methods that can tackle this sort of problem and it's useful having a library of such divisions to refer to.

Each of the constructions in this section shows you how to use four folds or fewer to obtain one of the points on the grid from which the other grid points can be determined. The horizontal divisions are shown beneath each construction.

$3 \times 3$






$27 \times 27$


First time: $\square$ $25 \mathrm{~cm} / 10^{\prime \prime}$

To-scale: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$
-

The Sheep is an example of a model with a colour change - one that deliberately exposes both sides of the paper on the outside of the finished model. The intention is for you to use paper with a white side for the wool and black for the skin - this creates the most distinctive look.

However, incorporating a colour change into a design tends to cause a compromise elsewhere - in this case there are many layers of paper around the forelegs of the Sheep, so use strong but thin paper.

Note that the crease pattern on the right doesn't have the upper raw corners in a paler colour, even though they are folded away and not
 used structurally, because this paper is used in the colour change for the forelegs.


3. Fold the top right raw corner to lie on the vertical crease so that the foldline goes through the top left raw corner.


Step 3 shows the standard method of folding a $30^{\circ}$ angle in a square. This is due to the equilateral triangle being implicitly used, which is highlighted in yellow.

4. Fold and unfold.

5. Unfold step 3.

6. Fold the top right raw corner to lie on the vertical crease so that the foldline goes through the top left raw corner.

8. Unfold step 6.

9. Fold and unfold the top raw edge to the creases made in steps 3 and 6.


Fold and unfold.

10. Collapse the paper - this is very similar to a preliminary fold.

11. Fold the top corner down so that the valley fold goes through the intersections shown.

12. Petal-fold the centre of the raw edge upwards on existing creases.

14. Fold and unfold the edges to the horizontal crease.

16. Unwrap paper on the left and right, flattening it upwards.

17. Squash-fold the white triangle.

13. Fold the corner back down.

15. Rabbit-ear the triangle, swinging the corner to the right.

18. Reverse-fold the raw edges. Note that the mountain folds lie directly above the valley folds (rather than meeting in the centre).

19. Valley-fold the raw corners as far as they will go (without folding any other parts of the paper).

20. Turn the paper over.

21. Fold the raw edges to the centre and unfold.

23. Swivel-fold the right hand flap upwards while bringing the raw edge upwards. The paper won't lie flat until step 25.

26. Fold and unfold so that the three lengths shown are equal. This doesn't have to be exact.

24. Flatten the paper by bringing the raw edge to the vertical centre line.

27. Reverse-fold the corners using lines you just made.

22. Fold using existing creases. You will have to reverse the orientation from a mountain fold to a valley fold.

25. Repeat steps 23 and 24 on the left.

28. Fold the raw corners diagonally. You will need to squash-fold the raw edges.

29. Fold the raw edges outward, so that the fold-lines goes through the intersections shown.

30. Pull out the paper and swivel-fold it behind using a mountain fold, on both the left side and the right side.

31. Bisect the hidden angle and unfold.

34. Fold the raw corners to the creases as shown.

37. Fold the corners up as shown.
36. Fold and unfold the paper through all layers, so that the fold-line aligns with the edge you just made.

38. Turn the paper over.
39. Mountain-fold the corner underneath so that the fold-line goes between the intersections shown.

40. Fold the corners down, ensuring that they stay underneath the white region.

41. Swing both the top section down and the two flaps on the rear upwards. Hold the paper up to a light to see the intersection shown.

42. Fold vertical lines and unfold.

43. Pinch a mountain fold between the intersections shown.

44. Fold and unfold.

47. Fold and unfold.

45. Fold the left flap down.

46. Swivel-fold.

50. Fold the flaps over on creases made in step 47, but do not squashfold the paper flat - the paper won't lie flat until step 54.

