Origami
New Worlds

Alessandro Beber
Origami New Worlds

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Alessandro Beber

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Origami New Worlds
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Prepare to be addicted!

Alessandro Beber is one of the world’s premiere practitioners of the geometric genre within origami known as tessellations. In the broader mathematical world, a tessellation is a subdivision of the plane by lines. In the origami world, a tessellation is a subdivision of a sheet of paper by folds. As in their mathematical kin, in origami tessellations the folds interact to create complex arrangements of tiles that exhibit an unexpected combination of precision, regularity, and surprise. It is the deviations from regularity that create the aesthetic appeal of tessellations (and so much other art): a mixture of order and disorder.

In Alessandro’s work, that mix of order and disorder is combined with elements that often seem impossible: 2-D shapes, and 3-D illusions that appear to float above a background. To see some of his tessellations, the observer thinks “that can’t possibly be a single sheet! You must have glued that together!” But not only is it a single sheet, each of these can be readily folded with the clear instructions in this book.

While Alessandro’s talents span a wide range of tessellation structures using different symmetries and tilings, all of the patterns here are based on a simple triangular grid and are folded from a single hexagon. Elegantly, you start and end with a hexagon. But when you finish, that hexagon is lined, subdivided, decorated, with background, foreground, and elements that seem to float with no means of support.
I mentioned addiction. The techniques of Alessandro’s simpler tessellations are accessible even to those with limited folding experience, while in the later designs, there is plenty of challenge for even the most die-hard tessellation folder. He also provides guidance on designing one’s own tessellations. I folded one of these at an origami convention—my first grid-based tessellation—and I must say, I was drawn into the beauty and mathematical purity of the genre. My prediction is that you will, too.

Happy folding!

Robert J. Lang
April, 2017
Preface

<About the Book>

My journey in the world of folded paper was subject to big changes in recent years. In particular, in 2012 I began exploring geometric origami and tessellated patterns more deeply than ever before, finding a kind of beauty usually not visible in representational origami, but hidden in their structures. As a result, I designed a number of intricate patterns of twist-folded paper, taking a distance from figurative origami. In late 2013 I published all those results in my first booklet, “Fold, Twist, Repeat” (CDO 2013, e-book 2014), taking this chapter to an end.

Then, I immediately began exploring a new technique, which is the subject of this new book. It consists of a mix of two origami tessellation techniques: flagstone parts are used to highlight shapes formed by blocks, and classic twist-folds are used to obtain a flat background. As you will see, this technique allows the design of many impossible and 3D-like objects, creating great visual effects.

In late 2014 I taught a simple example of these works during a workshop in Lyon, France, and began struggling on how to explain in details how to use this technique. While continuing the exploration of this area, by designing and folding more complex shapes, I collected a number of ideas and tricks, and decided to document these results in a new book, explaining how to fold my designs as well as how to design new works using these methods.

Eventually, this new chapter also came to an end. What will come next?!?
<Acknowledgements>

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Part I:
Basic Steps
Introduction

<About Origami Tessellations>

Tessellations are geometric patterns made by repeated shapes, or tiles, that can cover an infinite plane. Origami tessellations are single-sheet folded patterns, where the repeating tiles are made of folds, forming intricate beautiful decorations. Since the tiles can cover an unlimited sheet of paper, this means that the edges of paper are not used, but instead each tile accumulates some layers of paper in different places while shrinking by the folds. Often, the different number of layers stacked in different areas of each tile, form an unexpected mesmerizing pattern when the origami tessellation is displayed backlit. Moreover, since tiles are placed next to each other in order to cover the plane, this means that the folds of a tile must match with the folds of its adjacent ones.

The tessellations contained in this book are no exception. However, they aim at a different goal than just creating beautiful patterns: their purpose is to represent three-dimensional and impossible shapes through a flat-folded object. Therefore these designs can be examples of both geometric and representational origami at the same time, challenging the common classifications of origami designs.

Moreover, this book does not teach how to fold some designs through step-by-step instructions. Instead, it shows how to use the technique I developed for designing these artworks, for folding the illustrated designs as well as for designing new pieces.