51. Pop out the paper by folding on creases made in steps 42 and 43.

49. Repeat steps 45 to 48 on the right.
48. Fold two coloured layers over together - don't fold the white region.

53. Mountain-fold the model in half and flatten the top section, then rotate the paper.

54. Steps 54 to 57 show the far foreleg. Untuck the paper shown.

57. Fold the edge across.


58. Repeat steps 54 to 57 on the near foreleg.

59. Fold the raw corners up so that the fold-line is horizontal.

61. Fold and unfold.

62. From this step onwards, folds don't have to be exact. Fold parallel to the previous fold, then unfold.

65. Reverse-fold in order to thin the
tail and round the rump. Repeat behind.
60. Mountain-fold the legs. Repeat behind.

63. Crimp the back section around the front section using the folds made in steps 61 and 62.
66. Reverse-fold. Repeat behind.



64. Crimp the back corner down between the front and rear layers.


67. Round the back and open the model a little.


First time: $\square$ $25 \mathrm{~cm} / 10^{\prime \prime}$

To-scale: $\square$ $15 \mathrm{~cm} / 6^{\prime \prime}$


A Lamb can be made using almost entirely the same reference points as with the Sheep - only the tail is in a slightly different place, and this is as much for convenience of folding as necessity. This is a rarity in origami - normally animal forms need substantial modifications to be made into the shape of their young. The reason large modifications aren't needed here is because the extra paper needed for the spindly legs and longer neck is directly compensated for by losing the voluminous wool of the original Sheep design.

The advantage of this peculiarity is that some of the steps in the Sheep diagrams can be used for the Lamb too, making this one of the easier crease patterns to fold from in this half of the book.

Here is one way you might approach folding the crease pattern to obtain the base:

- Pre-crease the vertical lines (these will be used in open sinks later)
- Pre-crease the slanted folds near the bottom raw edge (these slanted creases don't actually require any exact reference points).
- Follow the first 25 steps of the Sheep (though you may wish to modify steps 16 and 17 to hide the white flap).
- Perform a multi-sink on each side to narrow the legs and body and another to narrow the head and neck.

Ensure you use thin paper for this model to accommodate the thin body and legs.


You may wish to deliberately make this section disproportionately large, to compensate for paper creep.


First time: $\square$ $25 \mathrm{~cm} / 10 "$

To-scale: $\square$

## Folded and photographed by Sampreet Manna



The donkey uses a colour change to allow for a grey coat with a white belly and face. Grey and white doublesided paper is a surprisingly uncommon colour pairing, so you may wish to prepare your own paper for this model by gluing a grey sheet onto a white sheet. If you do so, use thin paper for each sheet, such as tissue or rice paper.


1. Fold and unfold the diagonals.

2. Fold and unfold the corners to the centre.

3. Fold and unfold between the marked intersections and the bottom raw corner.

4. Fold and unfold.

5. Turn the paper over.

6. Fold and unfold the horizontal line which goes through the intersections shown.

7. Fold lines which are perpendicular to the edge of the paper and go through the intersections shown, then unfold.

8. Turn the paper over.

9. Fold the raw edges to creases made in step 11 and unfold.

10. Fold vertical creases to the diagonal and unfold.

11. Fold the raw edges to the centre and unfold.

12. Swivel-fold the raw edge to the right.

13. Collapse to bring the sides in, and push the point shown backwards...

14. Swivel-fold the right-hand raw corner to the left until it touches the raw edge on the left. Don't use the existing valley crease, but make an entirely new fold-line.

15. Reverse-fold the raw corner so that the diagonal line is horizontal.

16. ...then flatten the paper.

17. Reverse-fold the raw edge in using existing creases.

18. Petal-fold the raw edge to the left.

19. Fold the raw-edge back to the right.

20. Reverse-fold the edge underneath.


21. Fold the raw edges to the horizontal line and unfold.

22. Fold the raw corner up so that the fold-line goes through the crease intersections.

23. Reverse-fold.

24. Fold the raw corner down and swivel-fold the raw edges outwards.

25. Fold the lower section upwards.
26. Pull out the flap underneath.

27. Swivel-fold the paper to the right.


28. Valley-fold diagonally.

29. Pull out the hidden paper out.


30. Fold the corner up.

31. Bring the flap back down.

32. Squash-fold the flap.

33. Fold and unfold.

34. Fold the top flap across to the left.

35. Fold the raw edges to the centre line and unfold.

36. Open-sink the corner in.

37. Repeat steps 36 to 39 on the right.

38. Fold the raw corner so that it lies at the same level as the top of the creases you just made.

39. Fold the tip of the raw corner underneath. There are no exact references.

40. Spread the two points apart and open out the top of the paper.

41. Reverse-fold the corners in.

42. Sink the top corner. There are no exact references.

43. Sink the central section in and out as shown, and lift the two points upwards.

44. Unfold the paper slightly and wrap the raw edge around the vertex a little on each side.

45. Reverse-fold the edge in.

46. Mountain-fold along the two long points and bring them back to lie along the centre line.

47. Turn the paper over.

48. Fold the corner down, so that the fold-line aligns with the crease below.

49. Fold to the point then unfold.

50. Fold the flap down.

51. Rotate the model.

52. Fold the edges in and unfold.

53. Fold angle bisectors and unfold.

54. Mountain-fold the whole model in half. Be careful not to rip the paper.

55. Reverse-fold a hidden corner.

56. Open-sink the edges in.

57. Fold the bottom edge upwards while thinning the corners.

58. There are no exact references from this point onwards. Outside-reverse-fold the top section.

59. Unsink the central section of paper.

60. Stretch the tail down. You may need to readjust the layers to allow for this.

61. Hold the ears and crimp the head.

62. Round the neck with soft mountain folds.


63. Double-rabbit-ear the tail.

64. Crimp the neck.

65. Thin the forelegs with closed sinks.

66. Thin the forelegs with mountain folds below the knees.

67. Round the rump and back.


## Donkey Crease Pattern

You may have noticed that there was no crease pattern provided for the Donkey on page 170. That's because the extension here is for you to draw your own crease pattern of the model - making your own is one of the best ways of learning to read and fold from other crease patterns.

The diagram in step 14 is the best starting point here, because this step shows a number of important creases and all other lines can be drawn relative to these. Step 14 is repeated again below for your convenience.

Partially folding the model, then unfolding it and examining the creases is also a reliable method for drawing a crease pattern. The Donkey is an example of a model which uses $22.5^{\circ}$ geometry, so the angles of the creases are all easy to determine (though it's up to you to decide which folds constitute the base of the model here).

Use Maekawa's theorem (page 80) and Kawasaki's theorem (page 81) to help fill in sections and check that the crease pattern is correct.

To gain a thorough understanding of the structure of this model, you may also wish to investigate steps 1 to 6 specifically. Once you have a crease pattern, perform a similar analysis to the example shown on page 183 to work out why those particular folds are used. The creases from steps 1 to 6 are shown below too.


Step 14.


Creases from steps 1 to 6 .



# $22.5^{\circ}$ Geometry 

## What is $22.5^{\circ}$ geometry?

As we saw with box pleating on page 131, limiting our design geometry to specific angles can be an extremely powerful origami design tool. One common set of angles are the multiples of $22.5^{\circ}$ - these can easily be formed by halving a raw corner $\left(90^{\circ}\right)$ twice. These angles will feel very natural to you if you are familiar with the most common traditional origami bases.


A very common sequence for narrowing a corner.

Origami that limits itself primarily to these angles is using $22.5^{\circ}$ geometry. Pure-box-pleated models, where angles are limited to multiples of $45^{\circ}$, form a strict subset of the models that use $22.5^{\circ}$ geometry. So you can think of $22.5^{\circ}$ geometry as a relaxation of the pure-box-pleating rules, where angles are allowed to be halved.

With both pure box pleating and $22.5^{\circ}$ geometry, lines are specified by starting with one endpoint and an angle - the other endpoint is determined by an intersection with another line. Generalised box pleating is not a subset of $22.5^{\circ}$ geometry, because diagonal lines in generalised box pleating are specified by their two endpoints, which means the angle of the line cannot be a multiple of $22.5^{\circ}$ (see below).

As with box pleating, it's most helpful to think of $22.5^{\circ}$ geometry applying to certain regions of the paper, or to a particular base or crease pattern, rather than to an entire model.


Finding endpoints in $22.5^{\circ}$ geometry: Left: One endpoint and an angle determine the location of the other endpoint. Right: The endpoints of two lines are determined by their point of intersection.