In this and the following chapters you will find directions on how to start using this technique, some step-by-step folding exercises to practise, an introduction to the design process, and several designs as examples of possible results. Enjoy!
<Symbols>

Origami tessellations are not suited to be taught using the standard origami notation and step-by-step procedures. Instead, crease patterns (CPs) are widely used, representing all the fold lines on the unfolded sheet of paper, or part of it, in a single drawing. Moreover, since crease patterns can contain hundreds of fold segments, the standard style for fold lines (dashed lines for valley folds and dot-dash lines for mountain folds) would become very confusing.

In this book, valley folds will be represented with red lines, mountain folds with blue lines. All the crease patterns will refer to the colored side of your paper, even if most of them will not have the gray shading, for clarity. Every crease pattern will be followed by a drawing of the folded object as seen from the front and from the back, and folding procedures will be illustrated by sequences of progressive crease patterns, enhanced with arrows to help clarify the action needed to reach the following step. Some videos will also be available for further help.
Some drawings will also show other kinds of lines. Green lines are not creases, but are used to show the underlying pattern structure. Grey lines, instead, will be used to represent unfolded creases, like the grid lines on which all the patterns are based.

All the models designed using this technique are based on triangular grids, and the most convenient way to precrease them is by folding a regular hexagon of paper. The following sections will show you how to obtain hexagonal paper and fold accurate grids.

When folding a crease line on any kind of paper, especially with thicker ones, a small bump will appear on the mountain side of the crease, while a small depression will appear on the valley side, as you can see in the diagram on the left.

Even if you switch the crease orientation (in the diagram: from valley to mountain), a slight bump remains on the original mountain side and a slight depression remains on the original valley side.

I strongly suggest to fold each grid bidirectionally: folding each crease on one side of the paper, and then backcreasing it by switching its crease orientation. Every crease line will have a neutral orientation, and it will be easier to collapse the model as it requires small segments of each crease line in different directions. It will also be helpful for folding accurate grids. Pay special attention to have the same original orientation for all the
crease lines, in order to have a neat folded result. You can choose which side of your grid you want to appear on the front. When using coloring effects, I usually prefer keeping the mountain side of the grid as the front (visible) side, as it better absorbs the solution used. Otherwise, when using plain paper, I usually prefer keeping the valley side of the grid as the front (visible) side, as the flagstone blocks will remain slightly raised upon the background. These behaviors strongly depend on the paper you are using, therefore some practice with different kinds of paper will be extremely helpful.

<About Papers>

The best results can be achieved by using thick papers, weighting around 80-120 gsm. I personally prefer using Elephantenhaut paper and Efalin paper, produced by Zanders, and StarDream paper, produced by Cordenons, but other 100-110 gsm papers work just as fine. Interesting results can be obtained by coloring or decolorizing the paper after folding or precreasing. I tried using a solution of ink+water or bleach+water, applying it either on the precreased sheet of paper or on the folded model using a sponge. Each fold slightly weakens the paper fibers along the crease line, hence the paper will absorb more along the creases, creating unexpected appealing shades. The solution must be adjusted for the paper you are using, in order to be strong enough to create an interesting effect, but without damaging your paper. Moreover, some papers are not suited to this process, as they may absorb evenly also on the uncreased areas, or may not react to bleaching, or may not allow to see the added color. Just try!
This is probably the simplest method for obtaining an accurate regular hexagon. Most rectangular papers come in 1:√2 or similar proportions. If the short side measures 1, then the longer side must be at least $2/\sqrt{3} = 2\cdot\sqrt{3}/3 = 1.1547$. 

https://youtu.be/LlmAzxZXKW4
Using this method, you can construct the largest regular hexagon inscribed in a square.

https://youtu.be/U1p2H-TcYc0
Folding Grids

<Triangular Grids>

Grids of triangles are made by three sets of parallel, equally-spaced lines, arranged in three directions forming angles of 60° and 120° among them. It is easy to fold such a grid from a regular hexagon of paper: you just need to divide its height (the distance between two opposite edges) into an equal number of parts, and then repeat the process on the other two directions.

Pay attention at how the grids are named. The number indicating the divisions in n-ths will always refer to the distance between two opposite sides. Diagonals will be also divided in n equal segments, as they run all the way through the hexagon. Instead, edges will be divided into n/2 equal segments, because their length is half that of the diagonals, and at each iteration only half of the new crease lines will meet a given edge.