Generalised box pleating: Two endpoints can determine the angle of the line, allowing any line with a rational gradient. A $22.5^{\circ}$ line has an irrational gradient, so cannot be created with box pleating.

## Designing with $22.5^{\circ}$ geometry

As with box pleating, the benefits of designing using $22.5^{\circ}$ geometry include:

- References which are relatively easy to define;
- Easier folding sequences than designs using arbitrary angles;
- A consistent geometric aesthetic.

Using $22.5^{\circ}$ geometry also has an efficiency benefit, especially compared to pure box pleating. Relaxing the rules from pure box pleating to include more angles means that the minimal flap shape in $22.5^{\circ}$ geometry is not a square, but an octagon. (Recall that the minimal flap shape isn't necessarily attained by any particular flap.) An octagon is a much closer approximation of a circle than a square, which means that flaps in $22.5^{\circ}$ geometry can be packed together much more tightly than in pure box pleating, allowing for more flexibility and more efficient paper usage.

The greater variety of available angles also means that $22.5^{\circ}$-geometry models don't fall prey to the overly boxy look that afflict box-pleated models, and that flap lengths don't have to fall rigidly into discrete integer multiples of a unit. These visual benefits mean $22.5^{\circ}$ geometry is a particularly popular choice for designers when making animals, especially mammals.

As a bonus, models designed using $22.5^{\circ}$ geometry can also draw upon the familiar structures of traditional bases, which makes them much easier to fold. For example, it is not uncommon for a diagram to instruct the folder to "make half a bird base out of this triangle", or similar (though diagrammers should be careful not to make their instructions inaccessible to less experienced folders).


Left: The frog base and its tree. Right: The minimal flap shape for $22.5^{\circ}$ geometry is an octagon.


Left: The frog base crease pattern packed with circles. Right: The underlying design geometry of the frog base is $22.5^{\circ}$ geometry, therefore the flaps can be represented by non-overlapping octagons.

## Exact References

The traditional origami bases are very symmetric, and finding the references needed to fold them is quite straightforward. With designs that use non-traditional bases, however, finding references is not always so straightforward: once a designer has constructed a crease pattern, it is often not immediately obvious where to begin folding to get any of the required lines.

As a very simple example, suppose you have a crease pattern which contains only a single vertical line which is $2 / 3$ of the way along the square.


A crease pattern, but with no folding sequence for how to fold the line.
There are multiple ways to solve this problem, the least sophisticated of which is to simply guess where to fold. This is fine if the exact location of the fold doesn't matter, but won't work for any complex origami designs where folding inaccuracies can have drastic knock-on effects.

Another option is to measure the paper, calculate the length needed, and mark the folding references on the paper. This method has the benefit of not making any folds in the paper before the reference is known, so might be preferred when making smooth models for a display. If you do this, make sure to mark the side of the paper which won't be visible!

However, within the medium of origami, folding is the natural way forward. There are two main ways to assess the effectiveness of a folding sequence when trying to locate a given reference: accuracy of the sequence and length of the sequence.

Not all points on a square can be constructed exactly. For example, it is impossible to construct a point which divides the bottom edge in the ratio $1: \pi$ exactly. For such points, the best that can be done is to use an approximate sequence. There are some general algorithms for making a folding sequence which gets arbitrarily close to any point.

Make sure not to confuse exact sequences with referenced folds (page 134). All exact sequences use referenced folds, but the example below shows a sequence with referenced folds, but which is not exact.


An example of an approximate, referenced sequence for folding a line $2 / 3$ of the way along the paper. Fold this sequence, then on the same sheet of paper fold an exact sequence (for example, use page 158). Can you tell that one of them is an approximation?

In origami design, the types of points that need finding are almost always constructible - you aren't likely to come across a design that needs you to divide an edge into the ratio $1: \pi$. Therefore, you should generally aim to find an exact sequence, provided it isn't too long. Crease patterns which use $22.5^{\circ}$ geometry usually have references which can be found exactly using a relatively short sequence (see page 205).

## Example of finding an exact references using $22.5^{\circ}$ geometry

As an example, consider the crease pattern below.


Crease pattern and base using references from the Ox and the Two-humped Camel.

We could find a folding sequence to get references for this crease pattern using an algebraic method. This would require us to calculate the length of some of the lines in the crease pattern. In general, we would then have to use an algorithm for constructing one of the points.

This isn't too hard, but it's more accessible to use a purely visual, geometric route. A visual method will also find some of the lines needed for the folding sequence along the way, making it more practical than an algebraic method.

Because of the limited angles available in $22.5^{\circ}$ geometry, if we work out how to locate just one of the folds on that crease pattern, then the rest can be located easily. This particular crease pattern will be simpler to work with if we rotate the diagram, remove valley or mountain assignments, and add or extend creases to make the new figure below.

Aside from the edges of the square, the upward diagonal of the square is the only line we can locate immediately. As such, we should attempt to use this line when finding references.

After considering the various shapes in this diagram, we should be drawn to the orange triangle shown in the diagram on the right below. This is because, given the orientation and shape of this triangle, its position is completely determined by known lines: the bottom edge of the square, the right edge of the square, and the upward diagonal. That makes this orange triangle the easiest place to start.


Left: A cleaned-up version of the previous crease pattern. New and extended lines are shown as dotted lines. Right: The location of the orange triangle is completely determined by known lines.

Having identified this starting point, we can remove unneeded creases from the diagram to focus on the orange triangle, as shown below. Now, if we draw a new line from the bottom right corner of the square through the top left corner of the orange triangle, that line crosses the upward diagonal at one of the reference points we are seeking (see left diagram, below).

What's particularly useful about this new line is that all enlargements of the orange triangle about the bottom right corner of the square also have their top left corners on this line (see centre diagram, below). So now, we just need to locate this line, or equivalently, any of the enlargements of the orange triangle. In other words, we've just expanded our possible options for locating references from one triangle to an infinite number of triangles. This is the key step in this example, and generally works for locating references when using $22.5^{\circ}$ geometry.


Left: Finding the bold diagonal line is equivalent to finding the orange triangle.
Centre: The top left corners of all enlargements of the orange triangle lie on the bold diagonal line.
Right: The yellow triangle is the easiest enlargement to find.

In fact, we can fold one of these triangles fairly easily: the largest such triangle that fits in the square, shown in yellow in the diagram on the right above. Typically with a process like this, the largest enlargement is at least as readily foldable as the one we began with, since the largest one by necessity touches another edge of the square (i.e. a line which is already located).

One folding sequence for finding this yellow triangle is shown below.


An exact folding sequence to construct the top left point of the yellow triangle. Notice how the centre and right diagrams utilise $22.5^{\circ}$ angles.

All that remains now is to fold the preceding steps in reverse order to locate the orange triangle. See the $0 x$ on page 185 or the Two-humped Camel on page 194 for a full sequence.

Many other exact references can be found with a very similar method using $22.5^{\circ}$ geometry. For practice, see page 179 , where part of the extension is to apply similar reasoning for the folding sequence for the Donkey.


First time: $\square$ $20 \mathrm{~cm} / 8^{\prime \prime}$

To-scale: $\square$ $40 \mathrm{~cm} / 16^{\prime \prime}$
 reference points made in steps 1 to 4 .