<Grids in 2”>

The most simple divisions for a grid are numbers that are powers of two. In this case, at each iteration the distance between two existing parallel lines is halved, by placing a new crease in between them. This step is repeated for all the successive parallel lines from the previous iteration, and all this is repeated in the three directions.

You will obtain grids in 2, 4, 8, 16, 32, 64, ...
Part I: Basic Steps

16ths

32nds

64ths
Other grid divisions commonly used are 6, 12, 24, 48, 96, ...
Part I: Basic Steps

24ths

48ths

96ths
Part I: Basic Steps

Folding Accurate Grids

Although folding triangular grids is done by a very simple method, folding accurate ones is not so easy and requires some practice. Errors and inaccuracies can be introduced by the folding procedure used, the behavior of the paper, and by the limitations of our sight.

As we saw before, each crease leaves a small bump on its mountain side. This means that each crease accumulates a little bit of paper along its line, especially when using thicker papers. Although this amount will be unimportant in most cases, when folding dense grids it can give raise to considerable problems. If you tried folding a grid before, you may have noticed that, after folding, the sheet of paper will not measure as before!

The following procedures are used to overcome these problems by minimizing them. Moreover, these methods can be applied to any similar kind of folding, for instance when folding grids of squares or other types.

<Procedure #1 (left)>

The most obvious method consists in folding each distance in half in order, by placing the raw edge over an existing crease. It works fine at the beginning, but later an area containing many creases will be overlapped to an area with few ones, and a considerable error will be introduced.
<Procedure #2 (right)>
Instead, particular attention can be taken in making each fold by overlapping two areas containing the same number of creases. This method is useful for folding all-valleys grids, and a detailed sequence of steps having this property can be developed.

<Procedure #3 (left)>
Alternatively, each distance can be halved by switching one crease to mountain and placing it over the next one. It works well for bidirectional grids, but when the grid becomes denser, small errors may appear due to sight limits.

<Procedure #4 (right)>
The inaccuracy of the previous method can be minimized by switching one crease to mountain and placing it over the third (or fifth) next crease. This is the method I prefer, usually combined with concepts from Procedure #2.
How to Fold a Grid

Begin by valley folding one diagonal, then backcrease it to mountain, and repeat with the two other diagonals.

Then, valley fold two edges on a diagonal making 4ths, backcrease these lines to mountain, and repeat in the two other directions.

Do the same with 8ths, 16ths, 32nds, ... At each step valley fold the new lines, backcrease them, and repeat along the three directions.

Try folding a triangular grid out of a regular hexagon of paper, using the methods explained in the previous sections!

https://youtu.be/tFiv8xn91Qc
Twist Folding

<Twist Folds>

The building blocks of origami tessellations are pleat folds, pleat intersections and twist folds. A pleat fold is a pair of parallel folds, one mountain and one valley. When three or more pleats meet, a pleat intersection is formed by flattening the pleats together, and if those pleats are all arranged circularly around their meeting place, a twist fold will appear. A twist fold has a polygonal shape, with a pleat emanating from each of its sides, and when folding it, the central polygon twists with respect to the surrounding paper (or, alternatively, the surrounding paper whirls around the polygon).

Twist folds must follow certain rules in order to be flat-foldable and to be connected with adjacent twists, but this is not the topic of this book. For our purposes, we just need to know some simple twists and practice folding them before attempting to fold any of the following projects, although those twists will not be seen in the projects exactly as they are presented here.

<Triangle Twist, type 1>
This is the most simple kind of twist you can fold. It is made by flattening a 120° three-pleat intersection. Note that all the folds forming the pleats lie along grid creases, while the sides of the triangle do not. Triangle twists can be connected to each other forming a cluster of six, and if they are placed close enough, the triangles will touch each other and some creases connecting them will disappear.

Closed-Back Hexagonal Twist>

This hexagonal twist has all its folds along grid lines, and the six pleats all meet at a common point on the back side of the folded figure.
In this hexagonal twist, the pleat folds lie along grid lines, while the hexagon sides do not. Moreover, on the back side of this twist, the six pleats do not meet at the center of the polygon, as in the previous examples, but an open hexagonal frame appears.

The rhombic twist comes from the intersection of four pleats. Note that the four pleats do not meet at a common point on the back side of the folded object, but at two distinct points. This twist fold will be the essential element to form the blocks making the 3D-like “flagstone” figures in all the following projects.
This twist fold is made by the very same folds and angles as the Triangle Twist type 1, but placed differently on the grid. In fact, it has the central triangle made of grid segments, while the lines forming its three pleats lie along angle bisectors of grid lines. Despite the similarities, this is more tricky to fold than type 1, especially when coming in clusters or covering large areas with a pattern of those twists. I strongly recommend to practice folding a pattern of those twists because this structure is used to form the “background” areas in all the following projects. In the following chapter you will also find two different procedures to fold these patterns.
Part II: Exercises
<About the Exercises>

In the following sections you will find detailed procedures to fold some of the most simple elements used by this technique. I suggest you to practice them extensively, until you will find them easy to fold, before attempting to make the more advanced projects. These exercises give an idea of how this technique works. More details will be given in the next chapter. However, even the most complex designs of this kind follow the same rules, and they would be folded in a very similar way, with the added complexity of many more elements of the same few kinds covering a larger area, but also with the experience and practice gained in folding these simpler examples!

For each exercise you will start with a precreased grid, as you learnt in the previous chapter. Then, some more creases will be added, before beginning the collapse process. This last part is the most challenging, as the paper will be three-dimensional through several steps. Each step will show the crease pattern with all the fold lines used up to that point, and two figures of the folded object, as seen from the front and the back side. Those figures highlight the flattened sections, and leave out some of the wavy layers covering the areas you are working on, for clarity. In case of doubts, some videos are available at the URLs provided at the beginning of each sequence, to show the collapsing steps in action. Let’s start!
Background Exercise

Design: ---
Paper: a 20 cm regular hexagon or bigger
Grid: 16ths

Here are two different procedures for folding the background. They will both turn out useful when folding bigger projects.
Start by valley folding a first set of triangles, beginning in the middle. Note that all these triangles are aligned, therefore you can fold all the creases on the same line in a single step, by folding one segment, skipping one, folding one, and so on.

Valley fold a second set of triangles. These are again all aligned among them, but not with the previous ones.

Valley fold a third set of triangles. These are again all aligned among them, but not with the previous ones.
Part II: Exercises

<Background Collapsing #1>

https://youtu.be/Z-yum3QrkN8

(1) Here on the left is the precreased hexagon. Note the direction of the arrows, indicating where all the pleats will run. Start by collapsing the triangle in the middle. The model will not lie flat.

(2) Collapse one more triangle next to each side of the central one, extending its pleats outward. The model will not lie flat.
(3) Collapse three more triangles, by pushing their inner edge with a valley fold, and pushing at their sides forming two mountain folds. You can flatten the first ring of triangles.

(4) Form three more mountain folded triangles, as in step 2. The model will not lie flat, again.

(5) Collapse one more triangle at each side of the previous ones, as in step 3. The model still does not lie flat.
(6) Collapse three more triangles as in step 3, flattening a second ring of triangles.

(7) Continue, as with steps 2 and 4.

(8) Continue, as with step 5.
(9) Continue, as with step 5.

(10) Flatten the third ring of triangles, as with steps 3 and 6.

(11) Flatten the paper along the raw edges, using the half-triangles forming the background borders. The model will be completely flat now.
(12) Adjust the background by pushing all its corners from the front to the back side, starting with the ones closer to the center.

This action is done by finding the protruding corners on the front side, pushing them to the back side, and flattening them again, with some of their creases inverted. All the precreased triangles will be mountain folded and interlocked together, looking as hexagons on the front. On the back side, many small triangle twist folds will appear, made by grid segments.
(1) Here on the left is the precreased hexagon. Note the direction of the arrows, indicating where all the pleats will run. Start by collapsing the triangle in the middle. The model will not lie flat.

(2) Collapse one more triangle next to each corner of the central one, extending its pleats outward. The model will not lie flat.
(3) Collapse three more triangles, by pushing from behind, making their inner edges valley folded, and their outer edge mountain folded.

(4) Form three more triangles, as in step 2. The model will not lie flat, again.

(5) Collapse more triangles by pushing each valley folded pleat from behind, as in step 3. The model still does not lie flat.
(6) Collapse three more external triangles, as in steps 2 and 4.

(7) Continue, as with steps 3 and 5.

(8) Continue, as with steps 3, 5, 7.
(9) Continue, once more.

(10) Now you can flatten the paper along the raw edges.