1. Fold and unfold the vertical diagonal.

2. Fold the raw edges to the diagonal and unfold.

3. Fold to the intersection shown and unfold, but only crease firmly where the fold-line hits the raw edge.

4. Fold between the references shown, creasing firmly only over the diagonal crease. Then unfold.

5. Fold the raw edge to meet the vertical edge.

6. Unfold the paper, but keep steps 7 and 9 in place.

7. Fold the raw corner to meet the intersection you just made.

8. Fold the right raw corner over so that the raw edges align.

9. Fold the raw corner down and unfold.

10. Fold the left raw corner over so that the raw edges align.

11. Fold the raw edge to meet the vertical edge.

12. Fold the raw edges to meet the crease you just made. The fold-lines should cross at the intersection shown.

13. Rabbit-ear the raw corner down and to the right.

14. Fold and unfold angle bisectors through both layers.

15. Squash-fold the flap.

16. Repeat 18 to 21 on the right.

17. Fold the flap to the left.

18. Fold and unfold angle bisectors through both layers.

19. Turn the paper over.

20. Swivel-fold the corner up.

21. Fold the raw corner up.


22. Fold the raw corner down, so that the fold-line goes through the intersections shown.

23. Open the flaps and flatten them to the sides.

24. Turn the paper over diagonally.

25. Reverse-fold the large flap, so that the fold-line goes between the points shown.

26. Reverse-fold the corners.

27. Fold the top section down while thinning the corners.

28. Pleat the paper. The mountain fold was made in step 27.

29. Fold and unfold the lower layer.

30. Reverse-fold the raw edges.

31. Mountain-fold the paper in half, but only fold softly.

32. Unfold the raw corner.

33. Swivel-fold the flap, pivoting about the corner on the left.


34. Sink in and out.

35. Swivel-fold the paper to cover the white triangle. Repeat behind.

36. Fold and unfold.

37. Fold the raw corner down and unfold.

38. Reverse-fold the raw corner on the crease you just made.

39. Crimp the top section of the paper.

40. Reverse-fold the raw corner down.

41. Thin the point with two reverse folds.

42. Fold the tip out.

43. Fold and unfold. Repeat behind.

44. Fold and unfold through both layers.

45. Crimp the paper symmetrically.

46. Reverse-fold. Repeat behind.

47. Mixed-reverse-fold the edge and spread the triangle with the raw corner. The paper won't lie flat.

48. Push the paper between the layers below.
49. Simultaneously thin the points of the
quadrilateral and fold the left edge to the right.

50. Crimp the head down. There are no reference points.

51. Fold the flap forwards. The paper should lie flat. Repeat behind.

52. Crimp the top layer up again. There are no reference points.

53. Unwrap a layer of paper around the ear. Repeat behind.

54. Fold the flap back again to form the ear. Repeat behind.

55. Wrap the corner over to form the nose.

56. Shape the horns.

57. Fold and unfold using a fold-line which is perpendicular to the back edge and which goes through the intersection shown.

58. Lift the paper on the hind leg.

59. Pinch a mountain fold between the two intersections shown.

60. Begin to swivel-fold the point upward. The paper won't lie flat.

61. Complete the swivel-fold to flatten the paper.

62. Fold the point back down.

63. Repeat steps 64 to 68 behind.

64. Crimp the hind legs in and out.

65. Push the middle layer of paper forwards to form the front feet.



First time: $\square$ $25 \mathrm{~cm} / 10^{\prime \prime}$

Folded and photographed by Sampreet Manna

To-scale: $\square$ $25 \mathrm{~cm} / 10 "$


The head is perhaps the most distinctive feature of the $0 x$, and can be mostly reused in a number of cow-adjacent models by grafting extra paper onto the opposite corner of the square. The amount of paper grafted can be fine-tuned to create the desired proportions of the subject. This is done most elegantly when the amount of added paper is not arbitrary, but calculated to maintain the same $22.5^{\circ}$ geometry used in the Ox.

As an example, on the right is the crease pattern for a Calf. Compared to the $0 x$, more paper is needed to both create thin legs and a larger body relative to the size of the head. The neck is also much longer, allowing for more freedom when posing the model.

Start by pre-creasing the lines in the crease pattern, then fold the lines together to form the base shown here too. This is more complex than with the 0 x , because much of the base has to be collapsed in one step. From then on, the folding sequence for the Calf is very similar to the $0 x$ from steps 30 onwards, but the paper used for the horns of the 0 x is used for ears. As ever, try to make your own Calf rather than exactly replicating the model shown here.



First time: $\square$ $35 \mathrm{~cm} / 14$ "

To-scale: $\square$ $45 \mathrm{~cm} / 18^{\prime \prime}$


The legs of the Two-humped Camel are (relative to their length) narrower than any edge flaps of all other models in this book. While this might suggest thin paper is best, step 58 (which is optional) has a high tearing potential, and the model will not stand freely unless the paper is sturdy.

Therefore, the best results are achieved with thick paper which is wetfolded to keep the legs in shape - this was done with the model in the photograph above.