(11) Flatten the paper along the raw edges, using the half-triangles forming the background borders. The model will be completely flat now.
(12) Adjust the background by pushing all its corners from the front to the back side, starting with the ones closer to the raw edges.

This action is done by finding the protruding corners on the front side, pushing them to the back side, and flattening them again, with some of their creases inverted. All the precreased triangles will be mountain folded and interlocked together, looking as hexagons on the front. On the back side, many small triangle twist folds will appear, made by grid segments.
1-Cube Exercise

Design: 18 May 2014
Paper: a 20 cm regular hexagon or bigger
Grid: 16ths

This exercise was designed as a simple example to practice the technique.
Part II: Exercises
(1) First, precrease one set of triangles for the background with valley folds on your grid, starting at the corners of the green rhombi.
(2) Then, mountain fold the edges of the rhombic tiles.

(3) & (4) Precrease two more sets of triangles for the background, paying attention not to add any new crease inside the rhombic tiles.
(1) Here on the left is the precreased hexagon, with green reference lines. Start by collapsing the three rhombi in the middle, by bringing their inner edges together, and letting their inner corners meet at the center. The model will not lie flat.

(2) Flatten the model by introducing pleat folds around the rhombi, following the grid lines only.
(3) Start forming the background, mountain folding one triangle next to the side of a rhombic tile. The model will not lie flat, again.

(4) Collapse the adjacent triangles in a similar fashion, aligning the pleats extending from the triangles. The model still does not lie flat.

(5) Repeat steps 3-4 all around, forming the first ring of background around the three rhombi. Most of the paper lies flat now.
(6) Repeat steps 3-4-5 completing a second ring of background around the cube. Most of the paper will lie flat.

(7) Flatten the paper along the raw edges, using the half-triangles forming the background borders. The model will be completely flat now.

(8) Adjust the background by pushing all its corners from the front to the back side, starting with the ones closer to the rhombi. See next page.
For the last step, note that not all the corners can be pushed from the front to the back side! In fact, the triangle edges “closer” to the rhombic tiles will remain valley folded, as you can see in the CP below, which shows only the background creases not lying on grid lines.
1-Star Exercise

Design: 27 Dec. 2015
Paper: a 20 cm regular hexagon or bigger
Grid: 16ths

This exercise was designed as a second example to practice the technique.
1-Star Precreasing

(1) First, precrease one set of triangles for the background with valley folds on your grid, starting at the corners of the green rhombi.

(2) Then, mountain fold the edges of the rhombic tiles.

(3) & (4) Precrease two more sets of triangles for the background, paying attention not to add any new crease inside the rhombic tiles.
(1) Here on the left is the precreased hexagon, with green reference lines. Start by collapsing the three rhombi in the middle, by letting their inner corners meet at the center. The model will not lie flat.

(2) Bring the inner corners of the three other rhombi to the center. The model still does not lie flat.
(3) Flatten the paper in between each pair of rhombi, by forming a valley folded triangle. The model still does not lie flat.

(4) Collapse the ring of triangles around the six rhombi, forming the background. Most of the paper lies flat now.

(5) Flatten the paper along the raw edges, using the half-triangles forming the background borders. The model will be completely flat now.
(6) Adjust the background by pushing all its corners from the front to the back side, starting with the ones closer to the rhombi.

For the last step, note that not all the corners can be pushed from the front to the back side! In fact, the triangle edges “closer” to the rhombic tiles will remain valley folded, as you can see in the CP at the left, which shows only the background creases not lying on grid lines.
Part III:
Design Process
As you probably noticed in the exercises of the previous chapter, the 3D-like figures were made by rhombi, and the rhombic tiles in the crease patterns were enclosed in green sections.

The shaded rhombus will form the visible block in the folded figure, and it can be placed in a unique position inside its tile, depending only on the orientation of the triangles forming the background. These rhombi are the essential elements on which this technique is based. By placing three rhombi close to each other, forming an hexagon, our perception tells us to see it as a cube, when in fact it is just a flat figure made by three rhombi! This is the trick on which all the 3D-like figures of this book are based on, and becomes particularly appealing when the figures we see as 3D cannot exist in our physical world, as with the famous Penrose Triangle and its many variations.
However, we do not need to count upon rhombic tiles only. We can also make triangles out of the same technique.

This one works in a very similar way as the previous rhombic tile. Note, however, that the tile, and the grey block contained in it, have the very same orientation as the triangles used for the background.

What if we want a triangle oriented the other way? We need a new tile!

This one works differently from the previous two. In fact, there are more fold lines, used to separate the triangular grey block from the adjacent tiles. Initially, I avoided using it, as you can see in the projects «42» and «JOAS». But later I introduced this tile to complete some of the more advanced projects. Keep in mind that if your figure has all the triangles oriented in the same direction, only one kind of triangle tile is needed.
Part III: Design Process

Designing New Figures

<1: Choosing the Design>

To design a new figure, you first need to draw it on a grid of triangles. Easy, isn’t it?

Note that the grid will be the same size you will need to fold for your new design, and that the tiles edges measure two “units” and not just one. Alternatively, you can draw 1-unit tiles on a simpler grid, and then use a folded grid with a double division for folding it.

<2: Choosing the Background Orientation>

Then, you need to choose the orientation of the triangles forming the background of your figure. At this step, one triangle will be added at each corner of the chosen tiles, and all these triangles must have the same orientation.
You have two possibilities for placing these triangles, and the placement of tiles blocks will depend on this choice. If your design is made by rhombi only, then the two are equivalent, as in the examples above. But if your design includes triangular tiles, one choice can be more convenient than the other!

If all the triangular tiles are oriented in the same direction, then you would choose to place the background triangles in the same way, in order to use the simpler triangular tile only (as in the pattern on the left). Otherwise, you will need to use the second, more complex triangular tile, at least for some triangles (as in the pattern on the right).
<3: Filling in Tiles>

Next, you need to fill in tiles in your pattern. This step is straightforward, as the position of blocks in each tile is given by the chosen background.

Note that, in the pattern on the left, there is the same distance between the shaded blocks of adjacent tiles, therefore only “regular” tiles will be needed (rhombic, and triangular oriented as the background). On the contrary, in the pattern on the right, some triangular shaded blocks seem to be too close to their neighbors: this is when the second kind of triangular tiles will be needed.

<4: Completing the Background>

The last step is also straightforward. You just need to add as many triangles as possible in order to complete the background, as in the following figure.
By assigning the correct orientation to each crease, you will obtain the complete crease pattern. I am not providing details for this step, but you can easily discover the underlying rules by folding the previous exercises and looking at the following examples.
Folding New Designs

1: Precreasing

Start by folding the grid on which your design is based. Then, valley fold the first set of triangles, starting at the corners of the green tiles.

These triangles are all aligned, and will cover the entire sheet of paper, unless you placed some tiles with odd distances among them.

Mountain fold the edges of the blocks inside each tile.
Valley fold a second set of triangles, paying attention not to add any new crease inside the tiles.

Valley fold a third set of triangles, paying attention not to add any new crease inside the tiles.

<2: Collapsing>

Collapse your model, using the tricks learnt by folding the exercises of the previous chapter.
<3: Fixing the Background>

Adjust the background by inverting some of its creases. Note that not all the corners can be pushed from the front to the back side! In fact, the triangle edges “closer” to the tiles will remain valley folded, as you can see in the following CP, which shows only the background creases not lying on grid lines.
Part IV:
Advanced Projects
Penrose Triangle #0

Design: 08 Oct. 2013
Paper: a 25 cm regular hexagon or bigger
Grid: 24ths

By removing the “hole” in the middle of the Penrose Triangle, the crease pattern fits in a grid in 24ths.
Part IV: Advanced Projects

Penrose Triangle #0
Penrose Triangle #2

Design: 06 Oct. 2013
Paper: a 30 cm regular hexagon or bigger
Grid: 32nds

The “hole” in the middle of this Penrose Triangle is two units wide. You can fold it smaller, or bigger by using a denser grid.
Design: 11 May 2014
Paper: a 30 cm regular hexagon or bigger
Grid: 32nds

Try extending this pattern on a denser grid!
Part IV: Advanced Projects
Part IV: Advanced Projects

Cubes II

Design: 11 May 2014
Paper: a 30 cm regular hexagon or bigger
Grid: 32nds

Try extending this pattern on a denser grid!
Menger Sponge level 1

Design: 27 Sep. 2014
Paper: a 30 cm regular hexagon or bigger
Grid: 32nds

By removing the cubes in the middle of each face from a 3x3x3 cube, you can get this Menger Sponge.
Rubik’s Cube