1. Fold and unfold the vertical diagonal.

2. Fold the raw edge to the diagonal and unfold, but only crease firmly where the fold-line hits the raw edge.

3. Fold the raw corner the mark you just made and unfold, but only crease firmly where the fold-line hits the raw edge.

4. Fold a line between the intersections shown. Only fold firmly over the diagonal crease.
5. Fold the raw corner to the intersection shown and unfold.


6. Fold the raw corner to the crease intersection and unfold.

7. Fold lines between the intersections shown.

8. Fold and unfold through the intersections shown.

9. Fold the raw edges to the creases made in step 6 and unfold. Only fold firmly below the crease you just made.

10. Turn the paper over.
11. Fold the raw edges to lie on the horizontal crease and unfold.

12. Fold the raw edges to lie on the horizontal crease and unfold. The fold-lines should meet at the crease intersections shown.

13. Fold between the intersections shown. The raw edges should align with creases made in steps 7 and 9.

14. Fold and unfold so that the raw edges meet.

15. Fold the raw edges to the crease you just made and unfold.

16. Fold the raw edges to the horizontal crease and unfold.

17. Fold between the creases intersections shown.

18. Fold the raw edges to the horizontal line and unfold.

19. Fold the raw edges to the diagonal and unfold.
20. Fold the raw edges to the creases you just made and unfold.

21. Fold and unfold angle bisectors.

22. Squash-fold the flap.

23. Swivel-fold on each side to sharpen the raw corner.

24. Rabbit-ear the raw corner up to the right.

25. Fold along a horizontal line.

26. Fold the raw corner to the left.

27. Double-swivel-fold each side to sharpen the point while folding the raw edges up.


28. Fold the corners to the intersection shown and unfold.

29. Make the folds shown simultaneously and begin to the bring the corners together. The paper won't lie flat.
30. Make the folds shown
31. Make the folds shown in the central region while folding the paper in half.



32. ...flatten the paper.

33. Fold and unfold to get the crease on the second layer. Repeat behind.

34. Fold and unfold angle bisectors. Repeat behind.

35. Reverse-fold in and out. Repeat behind.

36. Fold and unfold angle bisectors.

37. Reverse-fold in and out. Note these do not use all of the creases made in the previous step.

38. Fold the corner to meet the vertical line, but leave a small gap rather than folding an exact angle bisector.

39. Open-sink the corner.


40. Reverse-fold the corner so the edge aligns with the hidden layer.


41. Closed-sink two flaps.


42. Fold and unfold angle bisectors.

43. Flatten the flap.

44. Reverse-fold the flap down.

45. Mountain-fold the last flap and tuck it behind.

46. Reverse-fold in and out on creases you just made.

47. Fold the flap down.

48. Closed-sink the front flap.

49. Reverse-fold the front flap down. The spine should be the second of the three mountain folds.

50. Wrap the layer of paper around to the front. This is a tricky step and you will need to unfold much of the paper to achieve this. Optionally, you can ignore this step.

51. Repeat steps 48 to 58 on the back.
52. From this step onwards most folds have no reference points. Reverse-fold the narrow flap.

53. Wrap a layer around to the front.

54. Mixed-reverse-fold.
55. Mixed-reverse-fold symmetrically.

56. Mixed-reverse-fold.

57. Swivel-fold the raw edge to sharpen the ear. Repeat behind.

58. Mixed-crimp up and down. Don't lock the raw edge.

59. Reverse-fold the raw corner.

60. Open-sink the two corners.

61. Crimp and make the head threedimensional.

62. Open the eye. Repeat behind.

63. Closed-sink.

64. Outside-reverse-fold the tail.

65. Closed-sink.

66. Crimp the ears backwards.

67. Crimp the feet.

68. Narrow the neck with mountain folds.

69. Pinch the legs except at the knees.


First time: $\square$ $40 \mathrm{~cm} / 16{ }^{\prime \prime}$

To-scale: $\square$ $50 \mathrm{~cm} / 20 "$

The Camel with Toes is an example of a model which uses strip grafts see page 89 for an explanation with this model.

The crease pattern on the right is only needed to fold the strip grafts. Once you have done this, you can use the instructions for the Twohumped Camel on page 194, except the extra paper in the strip grafts will used to make the toes.