Design: 27 Sep. 2014
Paper: a 30 cm regular hexagon or bigger
Grid: 32nds

By restoring the missing cubes of the Menger Sponge you get a 3x3x3 Rubik’s cube. Try folding a 2x2x2 one, or bigger versions!
Design: 24 Jan. 2015
Paper: a 30 cm regular hexagon or bigger
Grid: 32nds

This pattern may resemble an “exploded” 2x2x2 cube. But the same small cubes may also lie all on a common plane!
Design: 26 Sep. 2014
Paper: a 35x50 cm rectangle
Grid: 32nds on the short side

The triangular portions of this figure were left empty. You need to use both kinds of triangular tiles if you want to fill them in. See «JOAS» for more possibilities.
Design: 24 Jan. 2015
Paper: a 40 cm regular hexagon or bigger
Grid: 48ths

This pattern may resemble an “exploded” 3x3x3 cube. But the same small cubes may also lie all on a common plane!
Design: 10 Jul. 2015
Paper: a 40 cm regular hexagon or bigger
Grid: 48ths

This pattern can be seen as an extension of Cubes II, but was created by removing the “holes” among the elements of Space #1.
Design: 10 Jul. 2015
Paper: a 40 cm regular hexagon or bigger
Grid: 48ths

This work introduces the possibility of including a frame into your designs.
Part IV: Advanced Projects
Part IV: Advanced Projects
Design: 30 Dec. 2015
Paper: a 40 cm regular hexagon or bigger
Grid: 48ths

By removing the “holes” from the Sierpinski-Penrose Triangle, the crease pattern fits in a grid in 48ths.
Part IV: Advanced Projects
Sierpinski-Penrose Triangle #2

Design: 27 Oct. 2013
Paper: a 50 cm regular hexagon or bigger
Grid: 64ths

The “holes” in this Sierpinski-Penrose Triangle are two units wide. You can fold them smaller, or bigger by using a denser grid.
Part IV: Advanced Projects

Sierpinski-Penrose Triangle #2
Design: 23 Jan. 2015
Paper: a 50 cm regular hexagon or bigger
Grid: 64ths

Alternating simple elements as the cube and star examples is not easy!
Part IV: Advanced Projects
Stars’n’Kubes II

Design: 23 Jan. 2015
Paper: a 50 cm regular hexagon or bigger
Grid: 64ths

By rotating the cubes with respect to the stars, you can get a different feeling.
Stars’n’Kubes II

Part IV: Advanced Projects
Space #1

Design: 14 Feb. 2015
Paper: a 50 cm regular hexagon or bigger
Grid: 64ths

The “holes” among the elements in this pattern are one unit wide. You can fold them smaller, or bigger by using a denser grid.
Design: 14 Feb. 2015
Paper: a 50 cm regular hexagon or bigger
Grid: 64ths

This was the first design in which I introduced the second kind of triangular tile, to avoid it looking unfinished.
Part IV: Advanced Projects
Paper: a 50 cm regular hexagon or bigger
Grid: 64ths

This design introduces a frame of a different kind. You can also change the bars interlocking order or the distance among them.
Part IV: Advanced Projects
Perceptions

Paper: a 50 cm regular hexagon or bigger
Grid: 64ths

This design can also be modified by changing the bars interlocking order or the distance among them.
Part IV: Advanced Projects

134 Perceptions
Part IV: Advanced Projects
Design: 30 Jun. 2014
Paper: a 35x100 cm rectangle
Grid: 32nds on the short side

The triangular portions of this figure were left empty. You need to use both kinds of triangular tiles if you want to fill them in.

You can fold any 3D writing or sequence of letters, digits and other symbols, by drawing each character made of small squares on a square grid, then using rhombi on a triangular grid instead of squares and choosing one of the eight following perspectives.
Menger Sponge level 2

Design: 29 Sep. 2014
Paper: a 70 cm regular hexagon or bigger
Grid: 96ths

The big “holes” of this Menger Sponge are left empty, to highlight the external surface.
Part IV: Advanced Projects
Menger Sponge level 2 II

Design: 30 Dec. 2015
Paper: a 70 cm regular hexagon or bigger
Grid: 96ths

Through the big “holes” of this Menger Sponge, part of the internal surface is visible.
Part IV: Advanced Projects