As with the Two-humped Camel, thick paper is the best choice to avoid ripping, and wet folding the legs will allow the model to stand.


If you've reached the end of this book and still want to learn more about origami design, here are some further topics to research.

## Tree Theory and Circle Packing

The pages in this book dedicated to circle packing only touch on the topic. There is a lot of depth to this subject to be found in other literature. Research this for a better understanding of the makeup of a crease pattern and its the relationship with the tree.

## Molecules

In short, molecules within origami design are shapes whose boundaries collapse down to lie on a single line. While molecules are interesting to study in and of themselves from a mathematical perspective, they also arise within the theory of circle packing as the shapes whose perimeters are comprised of line segments which connect the centres of two touching circles.

Sometimes designers will design a model by assembling molecules to make their desired crease patterns, rather than considering flap length directly. This is a particularly common approach for models designed using $22.5^{\circ}$ geometry.

## Box Pleating

One of the methods used in generalised box-pleating is to use right-angled triangles whose hypotenuse is an integer multiple of one unit of the grid. These triangles are Pythagorean triangles, and so the method is called Pythagorean stretching. Pythagorean stretches allow the paper used for individual flaps to be smaller and more complex polygons than squares.

Generalised box pleating allows the creation of any crease which has a rational gradient. These angles form a dense set within the set of all possible angles, which means that the minimal flap shape is a circle.
Equivalently, the only paper guaranteed to be in the polygon for a (leaf edge) flap is the same circle as would be used for circle packing.

Level shifters are a way of increasing flap width within box-pleated models. This enables box pleating to represent a wide variety of subjects more effectively.

## Hex Pleating

Pure hex pleating limits lines to be at angles which are multiples of $30^{\circ}$, in the same way that pure box pleating and $22.5^{\circ}$ geometry are limited to angles which are multiples of $45^{\circ}$ and $22.5^{\circ}$, respectively. Generalised hex-pleating simply which uses a triangular grid - this is the hexagonal lattice from which hex pleating gets its name.

The minimal flap shape with pure hex pleating is a hexagon, and with generalised hex pleating it is a circle, for the same reasons as with generalised box pleating.

## Incorporated Patterns

A more efficient way to include a pattern on a model than grafting, is to incorporate the required pleats into the structure of the model. This means pleats don't propagate across regions of the paper where they aren't used. This is frequently done with subjects like insects and crustaceans which have segments or plates along their backs.

Unlike with pattern grafts, there's no way to make an easier, unpatterned version of a model which has an incorporated pattern. This means there is a steep learning curve for folding models which include these sorts of patterns.

## Finding References Algebraically

The points which are constructible with origami are famously known for being a strictly greater set than the set of points than those constructible with the traditional straight edge and compass limitations. Notably, it is possible to trisect any angle exactly into thirds using folding alone.

When finding exact references with $22.5^{\circ}$ geometry, crease lengths are usually a surd in the form $a+\sqrt{2} b$ for some integers $a$ and $b$ (relative to a square of width 1). Reference points based off these lengths are relatively easy to find.

There are a number of methods and algorithms designed to fold exact sequences. Some of these fulfil desirable conditions, such as only leaving creases at the raw edge of the paper.

In practice, long exact sequences are not always more accurate than short approximate sequences, due to the folding error incurred with each fold. The short length of the sequences on page 158 is one of the reasons that they are particularly useful.

Certain folds are also more susceptible to a large error than others. Over time, folders develop an intuition for which types of folds to avoid, and which can be relied upon. You may wish to investigate error modelling in origami for more on this topic.

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## ＂In short，a near essential book for folders who have already learned the basics．＂

－Nick Robinson，professional paper artist and author of over 100 books on origami

With this book，you can fold over 30 original models designed by paper－folding expert Peter Buchan－Symons，to create your own origami Nativity scene！
Along the way，learn advanced folding techniques to become an origami master，and discover design principles so that you can learn to create your own unique origami models．

Designed for beginners and experts alike， enjoy hours of creative fun with A Complete Origami Nativity！


领领领


Over 1000 individual diagrams！


[^0]:    1. Begin with a $1 \times 4$ rectangle. Fold the paper in half lengthways and unfold.
